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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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INCREASING THE EFFICIENCY OF PASSENGER TRANSPORTATION IN CITY STREETS

By John A. Miller, Jr.,* Assoc. M. Am. Soc. C. E.

To Be Presented June 2, 1926

Synopsis

Many plans for the relief of traffic congestion are based on the general theory of providing more roadway area to be used in exactly the same way that the existing street area is used. Cutting through new thoroughfares, widening present roadways, arcading sidewalks, double-decking arterial routes, and placing the electric railways underground are among the proposals made. Enormous expense would attend the execution of any of these plans. While it cannot be denied that substantial benefits would result, it is extremely doubtful whether permanent relief from traffic congestion would be attained. Experience has shown in a number of instances that new streets cut through to relieve overcrowding on older streets soon become congested themselves, without any marked diminution of traffic on the other streets. It seems, therefore, that the problem might well be approached from a different angle. By stimulating the more intensive use of present streets, a far larger volume of traffic could be moved through them without the necessity for huge expenditures on improvements.

This paper shows that the present general practice of using roadway space for the storage of private automobiles is in reality neither a convenience to the automobilists nor a benefit to the merchant. The elimination of parking in congested districts would greatly increase the roadway available for moving vehicles. The use of attractive common carrier vehicles can be developed to handle a large part of the traffic now requiring single passenger or two-passenger vehicles. A material reduction in congestion will thereby be effected without excessive cost. The movement of all traffic will be facilitated to a degree that will be advantageous to the users of public and private conveyances.

Introduction

The congestion of traffic in the streets of the business districts of practically all large American cities has become acute. The problem thus created

Note.—This paper is issued before the date set for presentation and discussion. Correspondence is invited and may be sent by mail to the Secretary. Discussion on the paper will be closed in September, 1926, and, when finally closed, the paper, with discussion in full, will be published in *Transactions*.

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is threefold, involving the transportation of passengers and of merchandise, and the movement of pedestrians. Particular interest attaches to the subject of passenger transportation, however, because it affects directly the comfort and convenience of a larger number of people than either of the other phases of the problem.

Despite the present serious traffic congestion the actual capacity of existing streets for the transportation of passengers has by no means been reached. The volume of traffic that can be moved over a particular route in a given time depends on the effective size of the existing roadway and the efficiency with which the available space is utilized. Broadly speaking, both can be increased with comparative ease. To accomplish this, all of the roadway of important arteries should be made available for moving vehicles and should be put to the most intensive possible use. In other words, the most efficient vehicles should be used in the most efficient way.

Increasing the Effective Width of Streets

By the elimination of automobile parking, the effective width of existing streets can be increased for moving traffic from 30 to 200 per cent. A recent transportation survey of Washington, D. C., disclosed that 29% of the roadway area in congested districts was reserved for automobile parking. In other cities, where the streets are narrower, an even larger proportion of the roadway is used for parking. Often parking is permitted on both sides of streets having a width of 60 ft. or less between building lines. Deducting two 12-ft. sidewalks, about 36 ft. of roadway is left. A row of parked automobiles will occupy an 8-ft. strip on each side. Thus, there remains for moving vehicles only a 20-ft. roadway, capable of accommodating one stream of traffic in each direction. For all practical purposes, 50% of the street is being used for parking. On one-way streets it happens not infrequently that there is room for only a single stream of moving traffic between the lines of parked vehicles. Under such conditions, the area devoted to parking would approximate two-thirds of the total.

The use of the public streets for the storage of private automobiles is tolerated largely because of an exaggerated idea of their importance in the general transportation plan. Traffic counts made in various large cities, however, show that public transportation vehicles are carrying upward of 75% of the total traffic, although they constitute only about 15% of the total number of vehicles. A check made of 13 leading stores in New York during a busy shopping hour showed that of 35 252 customers only 813 came by private automobile or taxicab. The others came by street car, subway, or bus. To the users of public transportation agencies, who thus greatly outnumber the users of private conveyances, relief from congestion by the elimination of parking would be a great boon. For example, one of the effects of the passage of a no-parking regulation in the down-town section of Pittsburgh, Pa., was to reduce by several minutes the running time of street cars through this area. The average savings are shown in Table 1.

Opposition to an absolute ban on parking on a particular street, however, is likely to be made by retail merchants whose stores are located there. Usually they contend that their business will be injured unless an opportunity is given for customers to park their automobiles in the street in front of the stores. A preferable plan, it is asserted, would be to fix a time limit of 1 hour for parking.

TABLE 1.—TIME SAVED IN OPERATION OF STREET CARS, PITTSBURGH, PA., BY PARKING ELIMINATION.

Loop Number.	Number of cars during maximum hour.	Average decrease in time, in minutes and seconds.
1	26	1:01
2	84	2:47
3	81	3:51
4	27	5:21
5	57	2:55
6	28	0:25
7	20	0:56
8	52	0:35
9	159	8:17

Such an idea is a fallacy. With parking parallel to a curb, a frontage of 100 ft. would accommodate 5 or possibly 6 automobiles at a time. If these 6 automobiles were replaced by another 6 each hour throughout the business day, a total of 48 automobiles would have been accommodated. Probably they would not carry more than 1 actual customer per vehicle, and the net result of 1-hour parking would be to accommodate about 50 customers during the day. For a retail store with a frontage of 100 ft. on an important street this number of customers would be a comparatively unimportant part of the daily business. Analysis thus confirms the census taken in the New York stores, already mentioned, showing that the number of customers arriving by automobile was only a small fraction of the total.

Moreover, from the point of view of general traffic congestion, 1-hour parking is just as detrimental as unlimited parking, perhaps more so. Limited privileges of this kind are far more difficult of enforcement than an absolute ban. The capacity of a street for moving vehicles suffers a continuous reduction on account of the limited parking and there is also the added confusion due to automobiles pulling into and out from the curb line.

ELIMINATION OF PARKING WILL AID BUSINESS

Contrary to a prevalent opinion, prohibition of parking in the congested districts is likely to aid business rather than injure it. After the enactment of the no-parking ordinances in Pittsburgh, already mentioned, retail trade in the down-town district showed an increase in comparison with that of the outlying commercial centers.

Experience of Pittsburgh.—Comparisons of the buying power of the Pittsburgh district in 1924 and 1925 as shown by bank payrolls, the total retail sales, and the down-town retail sales, as represented by seven large department stores, are given in Table 2 by 3-month periods.

Industrial depression caused the slight decrease in all these figures for 1925 as compared with 1924. It is significant, however, that the retail sales for the entire district showed a decrease of 2.80% for the year, whereas the down-town sales decreased only 0.8 per cent. This indication that the down-town business suffered less than that in the outlying sections appears to refute completely the claim that parking restrictions injure retail trade.

TABLE 2.—CHANGE IN PITTSBURGH BUSINESS CONDITIONS IN 1925 BASED ON 1924.

Three months ending	Percentage change in buying power.	Percentage change in district retail sales.	Percentage change in down-town retail sales.
March 31, 1925	−9.01	−1.87	−2.53
June 30, 1925	−1.77	−5.90	−2.37
September 30, 1925	−3.67	−6.20	−2.77
December 31, 1925	−8.13	+1.50	+3.47
Total for year	−5.72	−2.80	−0.80

Effects of Congestion.—In a large number of cases analysis would undoubtedly show that the volume of trade in down-town stores has not kept pace in recent years with the development of a community. The 1924 population of a certain district, including a city of about 500 000 people, with perhaps 250 000 more in the adjacent suburban territory, represented nearly 130% of the 1914 figure (Fig. 1). Meanwhile, the wages received by the average individual had increased to approximately 205% of those received in 1914. Thus, the total purchasing power of the district in 1924 was 266.5% of that before the World War.

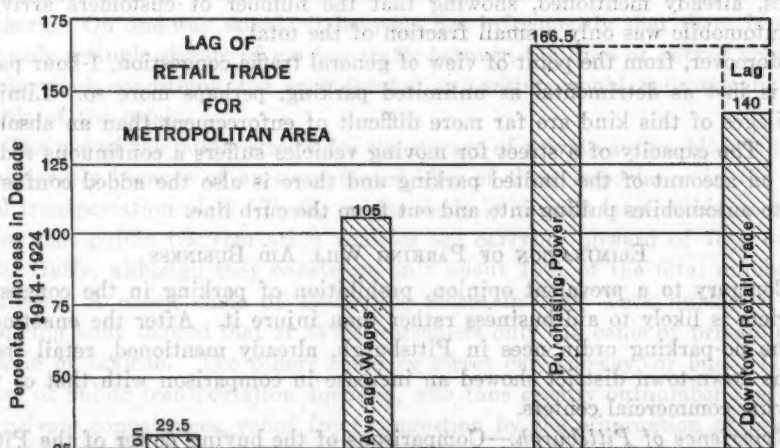

FIG. 1.

Investigation of the volume of business of a number of representative down-town retail stores in this particular city showed that, in 1924, it was only 240% of that 10 years earlier. Thus, it will be seen that the growth of retail trade in the down-town section lagged considerably behind the increase in the purchasing power (Fig. 1). On the other hand, in the case of one particular high-grade store, about 3 miles from the business center, the present volume of business is approximately 480% of that in 1914.

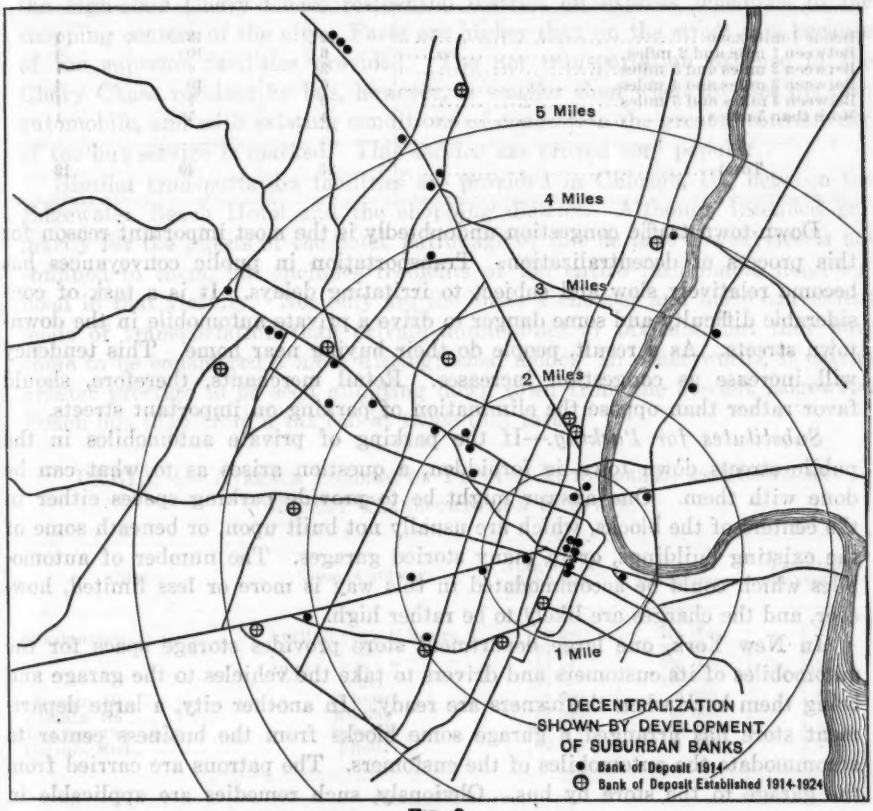

FIG. 2.

Banks as an Index of Business.—A realization of the remarkable increase that has occurred in the outlying commercial centers during the decade may be had from a study of the locations of banks of deposit (Fig. 2). In 1924, the number of such banks in the entire area was 49 as compared with 37 in 1914. During these 10 years, however, only one new bank was opened in the center of the city; all others were in the outlying districts. The zone from 1 mile to 2 miles from the center showed an increase of 4, as did also the zone from 3 to 4 miles. A summary of this is given in Table 3.

Figures of this kind do not show with mathematical accuracy the comparative growth of business in the city center and suburban areas. Nevertheless, a bank of deposit is rarely opened in a purely residential section, and

its presence may be taken to indicate fairly the existence of a business community. The surprising increase that has occurred in the outlying sections illustrates clearly how business has sprung up during the past ten years.

TABLE 3.—SUMMARY OF BUSINESS DEVELOPMENT AS INDICATED BY THE LOCATION OF BANKS OF DEPOSIT.

	1914.	1924.	Increase.
Inside 1-mile zone	12	13	1
Between 1 mile and 2 miles	6	10	4
Between 2 miles and 3 miles	3	4	1
Between 3 miles and 4 miles	6	10	4
Between 4 miles and 5 miles	6	7	1
More than 5 miles	4	5	1
Total	37	49	12

Down-town traffic congestion undoubtedly is the most important reason for this process of decentralization. Transportation in public conveyances has become relatively slow and subject to irritating delays. It is a task of considerable difficulty and some danger to drive a private automobile in the down-town streets. As a result, people do their buying near home. This tendency will increase as congestion increases. Retail merchants, therefore, should favor rather than oppose the elimination of parking on important streets.

Substitutes for Parking.—If the parking of private automobiles in the public streets down town is forbidden, a question arises as to what can be done with them. One answer might be to provide parking spaces either in the centers of the blocks, which are usually not built upon, or beneath some of the existing buildings, or in many storied garages. The number of automobiles which could be accommodated in this way is more or less limited, however, and the charges are likely to be rather high.

In New York, one large department store provides storage space for the automobiles of its customers and drivers to take the vehicles to the garage and bring them back when the owners are ready. In another city, a large department store has arranged a garage some blocks from the business center to accommodate the automobiles of the customers. The patrons are carried from the garage to the store by bus. Obviously, such remedies are applicable in only a few cases, as the smaller stores could not afford them.

The establishment of parking spaces on the outskirts of the city and transportation of passengers down town by public conveyance is a solution that is being tried in a number of cities. Several such parking spaces are operated by the Philadelphia Rapid Transit Company which carries the automobilists from the outlying sections into the business district by rail. A number of other electric railway companies have adopted similar methods, making the charge for parking to include transportation back and forth by street car. An arrangement of this kind not only solves the problem of where to store the private automobiles without wasting valuable roadway area, but it also promotes the efficiency of passenger transportation in the congested district by substituting one large vehicle for many small ones.

Economizing Roadway Space by Use of Large Capacity Vehicles

Advantage of Buses.—The substitution of public transportation vehicles of fairly large carrying capacity in place of individual automobiles holding on the average not more than two persons can be made advantageously in many instances. For example, *de luxe* bus service often will prove an entirely acceptable substitute for the use of numerous private automobiles. In Washington, D. C., the Capital Traction Company operates luxurious buses from the high-class Chevy Chase residential district on express schedules to the shopping centers of the city. Fares are higher than on the street cars because of the superior facilities provided. The net transportation expense to the Chevy Chase resident by bus, however, is smaller than when he uses his own automobile, and with existing conditions of congestion the greater convenience of the bus service is marked. This service has proved very popular.

Similar transportation facilities are provided in Chicago, Ill., between the Edgewater Beach Hotel and the shopping district. Although intended primarily for the guests of the hotel, patronage of this *de luxe* bus service is not confined to them, but includes residents of the entire fashionable neighborhood in that vicinity. Not only are the buses a cheaper and more convenient mode of transportation than private automobiles or taxicabs, but they have come to be considered a more distinguished mode. In other words, it carries greater prestige to be seen alighting down town from the 35-cent Edgewater Beach bus than from a taxicab at twice that fare.

TABLE 4.—Average Loading of Private Automobiles as Shown by Recent Transportation Surveys.

City.	Time of survey.	Number of vehicles.	Total passengers.	Passengers per vehicle.*
Washington, D. C.....	April 30 (in) 15 hours	100 284	171 051	1.71
"	" (out) " "	92 666	154 394	1.67
	June (in) " "	114 047	223 000	1.96
"	" (out) " "	112 957	218 988	1.94
Chicago, Ill.....	1 hour	2 600	4 700	1.81
Atlanta, Ga	All day	27 540	49 600	1.8
Los Angeles. Calif	11 hours	261 974	393 322	1.50
Detroit, Mich.....	1 hour	12 180	21 200	1.74

* Includes driver.

More general use of such collective transportation vehicles would go far toward increasing the efficiency with which the available street space is used. Traffic counts made in various cities show that private automobiles carry an average of less than 2 passengers per vehicle (see Table 4). This is equivalent to about 30 sq. ft. or more of street area per passenger. A bus of ordinary size can comfortably accommodate at least 20 persons. Assuming that it would average only three-quarters of a full load (15 people), about 16 sq. ft. of street area would be required per passenger. Inasmuch as the speed of the "limited stop" bus and the private automobile would be approximately the same in the congested district, the use of the bus to replace the automobile would double the efficiency with which the existing street area is utilized.

Theoretically, an even greater increase in efficiency might be attained by the substitution of the electric rail car, requiring only 3.5 sq. ft. of street space per passenger carried, for the private automobile. In actual practice, however, it is found that the character of the service rendered by these two modes of transportation is so different that it is impracticable to try to replace the automobile by the trolley. Well-operated bus service with commodious vehicles, on the other hand, can be made an entirely acceptable substitute for the use of private automobiles.

Taxicabs Inefficient.—Most wasteful of all vehicles used in passenger transportation is the taxicab. Investigations made by the writer at various locations in New York indicate that the average load is only 0.83 passenger. Approximately half the taxicabs were cruising without passengers; of those in use, however, many carried three and four passengers. As this vehicle occupies about 64 sq. ft. of street area, the space required per passenger carried is about 80 sq. ft. When it is realized that there are more than 17 000 taxicabs in New York, and correspondingly large numbers in other cities, the importance of this cause of congestion is evident.

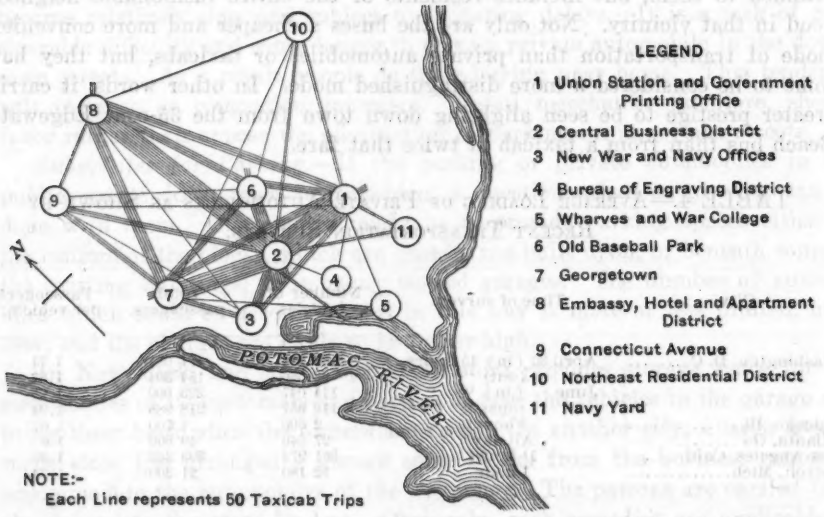

FIG. 3.—TAXICAB MOVEMENTS DURING A TYPICAL BUSINESS DAY IN WASHINGTON, D. C.

Channels of Taxicab Traffic.—Surprisingly enough, the greater part of the taxicab movements follow definite routes. In the transportation survey of Washington, D. C., previously mentioned, movements of 334 taxicabs operated by four taxicab companies were analyzed for 1 week day. The manifests made out by the drivers were abstracted to secure for each cab the origin and destination of each trip, the time at which the trip was commenced, and the time the trip was completed. These data were then analyzed as to the point of origin, point of destination, and time of day.

During this typical day, 7 924 taxicab movements were classified. Of this number about 5 000 movements were made along only 33 main routes. The three heaviest routes were between the Union Station and the business district,

600 trips, between the Mt. Pleasant residential district and the business district, 400 trips, and between Georgetown and the business district, 350 trips. Fig. 3 shows the number of taxicab movements on each of the 33 routes. On no other route in the entire city were there as many as 50 taxi movements during that day.

These figures indicate that in Washington about five-eighths of the taxicab movements are along certain definite routes. Substantially the same condition no doubt prevails in other cities. Why not replace these taxicab movements along definite routes by bus service requiring 16 ft. of street area per passenger carried instead of 80 sq. ft.?

Obviously, it would not be feasible to try to replace entirely the taxicab by the bus, any more than to try to replace altogether the private automobile. It is clear, however, that so much of both of these kinds of traffic flow along a comparatively few main routes, that a real opportunity exists to effect an important improvement in passenger transportation in congested districts by thus using one vehicle of high carrying efficiency to perform the work now being done by numerous smaller vehicles of low carrying efficiency.

Conclusion

The foregoing considerations point to a possible solution of street traffic problems by (1) the reduction or elimination of parking; and (2) the substitution of buses in many instances in place of the extravagant private automobile, and the taxicab. Such a solution would go far toward increasing the efficiency of passenger transportation in city streets without injury to the mercantile interests. It would also possess the advantages of small cost and speedy relief of present congestion.

ANALYSIS OF CONTINUOUS CONCRETE ARCH SYSTEMS

By C. S. Whitney,* M. Am. Soc. C. E.

Synopsis

A number of methods varying greatly in the labor involved and in degree of accuracy have been proposed for the analysis of concrete arches that form a series supported by elastic piers. In general, the more accurate methods are quite laborious and the approximate methods are quite easily applied. Little information is available as to the limits within which suitably accurate results may be obtained by the approximate methods, and it is hoped that this paper with its discussion will throw more light on this subject.

The writer presents herein a simple method, which he believes to be new, for analyzing a system composed of two arch spans with an elastic pier. This is an application of a general method for analyzing symmetrical statically indeterminate structures called the "Verfahren der Belastungsumordnung", several interesting examples of which are given by W. L. Andrée.† It consists in replacing a single eccentric load by two pairs of symmetrically placed loads which have the single load for their resultant. The analysis of the structure under this new loading is considerably simpler because of the symmetry. Advantage is also taken of the treatment of single arch ribs given in the writer's paper, "Design of Symmetrical Concrete Arches",‡ with which the reader is assumed to be familiar, and tables of influence-line ordinates are given which make the solution easy. By this method it is possible to study readily the effect on the adjacent spans of the elasticity of one pier and to determine the accuracy of approximate methods for different cases.

In addition, an easily applied approximate method is given for the analysis of a system of any number of spans, which the writer believes to be accurate enough for the majority of cases occurring in practice.

Analysis of Two-Span System

The following analysis shows the effect of the elasticity of a pier and the yielding of its foundation on the stresses in the adjacent arch ribs. The arch ribs are assumed to be similar and symmetrical. In order to simplify the equations, the outer ends of the arch ribs are assumed to be fixed but, if desired, the effects of elastic supports can readily be considered. The inner ends of the arch ribs are rigidly attached to the top of an elastic pier, the base of which is fixed against horizontal and vertical displacement, but permitted to rotate slightly by the elasticity of the supporting foundation. The

Note.—Written discussion on this paper will be closed in September, 1926. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

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† "Das B-U. Verfahren," by W. L. Andrée, R. Oldenbourg, Berlin, Germany.

‡ *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

effect of shortening of the pier due to vertical loading is not considered, as it is negligible in all cases. A bending moment in an arch rib is considered to be positive when it causes compression at the top and, in the pier, when it compresses the right-hand side. A horizontal thrust is considered to be positive when it tends to shorten an arch rib or to cause a displacement of the base of the pier toward the left.

A single concentrated vertical load placed on one of the ribs of a two-span system, as shown in Fig. 1, will produce a horizontal and vertical thrust and a moment at each of the two abutments and at the base of the pier, or a total of nine reactions the size of which cannot be determined by the three statical equations. The analysis can be greatly simplified by substituting for the single load two pairs of symmetrically placed loads which are equal to the single load. This substitute loading is shown in Fig. 1. If the loading of Case II be superimposed on that of Case I, the loads on the right-hand span are neutralized, and the sum of those on the left is equal to P. The determination of the reactions for the two cases taken separately is quite simple, and, if combined, they will give the reactions for the single load, P.

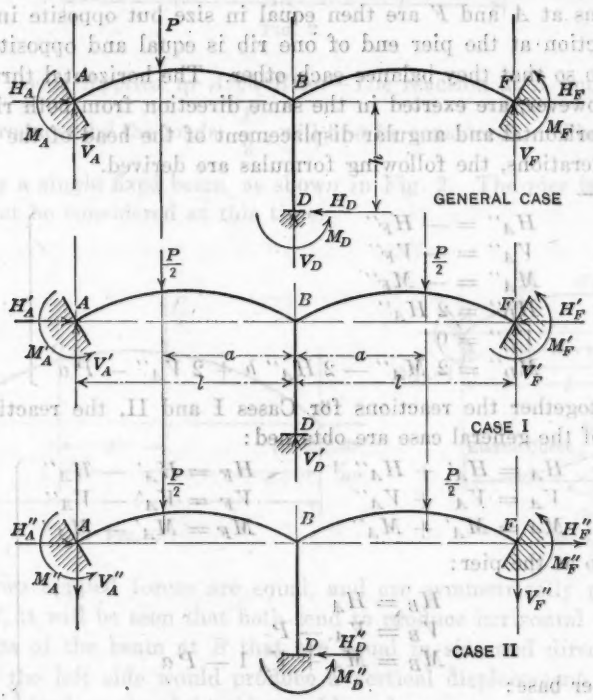


FIG. 1.

In Case I, the loads, $\frac{P}{2}$, both act downward and are equidistant from the center line of the pier. From symmetry, the reactions at A and F are the same. If the shortening of the pier due to direct compression is neglected, there is no displacement, horizontal, vertical, or rotational, of the head of the pier. The symmetrically placed loads produce at the springing line of

the two arch ribs next to the pier equal moments and thrusts, which exactly balance, the resultant being a vertical thrust down the axis of the pier. The reactions at the end of each rib are then the same as if the ribs were supported separately by rigid abutments. The reaction for Case I can be found, therefore, by considering the rib, *A-B*, to have fixed ends, and the following equations can be written.

Case I.—

$$\left.\begin{aligned} H_A' &= H_F' \\ V_A' &= V_F' \\ M_A' &= M_F' \\ H_D' &= 0 \\ V_D' &= 1 - 2\,V_A' \\ M_D' &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The solution of Case I can be effected by the methods given in the writer's paper, "Design of Symmetrical Concrete Arches", referred to previously.

For Case II, the loads are symmetrically placed but oppositely directed. The reactions at *A* and *F* are then equal in size but opposite in sign. The vertical reaction at the pier end of one rib is equal and opposite to that of the other rib so that they balance each other. The horizontal thrusts and the moments, however, are exerted in the same direction from both ribs and they produce a horizontal and angular displacement of the head of the pier. From these considerations, the following formulas are derived.

Case II.—

$$\left.\begin{aligned} H_A'' &= -\,H_F'' \\ V_A'' &= -\,V_F'' \\ M_A'' &= -\,M_F'' \\ H_D'' &= 2\,H_A'' \\ V_D'' &= 0 \\ M_D'' &= 2\,M_A'' - 2\,H_A''\,h + 2\,V_A'' - P\,a \end{aligned}\right\} \quad \ldots\ldots\ldots (2)$$

By adding together the reactions for Cases I and II, the reactions for the single load of the general case are obtained:

$$\begin{aligned} H_A &= H_A' + H_A'' & H_F &= H_A' - H_A'' \\ V_A &= V_A' + V_A'' & V_F &= V_A' - V_A'' \\ M_A &= M_A' + M_A'' & M_F &= M_A' - M_A'' \end{aligned}$$

In the rib at the pier:

$$\left.\begin{aligned} H_B &= H_A \\ V_B &= V_A - P \\ M_B &= M_A + V_A\,1 - P\,a \end{aligned}\right\} \quad \ldots\ldots\ldots (3)$$

At the pier base:

$$\left.\begin{aligned} H_D &= H_D'' \\ V_D &= V_D' \\ M_D &= M_D'' \end{aligned}\right\}$$

At the pier top:

$$\left.\begin{aligned} H_E &= H_D'' \\ V_E &= V_D' \\ M_E &= M_D'' - H_D''\,h \end{aligned}\right\}$$

Method of Analysis for Case II

Case II will be analyzed by considering first the pier base to be cut away from the foundation, leaving the two arch ribs acting as a curved beam fixed between the end abutments, A and F, with the pier as a cantilever projecting down from the center of the beam. Aside from the reactions at A and F, the forces acting on this beam are the couple formed by the two loads, $\frac{P}{2}$, and the two reactions at the base of the pier, M_D'', and H_D'', which will be applied as loads. The effects of these forces will be considered separately and the influence lines for M_D'' and H_D'' obtained.

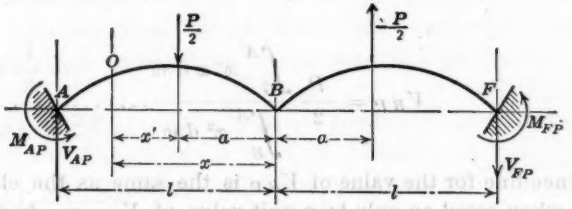


Fig. 2.

Effect of Loads Applied to Arch Ribs.—The reaction at A and F and the shear at B produced by the loads, $\frac{P}{2}$, will first be considered. The two spans are acting as a single fixed beam, as shown in Fig. 2. The pier is not shown, as it need not be considered at this time.

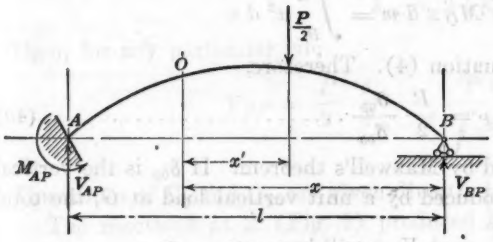


Fig. 3.

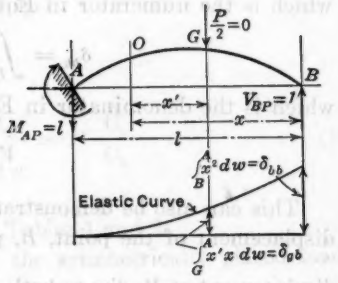


Fig. 4.

As the two applied forces are equal, and are symmetrically placed about the point, B, it will be seen that both tend to produce horizontal and angular displacements of the beam at B that are equal in size and direction. Also, the load on the left side would produce a vertical displacement of B downward, and the load on the right side would produce an equal vertical displacement upward. The resultant vertical displacement of B with both loads acting is, therefore, zero. Because of the symmetry, it will be sufficient to take only one rib from A to B and consider that the end, A, is entirely fixed while the end, B, is free to move horizontally and to rotate, but is fixed vertically as shown in Fig. 3.

As the vertical displacement at B is zero,

$$\int_B^A M_O\, x \frac{d\,s}{E\,I} = \int_B^A M_O\, x\, d\,w = 0$$

in which, $d\,w = \frac{d\,s}{E\,I}$ and M_O is the moment at any point, O, or,

$$M_O = V_{BP}\, x - \frac{P\, x'}{2}$$

Then,

$$\int_B^A M_O\, x\, d\,w = V_{BP} \int_B^A x^2\, d\,w - \frac{P}{2} \int_G^A x'\, x\, d\,w = 0$$

from which,

$$V_{BP} = \frac{P}{2} \frac{\int_G^A x'\, x\, d\,w}{\int_B^A x^2\, d\,w} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The influence line for the value of V_{BP} is the same as the elastic line for the rib, $A\,B$, when acted on only by a unit value of V_{BP} as shown in Fig. 4.

This is shown as follows: In Fig. 4, the vertical displacement δ_{gb} at the load point, G, due to a unit vertical force at B is, $\int_G^A M_O\, x'\, d\,w$, and as M_O in this case is equal to $V_{BP}\, x = x$,

$$\delta_{gb} = \int_G^A M_O\, x'\, d\,w = \int_G^A x'\, x\, d\,w$$

which is the numerator in Equation (4). Also, the displacement, δ_{bb}, at B is,

$$\delta_{bb} = \int_B^A M_O\, x\, d\,w = \int_B^A x^2\, d\,w$$

which is the denominator in Equation (4). Therefore,

$$V_{BP} = \frac{P}{2} \frac{\delta_{gb}}{\delta_{bb}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

This can also be demonstrated by Maxwell's theorem. If δ_{bg} is the vertical displacement of the point, B, produced by a unit vertical load at G, the total displacement at B, due to both $\frac{P}{2}$ and V_{BP} will be,

$$\frac{P}{2}\, \delta_{bg} - V_{BP}\, \delta_{bb} = 0$$

or,

$$V_{BP} = \frac{P}{2} \frac{\delta_{bg}}{\delta_{bb}}$$

As the displacement, δ_{bg}, at B produced by a unit load at G is equal to the displacement, δ_{gb}, produced by a unit load at B,

$$V_{BP} = \frac{P}{2} \frac{\delta_{gb}}{\delta_{bb}}$$

The values of δ_{gb} and δ_{bb} can be determined by graphical or numerical integration or, using the ribs as established in the writer's paper previously mentioned, they may be derived by direct integration. Equation (54),* given therein, is,

$$d\,w = \frac{l_1}{E\,I_c}[1 - (1 - m)\,z]\,d\,z$$

After integrating,

$$\int_B^A x^2\,d\,w = \delta_{bb} = \frac{l^3\,(7 + 9\,m)}{48\,E\,I_c}$$

in which,

$$l_1 = \frac{l}{2}.$$

$$z = 1 - \frac{2\,x}{l}.$$

$$m = \frac{I_c}{I_s \cos \phi_s}.$$

I_c = moment of inertia of rib at crown.

I_s = moment of inertia of rib at springing line.

$\cos \phi_s$ = cosine of angle between the horizontal chord and the tangent to the rib axis at the springing line.

This value of δ_{bb} can be taken from Table 1. The equation for the elastic curve of Fig. 4 has also been obtained by direct integration and, from this, Table 2 has been computed, giving the value of δ_{gb}:

$$\delta_{gb} = 2\,l\,\delta_3$$

Then, for any particular rib,

$$V_{BP} = \frac{P}{2}\frac{\delta_{gb}}{\delta_{bb}} = \frac{P\,l\,\delta_3}{\int x^2\,d\,w} = P\,\frac{C_2}{C_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The values of C_2 and C_1, are given directly by Tables 1 and 2.

The reactions at A (Fig. 2) produced by the symmetrically placed loads are then as follows, when $P = 1$,

$$\left.\begin{aligned} M_{AP} &= V_{BP}\,l - \frac{P\,(l - a)}{2} = V_{BP}\,l - \frac{l - a}{2} \\ H_{AP} &= 0 \\ V_{AP} &= \frac{P}{2} - V_{BP} = \frac{1}{2} - V_{BP} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (6)$$

Effect of Reactions at Base of Pier.—The effect of a moment, M_D, applied at the base of the pier after it has been cut free from its foundation, will be considered.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 983.

As the two arch ribs are equally stiff, this moment will be divided equally between the two ribs. The two ribs may be considered as if cut apart with the moment, $\frac{M_D}{2}$, applied to the end of each rib at B. The right end of the left-hand rib will be displaced horizontally to the left and, from symmetry, the left end of the right-hand rib will be displaced horizontally to the left an equal amount. When the ends are joined, there is, therefore, no horizontal restraint at B and hence no horizontal thrust is produced by the moment, M_D. There is, however, a tendency for the end of the left-hand rib to move down and that of the right-hand rib to move up an equal amount. As they are attached and are symmetrical about the pier axis, the ribs will restrain each other so that no vertical displacement of B occurs and a vertical shear, V_{BM}, will be produced.

The rib, $A\ B$, will be considered separately (see Fig. 6) and the reactions, M_{AM} and V_{AM}, at A produced by M_D will be determined. At A the rib is entirely fixed and at B restrained vertically, although free to move horizontally and to rotate. Therefore, the vertical displacement at B is,

$$\int_B^A M_O\, x\, d\, w = 0$$

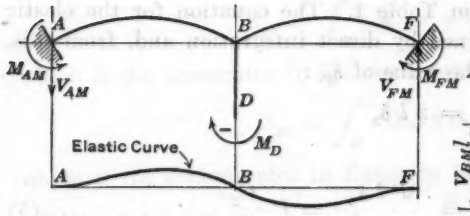


FIG. 5.

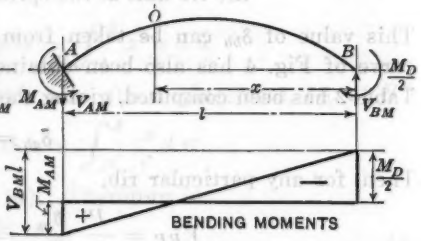


FIG. 6.

but,

$$M_O = \frac{M_D}{2} + V_{BM}\, x$$

so,

$$\int_B^A M_O\, x\, d\, w = \frac{M_D}{2} \int_B^A x\, d\, w + V_{BM} \int_B^A x^2\, d\, w$$

Then,

$$V_{BM} = -\frac{M_D}{2} \frac{\int_B^A x\, d\, w}{\int_B^A x^2\, d\, w} = -\frac{M_D}{2}\, \beta$$

in which,

$$\frac{\int_B^A x \, d\, w}{\int_B^A x^2 \, d\, w} = \beta$$

Then, when $M_D = -1$,

$$\left.\begin{aligned} V_{AM} &= -V_{BM} = -\frac{\beta}{2} \\ M_{AM} &= \frac{M_D}{2} + V_{BM}\, l = \frac{\beta\, l - 1}{2} \\ M_{OM} &= \frac{\beta\, x - 1}{2} \\ M_{BM} &= \frac{M_D}{2} = -\frac{1}{2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (7)$$

Considering the elasticity of the ribs and the pier, the angular displacement at D due to $M_D = -1$ will be,

$$\phi_{DM} = \int_D^A M \, d\, w = \frac{1}{2}\int_B^A (\beta\, x - 1) \, d\, w - \int_D^B d\, w$$

In order to take into consideration also the elasticity of the foundation, the reactions at D will be thought of as acting, not directly on the end of the pier, but on a rigid member attached to the end of the pier by a hinge that has the same elastic properties as those assumed for the foundation. It will be assumed further that a unit value of M_D causes this hinge to rotate through a small angle which will be called ε. The angle due to the yielding of the foundations will be $M_D\, \varepsilon$ and the expression for the total angular displacement produced by $M_D = -1$ at its point of application then becomes,

$$\phi_{DM} = \frac{1}{2}\int_B^A (\beta\, x - 1) \, d\, w - \int_D^B d\, w - \varepsilon \ldots\ldots\ldots\ldots (8)$$

Effect of Horizontal Thrust at D.—In so far as the effect on the arch ribs is concerned, the horizontal thrust acting at D (Fig. 7(a)) may be replaced by a thrust, H_D, acting through the elastic centers of gravity of the two ribs together with a moment at D equal to $-H_D\,(h + b)$. The resultant of this thrust and moment is the thrust, H_D, acting at D.

The thrust acting through the elastic centers of gravity of the two ribs produces only a horizontal displacement at B with no vertical or angular displacement. Because of symmetry, this thrust is equally divided between the two ribs, producing a tension in the right-hand rib and a compression in the left-hand rib.

Then,

$$H_{AH} = \frac{H_D}{2} \text{ and } M_{OH} = \frac{H_D\, y}{2}$$

This thrust produces moments at A and B equal to $\frac{H_D\, b}{2}$. The effect of

a moment at the pier base, as just determined, is given by Equation (7). From this, the combined effect of thrust and moment when $H_D = 1$ will be as follows,

$$\left.\begin{aligned} V_{AH} &= -V_{BH} = -\frac{\beta}{2}(h+b) \\ M_{AH} &= \frac{b}{2} + (h+b)\frac{\beta l - 1}{2} = \frac{(h+b)\beta l}{2} - \frac{h}{2} \\ M_{BH} &= \frac{b}{2} - \frac{h+b}{2} = -\frac{h}{2} \\ M_{OH} &= \frac{y}{2} + (h+b)\frac{\beta x - 1}{2} = \frac{(h+b)\beta x}{2} - \frac{h+b+y}{2} \\ H_{AH} &= \frac{1}{2} \end{aligned}\right\} \quad \ldots\ldots(9)$$

These equations can also be derived by considering the rib, $A\ B$, as fixed at A and restrained vertically at B as in Fig. 8. Placing the vertical displacement equal to zero,

$$\int_B^A M_O\, x\, d\, w = 0$$

in which,

$$M_O = V_{BH}\, x - \frac{H_D}{2}(b+y) - \frac{H_D\, h}{2} = V_{BH}\, x - \frac{H_D}{2}(b+y+h)$$

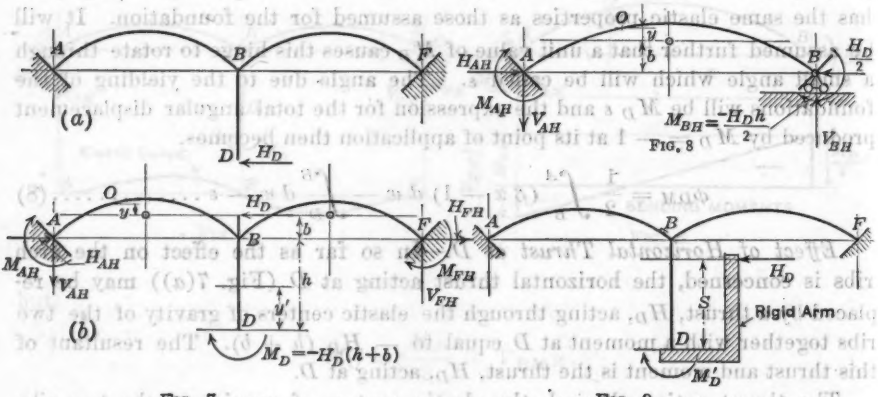


FIG. 7.

FIG. 8

FIG. 9.

Then,

$$\int_B^A M_O\, x\, d\, w = V_{BH}\int_B^A x^2\, d\, w - \frac{H_D}{2}(h+b)\int_B^A x\, d\, w - \frac{H_D}{2}\int_B^A x\, y\, d\, w = 0$$

As the axis of ordinates, y, was chosen through the elastic center of gravity of the rib, $\int x\, y\, d\, w$ and $\int y\, d\, w$ are zero. Hence,

$$V_{BH}\int_B^A x^2\, d\, w - \frac{H_D}{2}(h+b)\int_B^A x\, d\, w = 0$$

and,

$$V_{BH} = \frac{H_D}{2}(h + b)\frac{\int_B^A x\, d\, w}{\int_B^A x^2\, d\, w} = \frac{H_D\, \beta}{2}(h + b)$$

from which Equations (9) follow.

The angular displacement, ϕ_{DH}, at the foot of the pier, D, produced by a unit value of H will be,

$$\phi_{DH} = \int_D^A M_O\, d\, w = \frac{1}{2}\int_B^A [(h + b)\, \beta\, x - (h + b + y)] - \int_D^B y'\, d\, w$$

or,

$$\phi_{DH} = \frac{h + b}{2}\int_B^A (\beta\, x - 1)\, d\, w - \int_D^B y'\, d\, w \ldots\ldots\ldots\ldots (10)$$

The angular displacement at D produced by unit values of thrust and moment at D have now been determined, and an equation can be written for the total angular displacement from the two combined:

$$\phi_D = H_D\, \phi_{DH} + M_D\, \phi_{DM}$$

from which,

$$M_D = \frac{\phi_D - H_D\, \phi_{DH}}{\phi_{DM}}$$

If this angular displacement is equal to zero,

$$M_D = -H_D\frac{\phi_{DH}}{\phi_{DM}}$$

When $H_D = 1$,

$$M_D = -\frac{\phi_{DH}}{\phi_{DM}}.$$

This means that, if H_D acts on a rigid member connected to the base of the pier and is placed so that it produces at the base of the pier a moment equal to $-H_D\frac{\phi_{DH}}{\phi_{DM}}$, it will cause no rotation of this member. This condition will be satisfied when, in Fig. 9,

$$s = \frac{\phi_{DH}}{\phi_{DM}}.$$

The total moment M_D at D will then be,

$$M_D = M'_D - H\, s$$

As H_D thus placed produces no angular displacement at D, by Maxwell's theorem of reciprocal displacements, M'_D will produce no horizontal displacement of the point of application of H_D. These two forces, therefore, are independent of each other and can be considered separately.

Substituting Equations (8) and (10) in the expression for s:

$$s = \frac{\phi_{DH}}{\phi_{DM}} = \frac{\frac{h + b}{2}\int_B^A (\beta\, x - 1)\, d\, w - \int_D^B y'\, d\, w}{\frac{1}{2}\int_B^A (\beta\, x - 1)\, d\, w - \int_D^B d\, w - \varepsilon}$$

Placing

$$\frac{1}{2}\int_B^A (\beta x - 1)\, d w = \psi \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$s = \frac{(h+b)\,\psi - \int_D^B y'\, d w}{\psi - \int_D^B d w - \varepsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Then Equations (9) become:

$$\left.\begin{aligned} V_{AH} &= -V_{BH} = -\frac{\beta}{2}(h+b-s) \\ M_{AH} &= \frac{(h+b-s)\,\beta\, l}{2} - \frac{h-s}{2} \\ M_{BH} &= -\frac{h-s}{2} \\ M_{OH} &= \frac{(h+b-s)\,\beta\, x}{2} - \frac{h+b-s+y}{2} \\ H_{AH} &= \frac{1}{2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (13)$$

The values of β, and ψ, and the integrals for the pier can be obtained from the accompanying tables, as will be explained subsequently. The value of the angle, ϵ, can be computed if the size of the pier base and the elasticity of the supporting foundation are known. The latter value must be determined experimentally. The relative importance of the yielding of the foundation is indicated by the effect of ϵ on the value of s. If ϵ can be omitted with no appreciable effect on s, the yielding of the foundations can be neglected. It should be noted that the formulas herein derived can be used for the special case of a frictionless hinge at the base of the pier. In that case ϵ is infinitely great and s becomes zero.

Influence Line for M'_D.—The influence line for M'_D is the elastic curve of the rib, which gives the vertical displacement when $M'_D = -1$.

In Fig. 10, ϕ_{DM} is the angular displacement of the rigid member attached to the base of the pier produced by a unit value of M'_D. Let δ_m be the vertical displacement of the points where the loads are applied to the rib when $M'_D = -1$, which displacement, by Maxwell's theorem, is equal to the angular displacement at D produced by the two loads.

The total angular displacement at D is, therefore,

$$\phi_D = M'_D\,\phi_{DM} - \left[\frac{P}{2}\,\delta_m + \frac{-P}{2}(-\delta_m)\right] = M'_D\,\phi_{DM} - P\,\delta_m$$

Since $\phi_D = 0$,

$$M'_D = P\,\frac{\delta_m}{\phi_{DM}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

From Equation (8),

$$\phi_{DM} = \frac{1}{2}\int_B^A (\beta x - 1)\, d w - \int_D^B d w - \varepsilon$$

or,

$$\phi_{DM} = \psi - \int_D^B d\,w - \varepsilon \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The values of ψ and $\int_D^B d\,w$ are given in Tables 1 and 6, respectively.

The elastic curve for the vertical displacements of the rib is obtained from two elementary elastic curves. The bending moments in the rib are as shown in Fig. 11. (See, also, Equation (7).)

δ_1 is the vertical displacement of the rib when $M_A = M_B = -\frac{1}{2}$ and δ_2 is the displacement when $M_A = \frac{1}{2}$ and $M_B = 0$. The value of δ_m is then,

$$\delta_m = \delta_1 - \beta\, l\, \delta_2$$

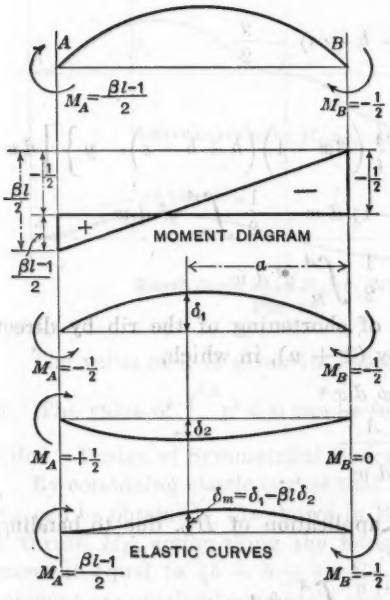


FIG. 11.

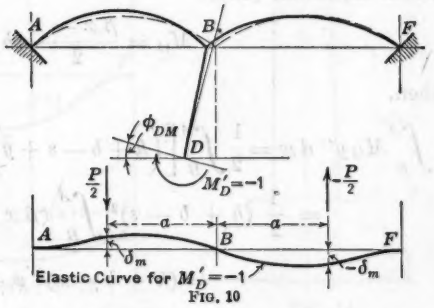


FIG. 10

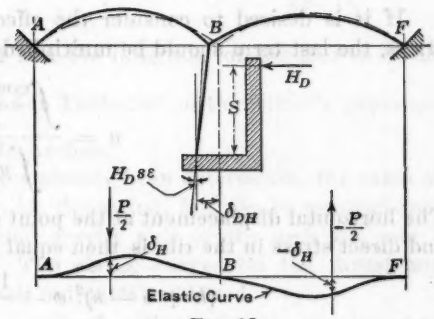


FIG. 12.

The values of δ_1 and δ_2 can be obtained from Tables 4 and 5 for any particular value of m, but δ_m is also given directly in Table 3. These values were obtained by integrating the differential equation for the elastic curves as explained for δ_3.

Influence Lines for H_D.—The influence line for H_D is the elastic curve of vertical displacements produced by $H_D = 1$. Let δ_{DH} be the horizontal displacement of D produced by $H_D = 1$, and let δ_H be the vertical displacement at the load point of the rib produced by $H_D = 1$, which, by Maxwell's theorem, is also the horizontal displacement at D produced by a unit vertical load on the

rib. Then, the total horizontal displacement at the point of application of H_D is,

$$\Delta_D = H_D\,\delta_{DH} + \left[\frac{P}{2}\,\delta_H + \frac{-P}{2}(-\delta_H)\right] = H_D\,\delta_{DH} + P\,\delta_H$$

or, as $\Delta_D = 0$,

$$H_D = -P\,\frac{\delta_H}{\delta_{DH}} \qquad (17)$$

The value of δ_{DH} is negative and can be obtained from the equation,

$$\delta_{DH} = \int_D^A M_O\,y''\,d\,w$$

in which, y'' is the ordinate measured from the line of action of H_D.

For the arch rib, from A to B,

$$y'' = (h + b - s + y)$$

From Equation (13),

$$M_O = \frac{\beta\,x - 1}{2}(h + b - s) - \frac{y}{2}$$

then,

$$\int_B^A M_O\,y''\,d\,w = \frac{1}{2}\int_B^A \left[\left(h+b-s+y\right)\left\{\left(\beta x-1\right)\left(h+b-s\right)-y\right\}\right] d\,w$$

$$= \frac{1}{2}(h + b - s)^2 \int_B^A (\beta\,x - 1)\,d\,w - \frac{1}{2}\int_B^A y^2\,d\,w$$

$$= (h + b - s)^2\,\psi - \frac{1}{2}\int_B^A y^2\,d\,w$$

If it is desired to consider the effect of shortening of the rib by direct stress, the last term should be multiplied by $(1 + u)$, in which,

$$u = \frac{\int \frac{\cos\,\phi\,d\,x}{E\,A}}{\int y^2\,d\,w}$$

The horizontal displacement at the point of application of H_D, due to bending and direct stress in the rib, is then equal to:

$$(h + b - s)^2\,\psi - \frac{1+u}{2}\int_B^A y^2\,d\,w$$

For the pier,

$$M_O = s - y' \text{ and } y'' = -(s - y')$$

hence,

$$\int_D^B M_O\,y''\,d\,w = -\int_D^B (s - y')^2\,d\,w = -s^2\int_D^B d\,w + 2\,s\int_D^B y'\,d\,w - \int_D^B (y')^2\,d\,w$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 945, Equation (20*a*).

For the elastic foundation,

$$\int M_O\, y''\, d\,w = -\,H_D\, s^2\, \varepsilon = -\,s^2\, \varepsilon$$

then the complete expression is,

$$\delta_{DH} = (h + b - s)^2\, \psi - \frac{1+u}{2} \int_B^A y^2\, d\,w - s^2 \left(\varepsilon + \int_D^B d\,w\right) - \int_D^B (y')^2\, d\,w + 2\,s \int_D^B y'\, d\,w \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

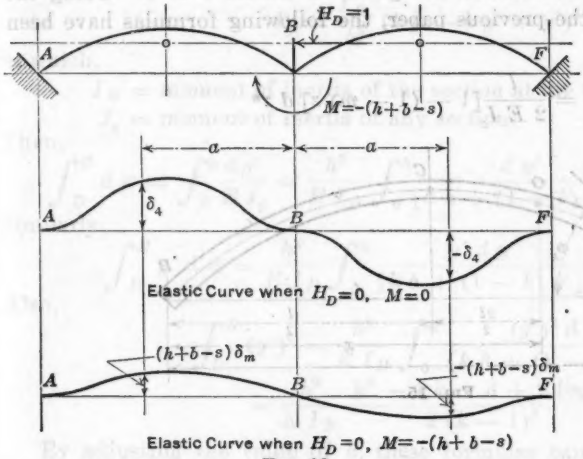


FIG. 13.

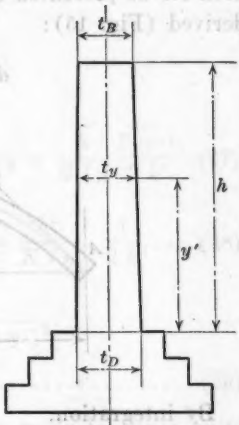


FIG. 14.

The value of ψ is given in Table 1 and the integrals for the pier in Table 6. The value of $\int_B^A y^2\, d\,w$ can be found in Table 18*, of the writer's paper entitled "Design of Symmetrical Concrete Arches."

By combining elastic curves that have already been determined, the value of δ_H can be obtained. As shown in Fig. 13, the thrust, H_D, can be resolved into a thrust, H_D, acting along the horizontal gravity axis of the arch rib, and a moment equal to $(h + b - s)\, H_D$. The elastic curves for the thrust and moment are obtained separately and their ordinates combined.

From Fig. 13 it is seen that this combined ordinate for the displacement produced by H_D is,

$$\delta_H = \delta_4 + (h + b - s)\, \delta_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The value of δ_m has already been obtained (Table 3) and δ_4 can be computed by means of Figs. 28 to 32 and Table 18 in the writer's paper previously mentioned, from the formula,

$$\delta_4 = \frac{C\, C_H\, r\, l^2}{2\, E\, I_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which, C is found from Table 18 and C_H from Figs. 28 to 32.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 991.

Influence Lines for Case II.—When the reactions at the base of the pier have been obtained for different positions of the load, the moments and thrusts at the desired sections, due to the loads and to M_D' and H_D, are to be computed from Equations (5), (6), (7), and (13). These values are to be added together in order to obtain the total moments or thrusts, as, for instance:

$$M_A'' = M_{AP} + M_{AM} + M_{AH}$$

Value of Integrals for Arch Ribs and Piers.—In order to facilitate the solution of the foregoing equations the values of the integrals for the ribs and the piers have been determined and are given in Tables 1 to 6. Using the arch rib as presented in the previous paper, the following formulas have been derived (Fig. 15):

$$d\,w = \frac{l}{2\,E\,I_c}[1 - (1 - m)\,z]\,d\,z^*$$

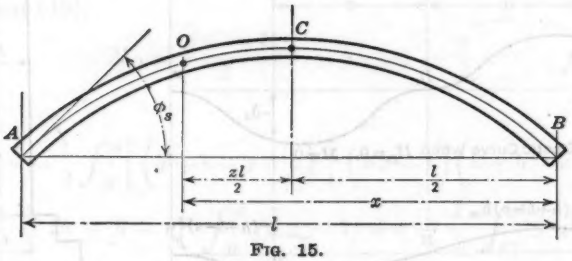


FIG. 15.

By integration,

$$\int_B^A d\,w = \frac{l}{E\,I_c}\,\frac{1+m}{2} \qquad (21)$$

$$\int_B^A x\,d\,w = \frac{l^2}{E\,I_c}\,\frac{1+m}{4} \qquad (22)$$

$$\int_B^A x^2\,d\,w = \frac{l^3}{E\,I_c}\,\frac{7+9\,m}{48} \qquad (23)$$

$$\beta = \frac{\int x\,d\,w}{\int x^2\,d\,w} = \frac{12}{l}\,\frac{1+m}{7+9\,m} \qquad (24)$$

$$\beta \int x\,d\,w = \frac{l}{E\,I_c}\,\frac{3\,(1+m)^2}{7+9\,m} \qquad (25)$$

$$\psi = \frac{\beta}{2}\int_B^A x\,d\,w - \frac{1}{2}\int_B^A d\,w = -\frac{l}{E\,I_c}\,\frac{1+4\,m+3\,m^2}{28+36\,m} \qquad (26)$$

Tables 2 to 5 give the elastic curves for the conditions indicated which were obtained by integrating the differential equation,

$$\frac{d^2\,\delta}{d\,z^2} = -\frac{M\,l^2}{4\,E\,I_c}[1 - (1 - m)\,z]\dagger$$

It was not thought necessary to give the resulting equations here.

* From Equation (54), *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 983.
† Equation (55), *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 983.

In order to derive formulas for the integrals for the pier, it has been assumed that the length of the pier is uniform from top to bottom and that the sides have a straight batter (Fig. 14). Then, placing $\frac{t_D}{t_B} = k$, it follows that,

$$\frac{t_y}{t_B} = 1 + (k-1)\frac{h-y'}{h} = \frac{1}{h}(k\,h - k\,y' + y')$$

Then assuming that,

$$\frac{I_y}{I_B} = \frac{t_y^3}{t_B^3} = \frac{(k\,h - k\,y' + y')^3}{h^3}$$

$$I_y = \frac{I_B}{h^3}(k\,h - k\,y' + y')^3$$

in which,

I_B = moment of inertia of the section at the top.

I_y = moment of inertia of any section.

Then,

$$\int_D^B d\,w = \int_0^h \frac{d\,y'}{E\,I_y} = \frac{h^3}{E\,I_B}\int_0^h \frac{d\,y'}{[k\,h + (1-k)\,y']^3} = \frac{h}{E\,I_B}\,\frac{1+k}{2\,k^2} \quad \ldots(27)$$

Similarly,

$$\int_D^B y'\,d\,w = \frac{h^3}{E\,I_B}\int_0^h \frac{y'\,d\,y'}{[k\,h + (1-k)\,y']^3} = \frac{h^2}{E\,I_B}\cdot\frac{1}{2\,k} \quad \ldots(28)$$

Also,

$$\int_D^B (y')^2 = \frac{h^3}{E\,I_B}\int_0^h \frac{(y')^2\,d\,y'}{[k\,h + (1-k)\,y']^3}$$

$$= \frac{h^3}{E\,I_B}\,\frac{k^2 - 4\,k + 3 + 2\log k}{2\,(k-1)^3} \quad \ldots(29)$$

By adjusting the value of k, these formulas can be made to apply quite closely when there is a batter at the ends of the pier as well as the sides.

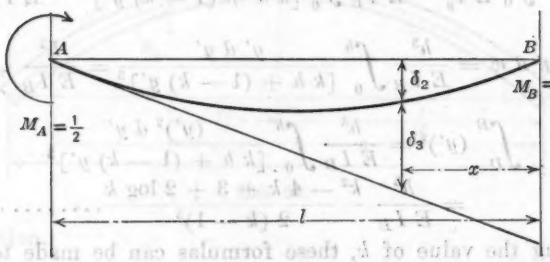
TABLE 1.—ARCH RIB CONSTANTS.

Value of m.	$\frac{E\,I_c}{l}\int d\,w$ (Equation (21)).	$\frac{E\,I_c}{l^2}\int x\,d\,w$ (Equation (22)).	$\frac{E\,I_c}{l^3}\int x^2\,d\,w$ (Equation (23)).	$\beta\,l$ (Equation (24)).	$\frac{E\,I_c}{l}\psi$ (Equation (26)).
0.15	0.575	0.2875	0.1740	1.652	−0.04991
0.20	0.600	0.3000	0.1883	1.637	−0.05455
0.21	0.605	0.3025	0.1852	1.633	−0.05546
0.22	0.610	0.3050	0.1871	1.630	−0.05638
0.23	0.615	0.3075	0.1890	1.627	−0·05730
0.24	0.620	0.3100	0.1908	1.625	−0.05821
0.25	0.625	0.3125	0.1927	1.622	−0.05912
0.26	0.630	0.3150	0.1946	1.619	−0.06003
0.27	0.635	0.3175	0.1965	1.616	−0.06094
0.28	0.640	0.3200	0.1983	1.614	−0.06185
0.29	0.645	0.3225	0.2002	1.611	−0.06275
0.30	0.650	0.3250	0.2021	1.608	−0.06366
0.31	0.655	0.3275	0.2040	1.605	−0.06456
0.32	0.660	0.3300	0.2058	1.603	−0.06547
0.33	0.665	0.3325	0.2077	1.601	−0.06636
0.34	0.670	0.3350	0.2096	1.598	−0.06727
0.35	0.675	0.3375	0.2115	1.596	−0.06817
0.36	0.680	0.3400	0.2133	1.594	−0.06906
0.37	0.685	0.3425	0.2152	1.592	−0.06996
0.38	0.690	0.3450	0.2171	1.589	−0.07085
0.39	0.695	0.3475	0.2190	1.587	−0.07175
0.40	0.700	0.3500	0.2208	1·585	−0.07264
0.45	0.725	0.3625	0.2302	1.575	−0.07709
0.50	0.750	0.3750	0.2396	1.567	−0.08152

Numerical Examples

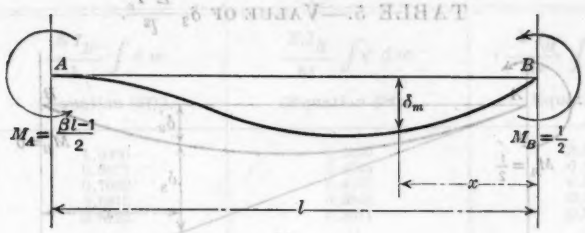
In order to show the application of the foregoing method of analysis and the effect of varying the proportions of the pier, two examples will be solved and the resulting influence lines compared with the influence lines for fixed arches. The proportions of the pier in the first example are more slender than would be used in bridge construction. For the second example, the ribs have been kept the same, but the pier has been doubled in thickness for the purpose of bringing the proportions within the limits for practical design. The second system has also been analyzed by an approximate method and the influence lines compared with the results of the more exact method in order to determine the importance of the errors introduced by the approximations.

TABLE 2.—Value of $\delta_3 \frac{E I_c}{l^2} = C_2$.



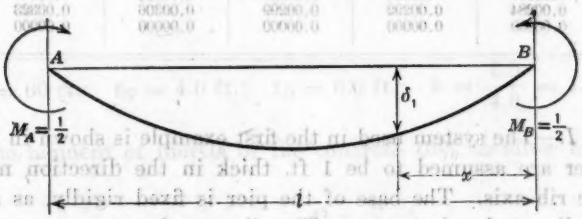
$\frac{x}{l}$	Values of m.					
	0.15	0.20	0.25	0.30	0.40	0.50
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.95	0.00011	0.00014	0.00017	0.00019	0.00026	0.00032
0.90	0.00050	0.00061	0.00073	0.00083	0.00106	0.00129
0.85	0.00125	0.00148	0.00173	0.00198	0.00245	0.00293
0.80	0.00242	0.00282	0.00324	0.00363	0.00455	0.00527
0.75	0.00408	0.00469	0.00529	0.00589	0.00709	0.00830
0.70	0.00628	0.00711	0.00798	0.00875	0.01039	0.01204
0.65	0.00907	0.01013	0.01119	0.01224	0.01435	0.01648
0.60	0.01245	0.01376	0.01507	0.01637	0.01898	0.02160
0.55	0.01646	0.01802	0.01960	0.02114	0.02427	0.02740
0.50	0.02109	0.02291	0.02474	0.02656	0.03021	0.03385
0.45	0.02634	0.03841	0.03050	0.03258	0.03675	0.04092
0.40	0.03210	0.03444	0.03680	0.03913	0.04383	0.04852
0.35	0.03626	0.04078	0.04351	0.04610	0.05135	0.05657
0.30	0.04477	0.04765	0.05056	0.05343	0.05922	0.06500
0.25	0.05151	0.06469	0.05787	0.06103	0.06738	0.07373
0.20	0.05844	0.07191	0.06588	0.06888	0.07576	0.08269
0.15	0.06549	0.07926	0.07302	0.07678	0.08430	0.09183
0.10	0.07262	0.08669	0.08075	0.08482	0.09295	0.10108
0.05	0.07979	0.08417	0.08855	0.09292	0.10166	0.11042
0.00	0.08698	0.09167	0.09635	0.10104	0.11042	0.11979

TABLE 3.—VALUE OF $\delta_m \frac{E I_c}{l^2}$.



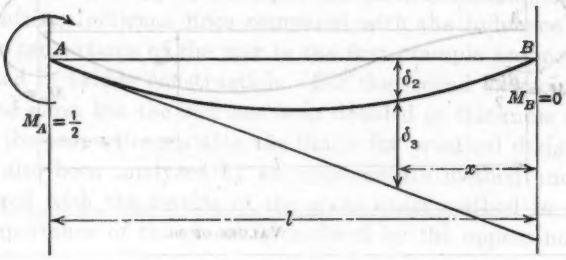
$\frac{x}{l}$	Values of m.					
	0.15	0.20	0.25	0.30	0.40	0.50
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.95	0.00007	0.00008	0.00010	0.00011	0.00014	0.00016
0.90	0.00031	0.00036	0.00042	0.00048	0.00058	0.00066
0.85	0.00074	0.00085	0.00097	0.00109	0.00129	0.00147
0.80	0.00138	0.00155	0.00174	0.00192	0.00226	0.00254
0.75	0.00221	0.00245	0.00271	0.00297	0.00343	0.00384
0.70	0.00321	0.00352	0.00385	0.00418	0.00477	0.00530
0.65	0.00434	0.00472	0.00511	0.00551	0.00621	0.00685
0.60	0.00553	0.00597	0.00642	0.00687	0.00769	0.00843
0.55	0.00671	0.00720	0.00771	0.00820	0.00911	0.00994
0.50	0.00779	0.00831	0.00886	0.00939	0.01037	0.01128
0.45	0.00866	0.00921	0.00977	0.01034	0.01138	0.01236
0.40	0.00924	0.00985	0.01040	0.01098	0.01206	0.01308
0.35	0.00946	0.01007	0.01064	0.01122	0.01235	0.01338
0.30	0.00930	0.00980	0.01046	0.01105	0.01216	0.01321
0.25	0.00871	0.00928	0.00985	0.01040	0.01148	0.01252
0.20	0.00771	0.00824	0.00876	0.00927	0.01028	0.01124
0.15	0.00630	0.00675	0.00721	0.00766	0.00852	0.00938
0.10	0.00451	0.00486	0.00519	0.00556	0.00622	0.00687
0.05	0.00238	0.00258	0.00279	0.00300	0.00337	0.00375
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 4.—VALUE OF $\delta_1 \frac{E I_c}{l^2}$.



$\frac{x}{l}$	Values of m.						$\frac{x}{l}$
	0.15	0.20	0.25	0.30	0.40	0.50	
1.00	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.00
0.95	0.007076	0.007858	0.007641	0.007923	0.008487	0.009052	0.05
0.90	0.013858	0.014367	0.014875	0.015388	0.016400	0.017417	0.10
0.85	0.020241	0.020925	0.02161	0.022294	0.023663	0.025031	0.15
0.80	0.026117	0.026933	0.027750	0.028567	0.030200	0.031833	0.20
0.75	0.031380	0.032292	0.033203	0.034115	0.035937	0.037760	0.25
0.70	0.035925	0.036900	0.037875	0.038850	0.040800	0.042750	0.30
0.65	0.039645	0.040658	0.041672	0.042685	0.044712	0.046740	0.35
0.60	0.042433	0.043467	0.044500	0.045533	0.047600	0.049667	0.40
0.55	0.044183	0.045225	0.046266	0.047306	0.049387	0.051469	0.45
0.50	0.044792	0.045833	0.046875	0.047917	0.050000	0.052083	0.50

TABLE 5.—VALUE OF $\delta_2 \frac{E I_c}{l^2}$.



$\frac{x}{l}$	Values of m.					
	0.15	0.20	0.25	0.30	0.40	0.50
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.95	0.00424	0.00444	0.00465	0.00486	0.00526	0.00567
0.90	0.00820	0.00856	0.00891	0.00927	0.00998	0.01069
0.85	0.01180	0.01227	0.01273	0.01319	0.01411	0.01504
0.80	0.01498	0.01551	0.01604	0.01657	0.01763	0.01869
0.75	0.01766	0.01823	0.01880	0.01937	0.02051	0.02165
0.70	0.01981	0.02039	0.02098	0.02156	0.02273	0.02390
0.65	0.02137	0.02196	0.02254	0.02312	0.02429	0.02545
0.60	0.02234	0.02291	0.02348	0.02404	0.02518	0.02632
0.55	0.02268	0.02323	0.02377	0.02432	0.02541	0.02651
0.50	0.02240	0.02292	0.02344	0.02396	0.02500	0.02604
0.45	0.02150	0.02200	0.02250	0.02299	0.02398	0.02496
0.40	0.02009	0.02056	0.02102	0.02149	0.02242	0.02335
0.35	0.01828	0.01870	0.01913	0.01957	0.02042	0.02129
0.30	0.01612	0.01651	0.01690	0.01729	0.01807	0.01885
0.25	0.01373	0.01406	0.01440	0.01475	0.01543	0.01611
0.20	0.01114	0.01142	0.01171	0.01200	0.01257	0.01314
0.15	0.00844	0.00866	0.00888	0.00910	0.00955	0.00999
0.10	0.00566	0.00581	0.00597	0.00611	0.00642	0.00673
0.05	0.00284	0.00292	0.00299	0.00306	0.00323	0.00338
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Example I.—The system used in the first example is shown in Fig. 16. The ribs and pier are assumed to be 1 ft. thick in the direction normal to the plane of the rib axis. The base of the pier is fixed rigidly, as are the outer ends of the ribs at the abutments. The ribs are the same as were used in the examples of the writer's paper on "Design of Symmetrical Concrete Arches", and detailed dimensions are given in Fig. 19* therein. The effect of rib-shortening has been neglected in these examples.

The properties of this rib are, as follows:

$l = 100$ ft.	$y_c = 3.15$ ft.	$I_c = 6\,399$ in.^{4} $= 0.309$ ft.^{4}	
$r = 15$ ft.	$d_c = 18$ in.	$I_s = 24\,560$ in.^{4} $= 1.185$ ft.^{4}	
$g = 4.70$	$d_s = 27$ in.	$b = 11.85$ ft.	
$m = 0.339$	$\cos \phi_s = 0.769$	$h + b = 71.85$ ft.	

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 961.

TABLE 6.—PIER CONSTANTS.

Value of k.	$\frac{E I_B}{h}\int d\,w$. (Equation (27)).	$\frac{E I_B}{h^2}\int y'\,d\,w$. (Equation (28)).	$\frac{E I_B}{h^3}\int (y')^2\,d\,w$. (Equation (29)).
1.0	1.0000	0.5000	0.3333
1.1	0.8677	0.4545	0.3100
1.2	0.7632	0.4167	0.2902
1.3	0.6815	0.3846	0.2727
1.4	0.6122	0.3571	0.2576
1.5	0.5555	0.3333	0.2437
1.6	0.5078	0.3125	0.2315
1.7	0.4674	0.2941	0.2205
1.8	0.4321	0.2739	0.2105
1.9	0.4017	0.2632	0.2014
2.0	0.3750	0.2500	0.1931
2.1	0.3514	0.2381	0.1855
2.2	0.3306	0.2273	0.1785
2.3	0.3119	0.2174	0.1720
2.4	0.2951	0.2083	0.1659
2.5	0.2800	0.2000	0.1604
2.6	0.2663	0.1923	0.1552
2.7	0.2538	0.1852	0.1503
2.8	0.2423	0.1785	0.1457
2.9	0.2319	0.1724	0.1414
3.0	0.2222	0.1667	0.1373

Then, from Table 1,

$$\beta\, l = 1.60\,; \quad \beta = 0.016$$

$$\frac{E\, I_c}{l}\,\psi = -\,0.06718$$

$$E\,\psi = \frac{-\,0.06718 \times 100}{0.309} = -\,21.73$$

For the pier,

$$h = 60 \text{ ft.}; \quad t_B = 4.0 \text{ ft.}; \quad t_D = 6.0 \text{ ft.}; \quad k = \frac{6.0}{4.0} = 1.5$$

Increasing the moment of inertia of the concrete 10% to allow for reinforcement,

$$I_B = 1.1 \times \frac{4^3}{12} = 5.87 \text{ ft.}^4$$

From Table 6,

$$E\int d\,w = 0.5555 \times \frac{60}{5.87} = 5.67$$

$$E\int y'\,d\,w = 0.3333 \times \frac{3\,600}{5.87} = 204.4$$

$$E\int (y')^2\,d\,w = 0.2437 \times \frac{216\,000}{5.87} = 8\,970$$

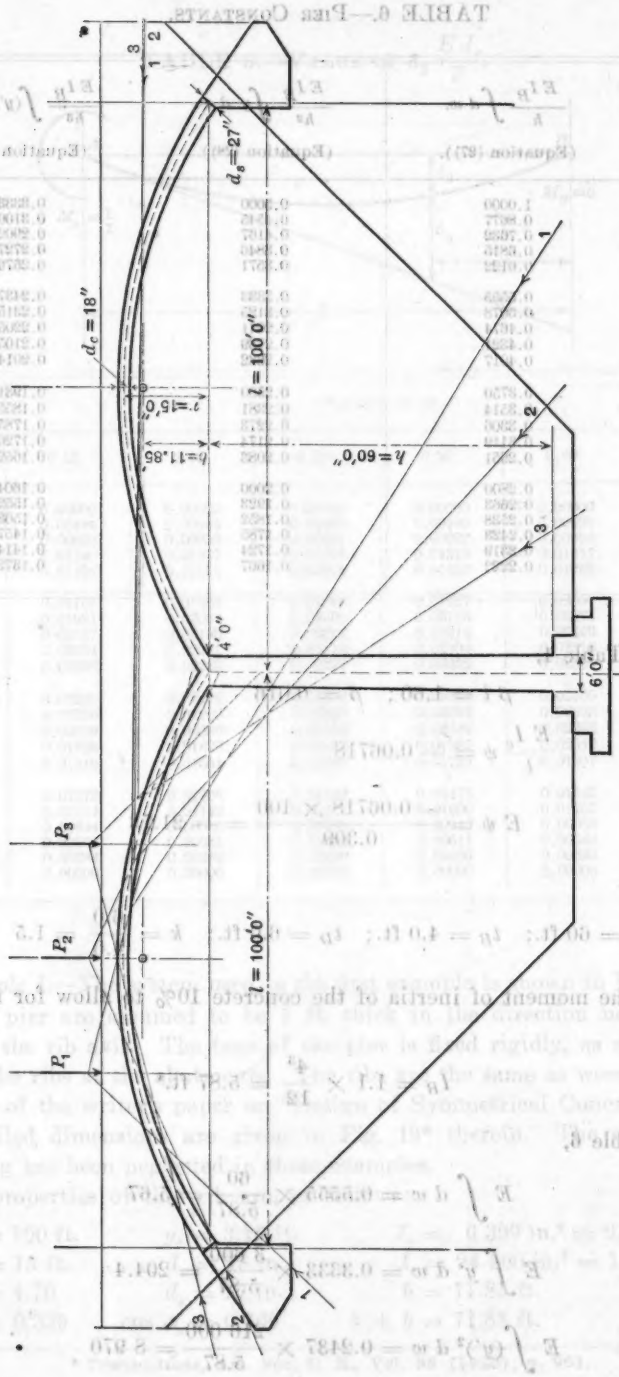


FIG. 16.

Then,

$$s = \frac{(h+b)\,\psi - \int_D^B y'\,dw}{\psi - \int_D^B dw - \varepsilon} = \frac{-(71.85 \times 21.73) - 204.4}{-21.73 - 5.67 - 0} = 64.4 \text{ ft.}$$

$$h + b - s = 71.85 - 64.4 = 7.45 \text{ ft.}$$

The reactions due to the loads can now be determined by Equations (5) and (6). Table 7 gives the computations. From Table 1, C_1 is 0.2094.

TABLE 7.—EFFECT OF LOADS.

Load point $\frac{x}{l}$.	C_2 (From Table 2).	V_{BP} $\frac{C_2}{C_1}$.	M_{AP} $\left(V_{BP}\,l - \frac{l-a}{2}\right)$.	V_{AP} $\left(\frac{1}{2} - V_{BP}\right)$.
10	0.00000	0.0000	—0.00	0.5000
9	0.00092	0.0044	— 4.56	0.4956
8	0.00399	0.0191	— 8.09	0.4809
7	0.00939	0.0448	—10.52	0.4552
6	0.01739	0.0831	—11.69	0.4169
5	0.02798	0.1337	—11.64	0.3664
4	0.04096	0.1956	—10.44	0.3044
3	0.05569	0.2660	— 8.40	0.2340
2	0.07153	0.3420	— 5.84	0.1580
1	0.08799	0.4200	— 3.00	0.0800
0	0.10470	0.5000	— 0.00	0.0000

The influence line for H_D will next be determined by Equations (17), (18), and (19):

$$H_D = \frac{-\delta_H}{\delta_{DH}}$$

$$\delta_{DH} = (h+b-s)^2\,\psi - \frac{1}{2}\int_B^A y^2\,dw - s^2\left(\varepsilon + \int_D^B dw\right) - \int_D^B (y')^2\,dw + 2s\int_D^B y'\,dw$$

$$(h+b-s)^2\,\psi = \frac{-7.45 \times 7.45 \times 21.73}{E} = -\,1\,205 \div E$$

$$-\frac{1}{2}\int_B^A y^2\,dw = -\,0.0408^* \frac{100 \times 225}{E \times 2 \times 0.309} = -\,1\,487 \div E$$

$$-s^2\left(\varepsilon + \int_D^B dw\right) = \frac{-64.4 \times 64.4\,(0 + 5.67)}{E} = -\,23\,530 \div E$$

$$-\int_D^B (y')^2\,dw = -\,8\,970 \div E$$

$$\text{Sum} = -\,35\,192 \div E$$

$$+2s\int_D^B y'\,dw = \frac{204.4 \times 2 \times 64.4}{E} = +\,26\,350 \div E$$

$$\delta_{DH} = -\,8\,842 \div E$$

$$\delta_H = \delta_4 + (h+b-s)\,\delta_m$$

* Value of C.

in which,

$$\delta_4 = \frac{C\, C_H\, r\, l^2}{2\, E\, I_c}$$

The value of C from Table 18* of the writer's paper previously mentioned, is 0.0408 and the values of C_H are taken from Figs. 28 to 32† of the same paper. δ_m is given by Table 3. The computation of H_D is given in Table 8.

TABLE 8.—COMPUTATION OF H_D.

Load point, $\frac{x}{l}$	C_H.	$E\,\delta_4 = \frac{C\, C_H\, r\, l^2}{2\, I_c}$.	$\delta_m \frac{E\, Ic}{l_2}$ (Table 3).	$(h+b-s)\,\delta_m\, E$.	$E\,\delta_H$.	$H_D = \frac{-\delta_H}{\delta_{DH}}$.
10	0.0000	0	0.00000	0.	0.	0.0000
9	0.0287	284	0.00052	125.	409.	0.0463
8	0.0972	965	0.00205	494.	1 459.	0.1651
7	0.1762	1 748	0.00441	1 062.	2 810.	0.3180
6	0.2385	2 365	0.00719	1 733.	4 098.	0.4635
5	0.2630	2 604	0.00977	2 358.	4 962.	0.5615
4	0.2385	2 365	0.01140	2 747.	5 112.	0.5780
3	0.1762	1 748	0.01148	2 765.	4 513.	0.5110
2	0.0972	965	0.00966	2 330.	3 295.	0.3725
1	0.0287	284	0.00582	1 404.	1 688.	0.1910
0	0.0000	0	0.00000	0.	0.	0.0000

The moments and thrusts produced by H_D are found from Equations (13), which were derived for unit values of H_D. These are given in Table 9.

TABLE 9.—EFFECT OF H_D.

Load point, $\frac{x}{l}$	$V_{AH} = -V_{BH} = \frac{-\beta}{2}(h+b-s)\,H_D$.	$H_{AH} = \frac{H_D}{2}$.	$= -\frac{M_{BH}}{h-s}\frac{}{2} H_D$	$M_{AE} = H_D \left[\frac{(h+b-s)\,\beta\, l}{2} - \frac{h-s}{2}\right]$.	$M_{DH} = H_D\, s$.
10	— 0.0000	0.0000	0.000	0.000	0.00
9	— 0.0028	0.0232	0.102	0.378	2 98
8	— 0.0098	0.0826	0.363	1.345	10.64
7	— 0.0182	0.1590	0.700	2.590	20.50
6	— 0.0276	0.2318	1.020	3.775	29.85
5	— 0.0334	0.2808	1.235	4.580	36.15
4	— 0.0344	0.2890	1.270	4.715	37.25
3	— 0.0304	0.2555	1.122	4.170	32.90
2	— 0.0222	0.1868	0.820	3.036	24.00
1	— 0.0114	0.0955	0.420	1.556	12.30
0	— 0.0000	0.0000	0.000	0.000	0.00

The influence-line ordinates for M_D' are next computed from Equation (14):

$$M_D' = P\,\frac{\delta_m}{\phi_{DM}}$$

$$\phi_{DM} = \psi - \int_D^B d\,w - \varepsilon = -\frac{21.73}{E} - \frac{5.67}{E} - 0 = -\frac{27.4}{E}$$

The computation of M_D' is given in Table 10; also, the effect of M_D' from Equation (7).

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 991.
† *Loc. cit.*, pp. 1004–1008.

TABLE 10.—COMPUTATION OF M_D'.

Load point, $\frac{x}{l}$.	$\delta_m \frac{E I_c}{l^2}$ (Table 3).	$M'_D = \frac{\delta_m}{\Phi_{DM}}$.	$V_{AM} = -V_{BM} = M_D' \frac{\beta}{2}$.	$M_{AM} = -M_D' \left(\frac{\beta l - 1}{2}\right)$.	$M_{BM} = \frac{M_D'}{2}$.
10	0.00000	— 0.00	—0.0000	0.00	—0.00
9	0.00052	— 0.62	—0.0049	0.18	—0.31
8	0.00205	— 2.43	—0.0194	0.73	—1.22
7	0.00441	— 5.21	—0.0417	1.56	—2.61
6	0.00719	— 8.52	—0.0680	2.55	—4.26
5	0.00977	—11.55	—0.0923	3.46	—5.78
4	0.01140	—13.50	—0.1080	4.05	—6.75
3	0.01148	—13.60	—0.1087	4.08	—6.80
2	0.00966	—11.42	—0.0913	3.43	—5.71
1	0.00582	— 6.88	—0.0550	2.06	—3.44
0	0.00000	— 0.00	—0.0000	0.00	—0.00

The final value of moments and thrusts for Case II loading are obtained by adding together algebraically the partial values for loading, H_D and M_D', as follows:

$$M_A'' = M_{AP} + M_{AM} + M_{AH}$$
$$M_B'' = M_{BP} + M_{BM} + M_{BH}$$
$$M_D'' = M_D' + M_{DH}$$
$$H_D'' = H_D$$
$$H_A'' = \frac{H_D}{2}$$
$$V_A'' = V_{AP} + V_{AM} + V_{AH}$$
$$V_B'' = V_{BP} + V_{BM} + V_{BH} = \frac{1}{2} - V_A''$$
$$V_D'' = 0$$

The moment at the crown section has also been computed from the formula,

$$M_c'' = \frac{V_A'' l}{2} - H_A'' r + M_A'' - \frac{P x'}{2}$$

All these final values are given in Table 11. For the right-hand span all values are the same as corresponding values for the left-hand span with the sign changed. The values given in Table 11 for Case I were obtained by dividing by 2 the corresponding values found for the fixed arch rib in the writer's previous paper.*

The sums of the values for Case I and Case II are the values for the general case as presented in Table 11. For comparison these values have been plotted in Figs. 18, 19, and 20, together with the corresponding influence lines for the two solutions of Example II and for the single-span fixed arch. Fig. 16 shows the reaction lines for a load at three different points on the left-hand arch rib.

Example II.—A complete solution has been made for a two-span system similar to that treated in Example I, except that the pier has been made twice

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), Table 19, p. 1001.

TABLE 11.—EXAMPLE I. COMPUTATION OF STRESSES.

(Comparison of Influence Lines for Examples I, II, and II(*a*), Is Given in Figs. 18, 19, and 20.)

$\frac{x}{l}$	H_A'.	H_A''.	H_A.	V_A'.	V_A''.	V_A.	V_B'.	V_B''.	V_B.	M_D.
1.0	0.0000	0.0000	0.000	0.500	0.5000	1.000	0.000	0.000	0.000	0.00
0.9	0.006	0.023	0.112	0.489	0.488	0.977	0.011	0.012	0.023	2.36
0.8	0.324	0.088	0.412	0.457	0.458	0.909	0.043	0.048	0.091	8.21
0.7	0.587	0.159	0.746	0.403	0.395	0.798	0.097	0.105	0.202	15.29
0.6	0.795	0.232	1.027	0.333	0.321	0.653	0.168	0.179	0.347	21.35
0.5	0.876	0.281	1.157	0.250	0.241	0.491	0.250	0.259	0.509	24.60
0.4	0.795	0.289	1.084	0.168	0.163	0.330	0.332	0.338	0.670	23.75
0.3	0.587	0.256	0.843	0.097	0.095	0.192	0.403	0.405	0.808	19.80
0.2	0.324	0.186	0.510	0.043	0.044	0.087	0.457	0.456	0.913	12.58
0.1	0.006	0.096	0.102	0.011	0.014	0.025	0.489	0.486	0.975	5.42
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.500	1.000	0.00
0.0'	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.500	1.000	0.00
0.1'	0.006	0.096	0.102	0.011	−0.014	−0.003	0.489	0.486	0.975	−5.42
0.2'	0.324	0.186	0.510	0.043	−0.044	−0.001	0.457	0.456	0.913	−12.58
0.3'	0.587	0.256	0.843	0.097	−0.095	−0.002	0.403	0.405	0.808	−19.80
0.4'	0.795	0.289	1.084	0.168	−0.162	−0.006	0.332	0.338	0.670	−23.75
0.5'	0.876	0.281	1.157	0.250	−0.241	−0.009	0.250	0.259	0.509	−24.60
0.6'	0.795	0.232	1.027	0.333	−0.321	−0.011	0.168	0.179	0.347	−21.35
0.7'	0.587	0.159	0.746	0.403	−0.395	−0.008	0.097	0.105	0.202	−15.29
0.8'	0.324	0.088	0.412	0.457	−0.458	−0.001	0.043	0.048	0.091	−8.21
0.9'	0.006	0.023	0.112	0.489	−0.488	0.001	0.011	0.012	0.023	−2.36
1.0'	0.000	0.000	0.000	0.500	−0.500	0.000	0.000	0.000	0.000	0.00

TABLE 11.—(*Continued*).

x	H_D	V_D	M_A	$M_{A''}$	M_A	M_B	M_B	$M_{C'}$	$M_{C''}$	M_C
1.0	0.046	0.000	0.00	0.00	0.00	0.00	0.000	0.000	0.000	0.00
0.9	0.165	0.023	3.19	4.00	7.19	0.55	0.76	0.15	0.04	0.11
0.8	0.318	0.087	3.36	6.02	9.38	1.47	2.31	0.39	0.51	0.08
0.7	0.463	0.196	1.59	6.37	7.96	1.81	3.69	0.27	0.38	0.71
0.6	0.561	0.337	0.94	6.36	4.42	0.87	4.07	0.58	0.33	2.81
0.5	0.578	0.500	1.07	5.80	0.50	1.44	3.10	2.46	4.33	6.68
0.4	0.511	0.668	3.69	4.15	2.54	4.53	0.94	0.58	2.00	2.67
0.3	0.373	0.805	2.31	1.97	4.28	7.23	1.59	0.27	0.77	0.50
0.2	0.191	0.913	0.76	0.67	1.43	9.24	3.36	0.39	0.11	0.28
0.1	0.000	0.978	0.00	0.32	0.32	9.81	3.19	0.15	0.13	0.28
0.0	0.000	1.000	0.00	0.00	0.00	9.00	0.00	0.00	0.00	0.00
0.1	0.000	0.978	0.76	0.00	0.76	8.19	0.00	0.15	0.13	0.28
0.2	0.191	0.913	2.31	0.67	2.98	7.23	1.59	0.27	0.77	0.50
0.3	0.373	0.805	3.69	1.97	5.66	5.66	3.36	0.58	2.00	2.67
0.4	0.511	0.668	3.10	4.15	7.25	4.53	0.94	2.46	4.33	6.68
0.5	0.578	0.500	0.94	5.80	6.74	1.44	3.10	0.58	2.00	2.67
0.6	0.561	0.337	1.59	6.36	7.95	0.87	4.07	0.27	0.77	0.50
0.7	0.463	0.196	3.36	6.37	9.73	1.81	3.69	0.39	0.31	0.70
0.8	0.318	0.087	3.19	6.02	9.21	1.47	2.31	0.15	0.04	0.19
0.9	0.165	0.023	0.00	4.00	4.00	0.55	0.76	0.00	0.00	0.00
1.0	0.046	0.000	0.00	0.00	0.00	0.00	0.000	0.000	0.000	0.00

as thick. The properties of the ribs remain the same and those of the pier are as follows:

$$h = 60 \text{ ft.}; \; t_B = 8.0 \text{ ft.}; \; t_D = 12.0 \text{ ft.}$$

$$k = 1.5; \; I_B = 1.1 \times \frac{8^3}{12} = 46.85$$

$$E \int d\,w = 0.5555 \times \frac{60}{46.85} = 0.710$$

$$E \int y'\,d\,w = 0.3333 \times \frac{3\,600}{46.85} = 25.6$$

$$E \int (y')^2\,d\,w = 0.2437 \times \frac{216\,000}{46.85} = 1\,123$$

$$s = \frac{-(71.85 \times 21.73) - 25.6}{-21.73 - 0.71} = 70.75$$

$$h + b - s = 71.85 - 70.75 = 1.10 \text{ ft.}$$

This value is very much smaller than that in the previous example. The size of $(h + b - s)$ may be used as a criterion of the importance of the effect of the elasticity of the pier. It should be noted that with a perfectly rigid pier,

$$\int_D^B d\,w = 0$$

$$\int_D^B y'\,d\,w = 0$$

and,

$$s = \frac{(h + b)\,\psi + 0}{\psi + 0} = h + b$$

Then,

$$h + b - s = 0$$

The final results of this analysis are given in Table 12. The Case I values are the same as those for the first example given in Table 11. Influence lines are plotted in Figs. 18, 19, and 20.

Example II.—Approximate Methods.—The arch system of Example II will be analyzed by a simple approximate method, which appears to give results that are reasonably close and err on the side of safety.

The effect on the arches of the elasticity of the pier is to permit the end of the arch ribs at the pier to move horizontally without vertical movement. This decreases the horizontal thrust in the loaded span by an amount, ΔH, and changes the bending moment by an amount, ΔHy. If H' and M_O' are the horizontal thrust and moment, respectively, at any point in the arch rib when its ends are fixed, the corresponding values with elastic supports will be, approximately,

$$\left.\begin{aligned} H &= H' + \Delta H \\ M_O &= M_O' - \Delta H\,y \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

This assumption is made for the unloaded as well as for the loaded span. The effect of the elasticity of the supports on the vertical reactions is neglected. In the case of the symmetrical two-span system, the decrease in horizontal

thrust in the loaded span will be equal to the increase in the unloaded span, and the thrust resisted by the pier will be,

$$H_D = H' + 2\,\Delta H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

This is true because the increase in length of the loaded span is the same as the decrease in the unloaded span, and, as their resistances are equal, the values of ΔH must be equal. This decrease in the length of the unloaded span is,

$$\delta_B = -\Delta H\,(1 + u) \int_B^C y^2\, d\, w$$

which is also equal to the movement of the top of the pier. The forces acting on the pier will be assumed as shown in Fig. 17, and the displacement will be computed at the springing line, B.

The reason for these assumptions should be noted. Actually, the resultant thrust from the loaded span does not follow the horizontal gravity axis of the ribs but intersects the pier axis at some point below, depending on the position of the load. For the position at the crown at which the load produces the greatest thrust, the resultant cuts the pier axis not far from the springing line. The average position of H, therefore, could be assumed to be at the springing line without great error.

Also, the displacement that effects the value of the horizontal thrust in the rib is not the displacement at B but that at B' which lies on the horizontal gravity axis of the rib. On account of the bending of the pier, the displacement at B' is greater than at B. It would be proper, therefore, to assume the thrust, H', to act at B, the thrusts, ΔH, to act at B' and to compute the displacement at B'. However, the displacement at B' produced by H' acting at B is the same as the displacement at B produced by H' acting at B'. It would, therefore, give the same results to use the displacement at B produced by H' at B' and $2\,\Delta H$ at B. For the sake of simplicity the thrust, $2\,\Delta H$, is also assumed to act at B', but this does not introduce a great error as ΔH is comparatively small. This error tends to compensate that introduced by assuming the thrust, H', to act at the springing line.

The thrust on the pier from the loaded span will be $(H' + \Delta H)$ and will be assumed to act along the horizontal gravity axis of the arch rib. The thrust from the unloaded span is $-\Delta H$, which likewise acts along the horizontal gravity axis of the corresponding rib. For the purpose of determining the deflection of the pier, the moment at any section of the pier will be,

$$M = (H' + 2\,\Delta H)\,(h + b - y')$$

The horizontal displacement at the top of the pier, B, due to bending of the pier will then be,

$$\delta_B = \int_B^D M\,(h - y')\, d\, w = (H' + 2\,\Delta H) \int_B^D (h + b - y')\,(h - y')\, d\, w$$

The horizontal displacement due to the elasticity of the foundation will be,

$$M_D\, h\, \varepsilon = (H' + 2\,\Delta H)\,(h + b)\, h\, \varepsilon$$

TABLE 12.—EXAMPLE II. COMPUTATION OF STRESSES.

(Comparison of Influence Lines for Examples I, II, and IIa, Is Given in Figs. 18, 19, and 20.)

$\frac{x}{l}$.	H_A'.	H_A''.	H_A.	V_A'.	V_A''.	V_A.	V_B'.	V_B''.	V_B.	M_D.	H_D.
1.0	0.000	0.000	0.000	0.500	0.5000	1.000	0.000	0.000	0.000	0.000	0.000
0.9	0.096	0.036	0.132	0.489	0.489	0.978	0.011	0.011	0.022	7.17	0.112
0.8	0.324	0.202	0.526	0.457	0.454	0.911	0.043	0.046	0.089	25.69	0.405
0.7	0.587	0.373	0.960	0.408	0.393	0.801	0.097	0.102	0.199	46.1	0.743
0.6	0.795	0.510	1.305	0.332	0.325	0.657	0.108	0.175	0.343	61.8	1.050
0.5	0.876	0.558	1.434	0.250	0.243	0.493	0.500	0.500	1.000	67.2	1.150
0.4	0.795	0.540	1.335	0.168	0.168	0.331	0.382	0.357	0.669	59.8	1.079
0.3	0.587	0.419	1.006	0.097	0.094	0.191	0.408	0.406	0.809	42.6	0.898
0.2	0.324	0.255	0.579	0.043	0.042	0.085	0.457	0.458	0.915	22.2	0.510
0.1	0.096	0.096	0.192	0.011	0.011	0.022	0.489	0.489	0.978	6.1	0.191
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.500	1.000	0.0	0.000
0.1	0.096	−0.096	0.000	0.011	0.011	0.000	0.489	0.489	1.000	0.0	0.000
0.2	0.324	−0.255	0.069	0.043	0.042	0.001	0.457	0.458	0.999	−5.1	0.191
0.3	0.587	−0.419	0.168	0.097	0.094	0.003	0.408	0.406	0.997	−23.2	0.510
0.4	0.795	−0.540	0.253	0.168	0.168	0.005	0.382	0.337	0.995	−42.6	0.898
0.5	0.876	−0.558	0.318	0.250	0.248	0.007	0.250	0.257	0.993	−59.8	1.079
0.6	0.795	−0.510	0.285	0.332	0.325	0.007	0.168	0.175	0.993	−67.2	1.150
0.7	0.587	−0.372	0.215	0.403	0.398	0.005	0.097	0.102	0.995	−61.8	1.050
0.8	0.324	−0.202	0.122	0.457	0.454	0.003	0.043	0.046	0.997	−46.1	0.743
0.9	0.096	−0.056	0.040	0.489	0.489	0.000	0.011	0.011	1.000	−25.69	0.405
1.0	0.000	0.000	0.000	0.500	0.500	0.000	0.000	0.000	1.000	−7.17	0.112
										0.000	0.000

TABLE 12.—(*Continued*).

$\frac{x}{l}$.	M_A'.	M_A''.	M_A.	M_B'.	M_B''.	M_B.	M_C'.	M_C''.	M_C.
1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.9	3.19	3.63	6.82	0.76	0.28	0.00	0.15	0.04	0.19
0.8	3.36	4.67	8.03	2.31	0.70	3.01	0.39	0.08	0.47
0.7	1.59	3.97	5.56	3.69	0.71	4.40	0.37	0.35	0.72
0.6	0.94	3.21	4.15	4.10	0.30	4.37	0.58	1.30	1.88
0.5	4.07	0.80	4.87	3.94	0.30	4.24	0.46	1.30	1.76
0.4	1.21	0.28	1.49	1.59	0.30	1.89	0.58	0.23	0.81
0.3	1.80	0.01	1.81	0.36	0.30	0.66	0.27	0.16	0.43
0.2	1.57	0.71	2.28	1.59	0.30	1.89	0.39	0.17	0.56
0.1	0.76	0.00	0.76	0.94	0.30	1.24	0.15	0.00	0.15
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.76	0.71	1.47	3.19	0.30	3.49	0.15	0.17	0.32
0.2	3.31	1.57	4.88	3.36	0.30	3.66	0.39	0.16	0.55
0.3	3.60	1.80	5.40	1.59	0.30	1.89	0.58	1.30	1.88
0.4	1.07	1.24	2.31	0.94	0.30	1.24	0.46	1.30	1.76
0.5	0.94	0.33	1.27	1.07	0.30	1.37	0.58	0.23	0.81
0.6	1.59	0.81	2.40	2.81	0.71	3.52	0.37	0.35	0.72
0.7	3.36	3.97	7.33	0.76	0.70	1.46	0.39	0.08	0.47
0.8	3.19	3.63	6.82	0.00	0.28	0.28	0.15	0.04	0.19
0.9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Adding this to the previous expression for displacement, the total due to elasticity of pier and foundation will be,

$$\delta_B = (H' + 2\,\Delta\,H)\left[\int_B^D (h + b - y')\,(h - y')\,d\,w + (h + b)\,h\,\varepsilon\right]$$

Placing,

$$\left[\int_B^D (h + b - y')\,(h - y')\,d\,w + (h + b)\,h\,\varepsilon\right] = K$$

$$\delta_B = H'\,K + 2\,\Delta\,H\,K$$

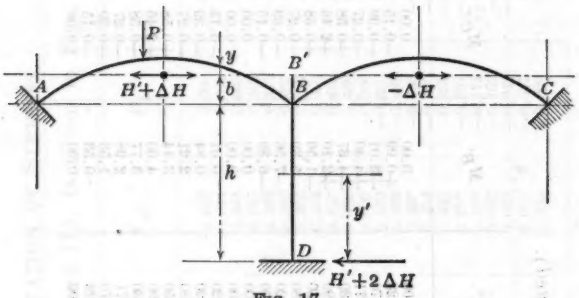


FIG. 17.

From consideration of the arch rib it was found that,

$$\delta_B = -\,\Delta\,H\,(1 + u)\int_B^C y^2\,d\,w = -\,\Delta\,H\,(1 + u)\int_A^B y^2\,d\,w$$

Then, as the end of the rib is attached to the end of the pier, these displacements are the same, and,

$$-\,\Delta\,H\,(1 + u)\int_A^B y^2\,d\,w = H'\,K + 2\,\Delta\,H\,K$$

from which,

$$\Delta\,H = -\,H'\,\frac{K}{(1 + u)\int_A^B y^2\,d\,w + 2\,K} \qquad \ldots\ldots\ldots\ldots (32)$$

The value of K reduces to,

$$K = h\,(h + b)\int_D^B d\,w - (2\,h + b)\int_D^B y'\,d\,w + \int_D^B (y')^2\,d\,w + (h + b)\,h\,\varepsilon \qquad \ldots\ldots\ldots\ldots (33)$$

The value of the integrals can be taken from Table 6.

The moment at the top of the pier will be approximately,

$$M_E = M_B' + 2\,\Delta\,H\,b \qquad \ldots\ldots\ldots\ldots (34)$$

and at the foot of the pier,

$$M_D = M_B' + 2\,\Delta\,H\,b + (H' + 2\,\Delta\,h)\,h$$
$$= M_B' + H'\,H + 2\,\Delta\,H\,(h + b) \qquad \ldots\ldots\ldots\ldots (35)$$

M_B' and H' are the values for the fixed-end arch.

Numerical Example of Approximate Method, Two Spans.—This approximate solution will be applied to the system of Example II so that the results may be compared with those given by the more exact method. It was found that,

$$\frac{1}{2}\int_A^B y^2\,dw = \frac{1\,487}{E}, \text{ or } \int_A^B y^2\,dw = \frac{2\,974}{E}$$

Also,

$$E\int_D^B dw = 0.710$$

$$E\int_D^B y'\,dw = 25.6$$

$$E\int_D^B (y')^2\,dw = 1\,123$$

Then,

$$\begin{aligned} K = +\,60 \times 71.85 \times 0.71 \div E &= +\,3\,060 \div E \\ -\,131.85 \times 25.6 \quad \div E &= -\,3\,375 \div E \\ &\quad +\,1\,123 \div E \\ \text{Sum} &= +\quad 808 \div E \end{aligned}$$

and,

$$\Delta H = -H'\frac{808}{2\,974 + 1\,616} = -0.176\,H'$$

$$H = 0.824\,H'$$

For a load at the crown,

$$H = 0.824 \times 1.752 = 1.443$$

The value obtained by the accurate method was 1.434. The calculation of influence line ordinates for the moments and horizontal thrust are given in Table 13. The vertical reactions are assumed to be the same as for the fixed arch.

The influence lines found by this approximate method are plotted in Figs. 18, 19, and 20, and show a fairly close agreement with those obtained by the other method. It should be noted that the proportions of the system analyzed are unusually slender. The pier is unusually high and the arch ribs have a low rise; also the width of the pier has been taken the same as that of the arch ribs. If separate ribs are used the pier will be wider than the ribs and the relative stiffness of the pier will be increased. As the relative stiffness of the pier increases and its effect on the stresses in the rib decreases, the accuracy of the foregoing approximate method increases. It is suggested that the value of $\frac{h + b - s}{b}$ might be used to indicate the magnitude of the effect of the elasticity of the pier. In Example No. 1,

$$\frac{h + b - s}{b} = \frac{7.45}{11.85} = 0.628$$

TABLE 13.—EXAMPLE II, APPROXIMATE METHOD.—COMPUTATION OF STRESSES.

(Comparison of Influence Lines for Examples I, II, and II (*a*), Is Given in Figs. 18, 19, and 20.)

$\frac{x}{l}$.	H_A'.	ΔH_A.	H_A.	M_A'.	ΔM_A.	M_B'.	ΔM_B.	M_B.	M_C'.	ΔM_C.	M_C.	$H_A' h$.	$2 \Delta H (h+b)$.	M_D.*	H_D.
1.0	0.000	0.000	0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	00.0	0.0	00.0	0.000
0.9	0.191	0.084	0.157	−0.38	−0.40	−1.52	−0.40	−1.12	−0.31	0.11	−0.20	11.5	4.9	08.1	0.123
0.8	0.648	0.114	0.534	−2.72	−1.35	−4.61	−1.35	−3.26	−0.77	0.36	−0.41	38.9	16.4	27.1	0.420
0.7	1.174	0.207	0.967	−5.18	−2.45	−7.38	−2.45	−4.93	−1.16	0.65	−0.51	70.5	29.8	48.1	0.760
0.6	1.590	0.280	1.310	−7.87	−3.32	−8.14	−3.82	−4.32	−1.16	0.84	−2.04	95.4	40.3	63.2	1.080
0.5	1.751	0.308	1.443	−8.19	−3.65	−6.19	−3.65	−2.54	−1.16	0.97	−5.80	105.6	44.3	68.2	1.185
0.4	1.590	0.280	1.310	−7.87	−3.32	−1.87	−3.32	−4.82	−1.16	0.88	−2.04	95.4	40.3	57.0	1.080
0.3	1.174	0.207	0.967	−5.18	−2.45	−3.18	−2.45	−7.38	−0.53	0.65	−0.08	70.5	29.8	37.5	0.760
0.2	0.648	0.114	0.534	−2.72	−1.35	−6.72	−1.35	−8.07	−0.77	0.36	−0.41	38.9	16.4	15.8	0.420
0.1	0.191	0.084	0.157	−1.53	−0.40	−6.88	−0.40	−6.78	−0.31	0.11	−0.20	11.5	4.9	0.2	0.123
0.0	0.000	0.000	0.000	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	00.0	0.0	00.0	0.000
0.0	.....	0.084	0.084	.....	0.00	.....	0.00	0.00	.....	0.00	0.00	00.0	0.0	00.0	0.000
0.1	.....	0.114	0.114	.....	0.40	.....	0.40	0.40	.....	0.11	0.11	11.5	4.9	− 0.2	0.123
0.2	.....	0.207	0.207	.....	1.35	.....	1.35	1.35	.....	0.36	0.36	38.9	16.4	− 15.8	0.420
0.3	.....	0.280	0.280	.....	2.45	.....	2.45	2.45	.....	0.65	0.65	70.5	29.8	37.5	0.760
0.4	.....	0.308	0.308	.....	3.32	.....	3.32	3.32	.....	0.88	0.88	95.4	40.3	57.0	1.080
0.5	.....	0.280	0.280	.....	3.65	.....	3.65	3.65	.....	0.97	0.97	105.6	44.3	61.5	1.185
0.6	.....	0.207	0.207	.....	3.32	.....	3.32	3.32	.....	0.88	0.88	95.4	40.3	63.2	1.080
0.7	.....	0.114	0.114	.....	2.45	.....	2.45	2.45	.....	0.65	0.65	70.5	29.8	48.1	0.760
0.8	.....	0.084	0.084	.....	1.35	.....	1.35	1.35	.....	0.36	0.36	38.9	16.4	27.1	0.420
0.9	.....	0.000	0.000	.....	0.40	.....	0.40	0.40	.....	0.11	0.11	11.5	4.9	08.1	0.123
1.0	.....	0.000	0.000	.....	0.00	.....	0.00	0.00	.....	0.00	0.00	00.0	0.0	00.0	0.000

* $M_D = H_A' h + M_B' - 2 \Delta H (h + b)$.

and in Example No. 2,

$$\frac{h + b - s}{b} = \frac{1.10}{11.85} = 0.093$$

The computation of this value will give some idea as to whether or not the approximate solution may be used.

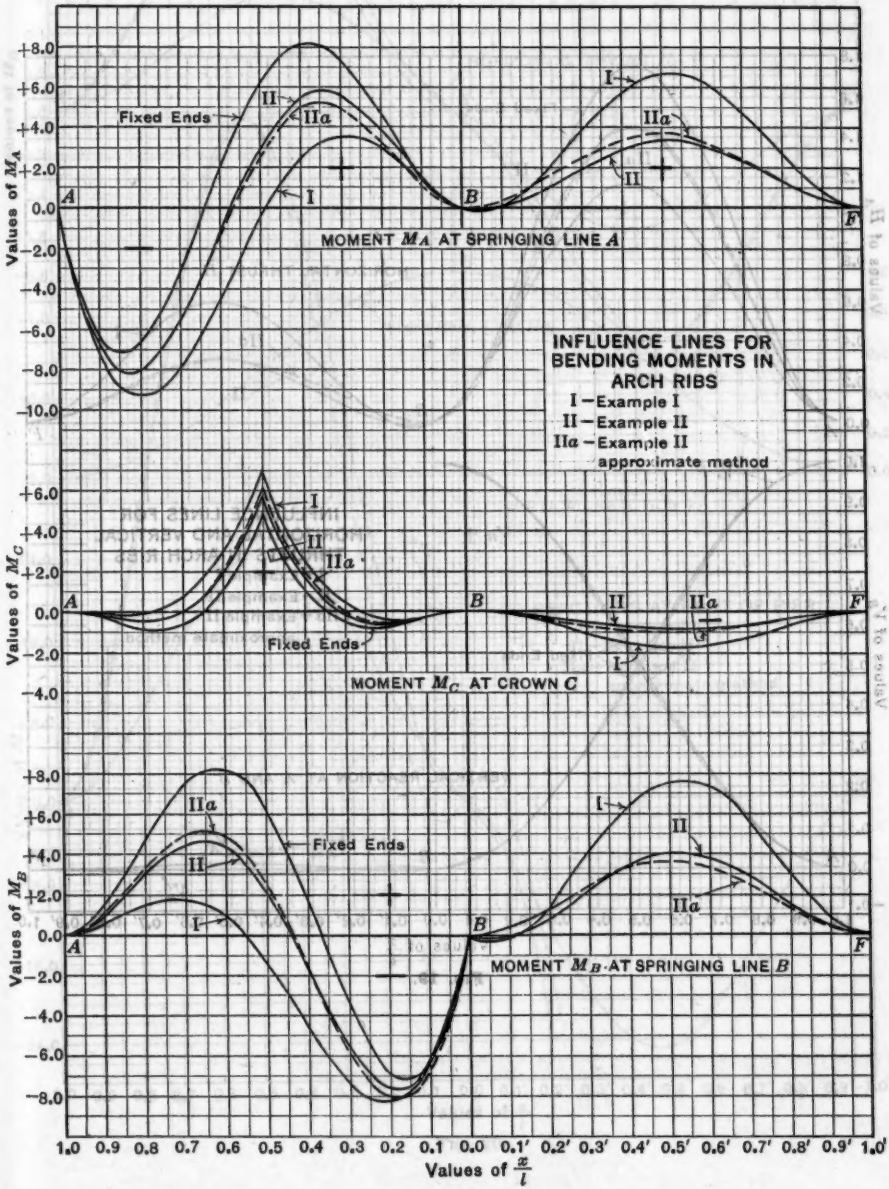


FIG. 18.

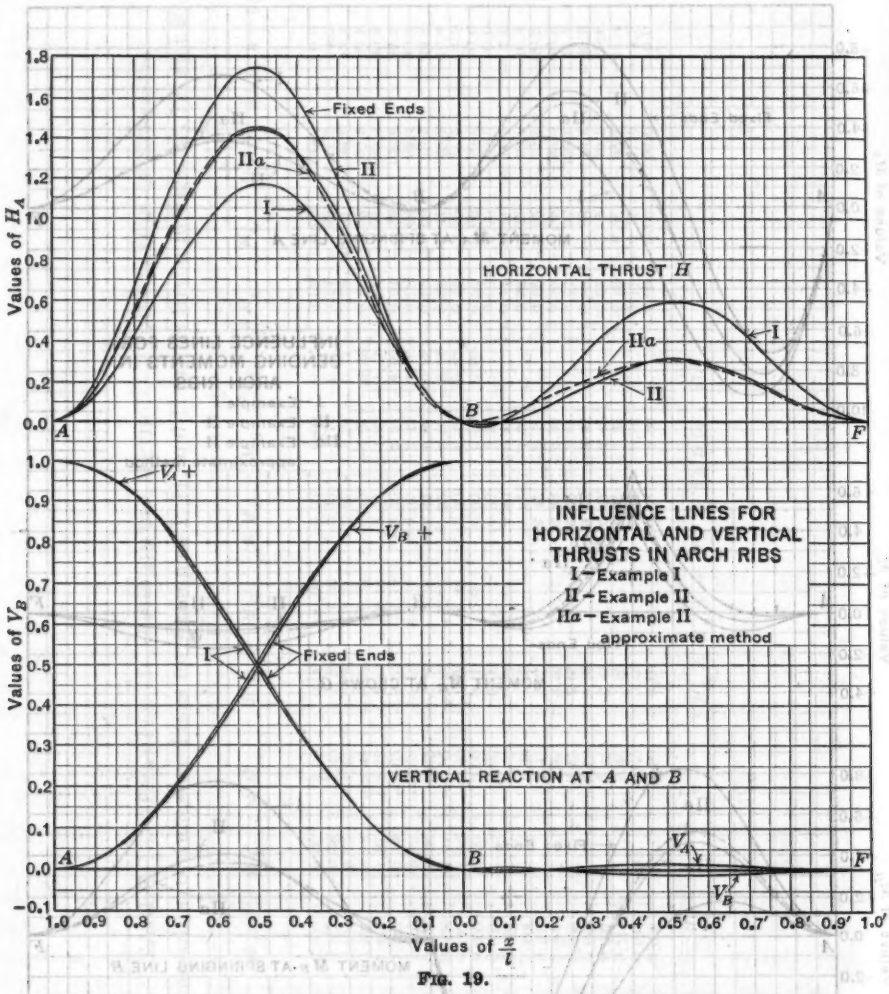

Values of H_A
Fixed Ends
IIa
II
I
HORIZONTAL THRUST H
A
B
F
Values of V_B
V_A +
V_B +
INFLUENCE LINES FOR HORIZONTAL AND VERTICAL THRUSTS IN ARCH RIBS
I – Example I
II – Example II
IIa – Example II approximate method
Fixed Ends
VERTICAL REACTION AT A AND B
V_A'
V_B'
Values of $\frac{x}{l}$

FIG. 19.

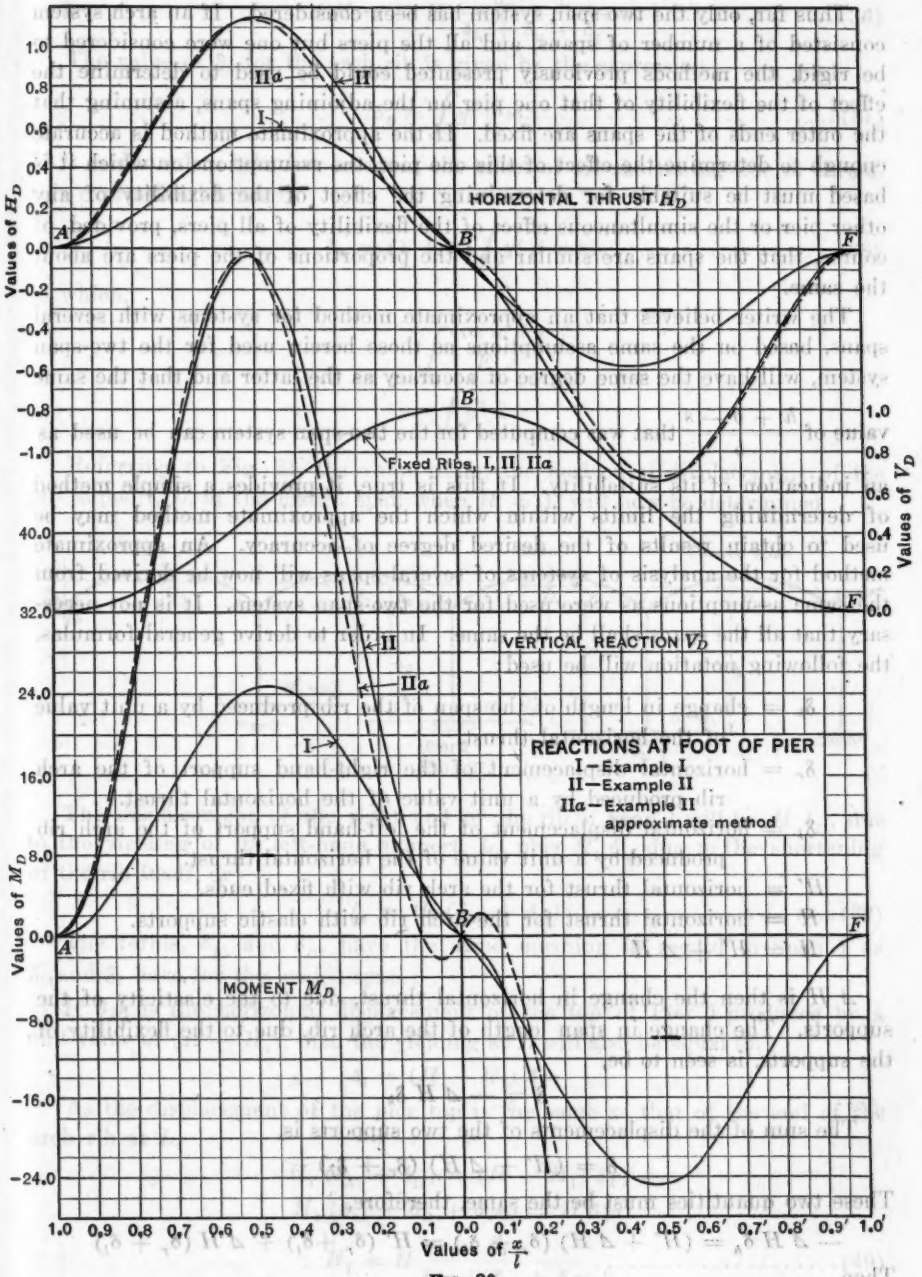

HORIZONTAL THRUST H_D
VERTICAL REACTION V_D
MOMENT M_D
Fixed Ribs, I, II, IIa
REACTIONS AT FOOT OF PIER
I—Example I
II—Example II
IIa—Example II approximate method
Values of H_D
Values of V_D
Values of M_D
Values of $\frac{x}{l}$
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0 0.1′ 0.2′ 0.3′ 0.4′ 0.5′ 0.6′ 0.7′ 0.8′ 0.9′ 1.0′

FIG. 20.

Approximate Method for Any Number of Spans

Thus far, only the two-span system has been considered. If an arch system consisted of a number of spans, and all the piers but one were considered to be rigid, the methods previously presented could be used to determine the effect of the flexibility of that one pier on the adjoining spans, assuming that the outer ends of the spans are fixed. If the approximate method is accurate enough to determine the effect of this one pier, the assumptions on which it is based must be suitable for determining the effect of the flexibility of any other pier or the simultaneous effect of the flexibility of all piers, provided, of course, that the spans are similar and the proportions of the piers are about the same.

The writer believes that an approximate method for systems with several spans, based on the same assumptions as those herein used for the two-span system, will have the same degree of accuracy as the latter and that the same value of $\frac{h + b - s}{b}$ that was computed for the two-span system can be used as an indication of its suitability. If this is true, it provides a simple method of determining the limits within which the approximate method may be used to obtain results of the desired degree of accuracy. An approximate method for the analysis of systems of several spans will now be derived from the same assumptions as were used for the two-span system. It is not necessary that all the spans shall be the same. In order to derive general formulas, the following notation will be used:

δ_h = change in length of the span of the rib produced by a unit value of the horizontal thrust.

δ_r = horizontal displacement of the right-hand support of the arch rib produced by a unit value of the horizontal thrust.

δ_l = horizontal displacement of the left-hand support of the arch rib produced by a unit value of the horizontal thrust.

H' = horizontal thrust for the arch rib with fixed ends.

H = horizontal thrust for the arch rib with elastic supports.

$H = H' + \Delta H$

ΔH is then the change in horizontal thrust, due to the elasticity of the supports. The change in span length of the arch rib, due to the flexibility of the supports, is seen to be,

$$\delta = -\Delta H \, \delta_h$$

The sum of the displacements of the two supports is,

$$\delta = (H' + \Delta H)(\delta_r + \delta_l)$$

These two quantities must be the same, therefore,

$$-\Delta H \, \delta_h = (H' + \Delta H)(\delta_r + \delta_l) = H'(\delta_r + \delta_l) + \Delta H (\delta_r + \delta_l)$$

Then,

$$-\Delta H (\delta_h + \delta_r + \delta_l) = H'(\delta_r + \delta_l)$$

and,

$$\Delta H = -H' \frac{\delta_r + \delta_l}{\delta_h + \delta_r + \delta_l} \quad \ldots\ldots (36)$$

The value of δ_h for the arch rib is given by the expression,

$$\delta_h = \int y^2 \, d\, w \quad \ldots\ldots (37)$$

If it is desired to take into account the effect of rib-shortening due to thrust, this expression becomes,

$$\delta_h = (1 + u) \int y^2 \, d\, w \, * \quad \ldots\ldots (38)$$

in which,

$$u = \frac{\int \frac{\cos \varphi \, d\, x}{E\, A}}{\int y^2 \, d\, w}$$

Referring to Fig. 21, the value of δ_l, the horizontal displacement of the left support, L, of the loaded span when $H = 1$, will now be determined.

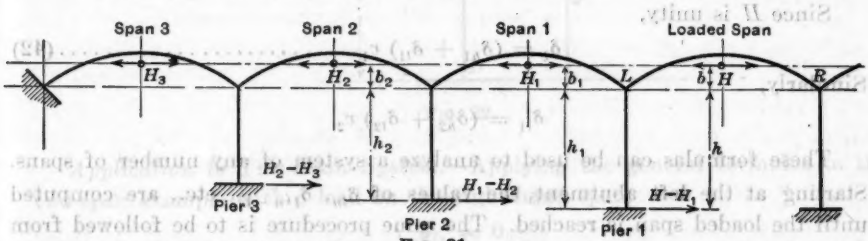


FIG. 21.

The displacement of the right end of the rib of Span 1 will be $H\, \delta_{l1}$, due to the yielding of its left-hand support, L_1, plus $H_1\, \delta_{h1}$ due to the shortening of the rib itself, or

$$\delta_l = H_1 (\delta_{h1} + \delta_{l1}) \quad \ldots\ldots (39)$$

The terms, δ_{h1} and δ_{l1}, have the same meaning in regard to Span 1 as δ_h and δ_l have for the loaded span.

If δ_{P1} is the horizontal displacement of the top of Pier 1 produced by a unit value of $(H - H_1)$, then the yielding at L will also be equal to,

$$\delta_l = (H - H_1)\, \delta_{P1}$$

As the displacement of the pier top is the same as that of the end of the arch rib at L,

$$H_1 (\delta_{h1} + \delta_{l1}) = (H - H_1)\, \delta_{P1}$$

$$H_1 (\delta_{h1} + \delta_{l1} + \delta_{P1}) = H\, \delta_{P1}$$

$$H_1 = H \frac{\delta_{P1}}{\delta_{h1} + \delta_{l1} + \delta_{P1}} \quad \ldots\ldots (40)$$

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 945, Equation (20*a*).

Similarly,

$$H_2 = H_1 \frac{\delta_{P2}}{\delta_{h2} + \delta_{l2} + \delta_{P2}}$$

Placing,

$$\frac{\delta_{P1}}{\delta_{h1} + \delta_{l1} + \delta_{P1}} = v_1$$

and,

$$\frac{\delta_{P2}}{\delta_{h2} + \delta_{l2} + \delta_{P2}} = v_2, \text{ etc.}$$

$$\left.\begin{aligned} H_1 &= H\, v_1 \\ H_2 &= H_1\, v_2 = H\, v_1\, v_2 \\ H_3 &= H_2\, v_3 = H\, v_1\, v_2\, v_3 \end{aligned}\right\} \quad \ldots\ldots (41)$$

If all spans and piers are identical, v_1, v_2, etc., are all the same, and,

$$H_n = H\, v^n \quad \ldots\ldots (41a)$$

Combining Equations (41) and (39),

$$\delta_l = H_1 (\delta_{h1} + \delta_{l1}) = H (\delta_{h1} + \delta_{l1})\, v_1$$

Since H is unity,

$$\delta_l = (\delta_{h1} + \delta_{l1})\, v_1 \quad \ldots\ldots (42)$$

Similarly,

$$\delta_{l1} = (\delta_{h2} + \delta_{l2})\, v_2$$

These formulas can be used to analyze a system of any number of spans. Starting at the left abutment, the values of δ_{ln}, $\delta_{l(n-1)}$, etc., are computed until the loaded span is reached. The same procedure is to be followed from the right abutment to obtain the value of δ_r. The value of H is then determined and the thrusts in the unloaded spans are computed successively back to the abutments.

The value of δ_{P_1} is derived as follows: The forces acting on the pier are shown in Fig. 22. When $(H - H_1) = 1$, the moment at any section of the pier is,

$$M_O = (h + b - y')$$

The displacement of the pier-head will be (considering also the elasticity of foundation),

$$\int_B^D M_O\, (h - y')\, d\, w + M_D\, h\, \varepsilon = \int_B^D (h + b - y')\, (h - y')\, d\, w + (h + b)\, h\, \varepsilon$$

which was placed equal to K. (See Equation (33).)

Then, $\delta_{P_1} = K_1$ and $\delta_{P_2} = K_2$, etc., and the equations for v_1, etc., become,

$$\left.\begin{aligned} v_1 &= \frac{K_1}{\delta_{h1} + \delta_{l1} + K_1} \\ v_2 &= \frac{K_2}{\delta_{h2} + \delta_{l2} + K_2}, \text{ etc.} \end{aligned}\right\} \quad \ldots\ldots (43)$$

After the horizontal thrusts have been obtained, the bending moments are found in the same manner as that used for the solution of the two-span system:

$$H = H' + \Delta H$$

$$M = M' - \Delta Hy.$$

Vertical reactions are assumed to be the same as for fixed ribs.

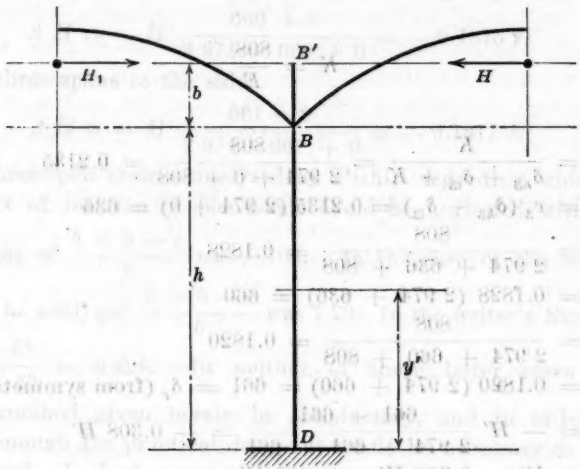


FIG. 22.

Application to Two-Span System.—Applying the general formulas to the two-span example with a load on the right-hand span,

$$\delta_{l_1} = 0$$

and,

$$v_1 = \frac{K_1}{\int y^2\, d\,w + 0 + K_1}$$

$$\delta_l = v_1 (\delta_{h1} + \delta_{l1}) = \frac{K_1 \int y^2\, d\,w}{\int y^2\, d\,w + K_1}$$

As the right support of the loaded span is fixed,

$$\delta_r = 0$$

Then,

$$\Delta H = -H' \frac{\delta_l}{\int y^2\, d\,w + \delta_l} = -H' \frac{K_1}{\int y^2\, d\,w + 2\,K_1}$$

which is the same as Equation (32).

Numerical Example.—The general method will now be applied to a system of seven spans having ribs and piers the same as those of Example II. All

values of $\int y^2\,d\,w$ and K are equal. The thrust produced by a load on the center span will be determined.

It was found that,

$$\int y^2\,d\,w = \frac{2\,974}{E}$$

and,

$$K = \frac{808}{E}$$

Then,

$$v_3 = \frac{K}{\delta_{h3} + \delta_{l3} + K} = \frac{808}{2\,974 + 0 + 808} = 0.2135$$

$$\delta_{l2} = v_3\,(\delta_{h3} + \delta_{l3}) = 0.2135\,(2\,974 + 0) = 636$$

$$v_2 = \frac{808}{2\,974 + 636 + 808} = 0.1828$$

$$\delta_{l1} = 0.1828\,(2\,974 + 636) = 660$$

$$v_1 = \frac{808}{2\,974 + 660 + 808} = 0.1820$$

$$\delta_l = 0.1820\,(2\,974 + 660) = 661 = \delta_r \text{ (from symmetry)}$$

$$\Delta H = -H' \frac{661 + 661}{2\,974 + 661 + 661} = -0.308\,H'$$

$$H = H' - 0.308\,H' = 0.692\,H'$$

$$H_1 = v_1 H = 0.1820 \times 0.629\,H' = 0.126\,H'$$

$$H_2 = v_2 H_1 = 0.1828 \times 0.126\,H' = 0.023\,H'$$

$$H_3 = v_3 H_2 = 0.2135 \times 0.023\,H' = 0.0049\,H'$$

From this it is seen that the effect on the end span is negligible. If there were five spans in the system, the thrust in the center span when loaded would be:

$$H = H' - H' \frac{660 + 660}{2\,974 + 660 + 660} = 0.6925\,H'$$

With three spans, it would be

$$H = H' - H' \frac{636 + 636}{2\,974 + 636 + 636} = 0.7005\,H'$$

This shows that, for a system with piers of the relative proportions assumed in the example, the horizontal thrust is little affected by the consideration of more than one span on each side of the loaded span. The value of ΔH is $-0.2995\,H'$ for the three-span system as against $-0.176\,H'$ for the two-span system, so that one might expect the errors introduced by neglecting the effect of the rotation of the pier-heads to be greater. It should be noted, however, that the thrust resisted by the pier in the former case is smaller than in the latter. There would, therefore, be less bending of the pier in the former case and the error would not be proportional to the value of ΔH. With somewhat stiffer piers, the value of ΔH would be smaller and the effect of the second span from the loaded span would also be reduced.

The conclusion drawn by the writer from this is that for any system to which this approximate method is applicable, it is necessary to consider the effect of only one span on each side of the loaded span, except when the end span is analyzed, in which case the effect of the two spans at one side should be considered.

For the end span loaded, considering only one additional span, ΔH was found to be $-0.176\,H'$. Considering two additional spans, it would be,

$$\Delta H = -H' \frac{660 + 0}{2\,974 + 660 + 0} = -0.1815\,H'$$

Considering three spans to the side,

$$\Delta H = -H' \frac{661 + 0}{2\,974 + 661 + 0} = -0.1817\,H'$$

For the three-span system analyzed by Pichl,* regarding which he states that the effect of rotation of pier-heads could be neglected without serious error, the value of $\frac{h + b - s}{b}$ was 0.198. In the case of the more flexible system which he analyzed, $\frac{h + b - s}{b}$ was 1.79. In the writer's Example I, the value was $\frac{7.45}{11.85} = 0.628$. In neither of these latter cases would the approximate method given herein be satisfactory, and in order to obtain results close enough for practical design it would be necessary to use a more accurate method. In both cases, the pier bases were considered fixed. The rotation of the pier about the base does not introduce as great an error as the bending of the pier itself, because the center of rotation is farther from the end of the rib and the horizontal displacement is proportionately larger. Therefore, if the effect of rotation of the pier base is considerable, a large value of $\frac{h + b - s}{b}$ is not necessarily an indication that the approximate method here presented is not suitable. This method might give very accurate results for a system with tall stiff piers on frictionless hinges at the base, although in that case the value of s would be equal to $(h + b)$.

For such a case the value of s computed with ϵ placed equal to zero would be some indication of the size of error introduced. The approximate method fails to take into consideration the bending moments produced in the arch ribs by the rotation of the pier-head; when $(h + b - s)$ is small, this rotation is slight.

Conclusion

The writer believes that the data in Tables 1 to 6 make the application of the accurate method presented herein for the analysis of two-span arch systems so simple that the use of an approximate method is unnecessary, except perhaps for the preparation of a preliminary design. This accurate method also provides an easy way to study the effect of the elasticity of a single pier of varying proportions so that the designer may form an idea of its im-

* "Der durchgehende gelenklose Bogen auf elastischen Stützen," by Ernst Pichl.

portance and may judge the accuracy of the approximate methods when applied to systems of several spans. The influence which the elasticity of the piers will have on the design of the arch ribs will depend on the relative importance of live and dead loads, as well as on the proportions of the piers and ribs, so that the preliminary design will be largely a matter of judgment. A fixed-end rib design may be used as a basis and the thickness modified to suit the particular condition.

As all arch-rib design is a matter of trial, the writer feels that the methods of this paper and of the one previously published* are of great value, because they permit a complete analysis to be made without designing the rib in each case. It is merely necessary to establish the span, rise, and loading and select suitable values of m, N, I_c, and I_s, which then fix all the characteristics of the rib. The design is completed after its suitability is determined. This is particularly advantageous in the case of multiple-arch systems in which the characteristics of a number of spans are involved.

The approximate method presented will probably be sufficiently accurate for most practical cases, but the designer should determine in each instance whether or not it is suitable. The test suggested by the writer, consisting of the computation of the value of $\frac{h + b - s}{b}$, should be helpful in this respect if properly applied. The large number of variables involved in bridge design makes it inadvisable to draw too definite conclusions, but the writer hopes that a discussion by engineers who have had experience in the design of multiple-span arch bridges will help to establish the limits within which the approximate method may be used.

For the analysis of systems of more than two spans when the piers are very flexible, or when there is doubt as to the sufficiency of the approximate method, the writer would suggest the use of the method given by Hardy Cross, M. Am. Soc. C. E.,† in connection with one similar to that presented by J. Melan.‡ Professor Melan gives a method of analysis of single-span ribs with elastic supports and Professor Cross shows how to determine the elastic properties of the supports of any rib in a series of any number of spans. Using the elastic characteristics of arch ribs as presented by the writer, these methods can be applied to obtain an accurate solution without an unreasonable amount of labor.

For the analysis of three-span systems the writer has developed a means of design in the same way as for two spans by the accurate method. This requires considerably more labor than the two-span solution, but is probably as simple as any other method by which full influence lines for three spans can be obtained.

The writer is indebted to Mr. W. J. Fuller, of the Extension Division of the University of Wisconsin, for checking over the manuscript.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 981.

† *Loc. cit.*, p. 1196.

‡ "Plain and Reinforced Concrete Arches," by J. Melan, translated by D. B. Steinman, M. Am. Soc. C. E., p. 71.

FIELD PROCEDURE OF ADJUSTING THE GREAT CIRCLE LINE TO THE RHUMB LINE

By N. B. Sweitzer,* Assoc. M. Am. Soc. C. E.

Synopsis

Technical literature on the various branches of surveying is rich on the subject of the geodetic problems of establishing various lines on the earth's surface, comprising boundaries, to connect points of known latitude and longitude. The methods of procedure for the actual field operations, however, are not summarized and for the field engineer to compile these results for actual field use covering all phases of this question is a complicated process. To summarize selectively for the needs of the field engineer, to supply shorter and simpler formulas giving results well within the limits of admissible error, and to supply data for field use, is the object of this paper.

Many boundary lines are rhumb lines varying from the parallel of latitude through the quadrant until they merge into the meridian. Ordinarily, such lines were first run by instruments with a solar attachment, or by the needle, but retracements for extreme accuracy are now generally run by the engineer's transit. To convert this great circle or transit line to the rhumb line, or loxodromic curve, a short practical method has been needed. The basis of these calculations is the geodetic proposition involving the lines connecting latitude and longitude positions. These are simplified by the use of the theories governing railroad curves, with all unnecessary refinements eliminated, care being taken that the limit of error in the calculations would be so small that it would not show in the field projections. The formulas given contain all data required in running a rhumb line by the use of the transit or great circle line and also for calculating the latitude and longitude arcs of the earth within the limits of field operation.

National boundaries as well as sub-divisions thereof, and even boundaries as minute as farms and city lots, in the great majority of cases, were originally surveyed and marked on the ground and their bearings platted on their respective maps, giving the courses as of record, and referred to the rhumb line (loxodromic curve), that is, to a line making equal oblique angles with all meridians. The final recorded plats of many railroads, canals, highways, etc., are also thus given. Calculations for closures, areas, etc., are also based on this curve; and the surveyor or engineer, especially in cadastral work, is continually making his calculations on this line as a basis. One of the main reasons for adopting the rhumb line so generally was on account

Note.—Written discussion on this paper will be closed in September, 1926. When finally closed the paper, with discussion in full, will be published in *Transactions*.

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of the instruments in wide use in former years and the consequent formulas and tables adopted for proper reduction and calculations based on this system. The first of these instruments for finding direction or bearing was the magnetic compass which ran an approximate rhumb line. About the middle of the Nineteenth Century, the solar compass came into use; this was a vast improvement in accuracy over the needle compass and ran a true rhumb curve. By the aid of these two instruments practically all the cadastral surveys in the United States were made and recorded on the official or commercial plats as rhumb lines.

On account of inaccuracies frequently caused by the inferiority of the magnetic compass in recording true bearings and also by carelessness in handling the solar compass, a review of early surveys often shows large errors in closure or in areas. This was, originally, not considered particularly detrimental on account of the small monetary value of lands, but as values increased and restorations for lost corner monuments became imperative a new element was introduced into cadastral work—the harmonizing of the error.

These increased land values demanded better and closer field work and better instruments. The legal side in regard to the re-location or re-establishment of these lost corners of boundary lines demanded an exact closure as regards area. The result was a proper legal and scientific treatment of the error.

This naturally brought prominently to the front the improved engineer's transit. In railroad and other engineering work of magnitude this instrument has always been the main reliance for the projection of lines and its supremacy for cadastral work of prime accuracy is now acknowledged. The engineer's transit, however, runs exclusively a great circle line, and in cadastral restoration work this will have to be referred to the rhumb line as originally run by the engineer or surveyor, who placed monuments that must be restored in their original positions.

The requirement to-day is to run a great circle line with the engineer's transit and to transfer this line on the ground by means of offsets and proper intersection angles to the original survey which was run as a rhumb line on the ground and recorded as such on the plats. Engineering literature has data and methods for running one kind of a rhumb line only, namely a parallel of latitude by the tangent or secant method, but there are many variations of a rhumb line from the parallel in practice, and the writer has been unable to find a general method treating all positions or courses of this line, and capable of use in field practice. The general rigid methods given in geodetic surveying for azimuth, with latitude and longitude known, are too involved for a rapidly moving field party.

Leonard S. Smith, M. Am. Soc. C. E., suggests the following:

$$\phi - \phi' = (-d\phi) = B K \cos Z + C K^2 \sin^2 Z + D h^2$$

$$\lambda - \lambda' = (+d\lambda) = A K \sin Z \sec L'$$

$$Z + 180° - Z = -dZ = D \phi \sin \tfrac{1}{2}(\phi + \phi')$$

$$A' = \frac{1}{N' \text{ arc } 1''}; \quad B = \frac{1}{R' \text{ arc } 1''}; \quad C = \frac{\tan \phi'}{2 R' N' \text{ arc } 1''}$$

$$D = \frac{\frac{3}{2} e^2 \sin \phi' \cos \phi' \sin 1''}{(1 - e^2 \sin^2 \phi')}; \quad m\,\phi = \text{mean latitude} = \frac{\phi + \phi'}{2}$$

in which,

h = $B\,K \cos Z$;
ϕ = latitude of a known point;
ϕ' = latitude of a second or unknown point = $\phi + d\,\phi$;
$d\,\phi$ = $\phi' - \phi$;
λ = longitude of a known point;
λ' = longitude of a second or unknown point = $\lambda + d\,\lambda$;
$d\,\lambda$ = $\lambda' - \lambda$;
Z = azimuth of the unknown point from the known, counting from the south point in the direction S. W. N. E.;
Z' = azimuth of the known point from the unknown, or the back azimuth = $Z + d\,Z + 180°$;
$d\,Z$ = $Z' - Z - 180°$;
K = length, in meters, of line joining the two points;
e = eccentricity of the earth's meridian section;
N = length of the normal, or radius of curvature of a section perpendicular to the meridian of the middle latitude, in meters;
R = radius of curvature of the meridian, in meters.

This formula is not exactly rigid, but is sufficiently accurate for the most exacting field engineer. The main trouble is that the geodesists have not wholly agreed on the form of the earth and give various values for its flattening or ellipticity, and, apparently, the arcs in the same latitude vary in different longitudes, and conversely. However, the surveyor is not directly interested in this question, as the magnitude of his operations will not be such as to cause his work to be materially affected by these discrepancies.

The tangent method of locating a parallel has its limitations. The main objection against it is that it departs from the parallel rapidly—as the square of the distance—but for short lines it is good, because of its freedom from complications. The secant method is accurate, but introduces many unnecessary complications. It is never tangent to the parallel, contact being dependent on accurate measurement and alignment; it is first south, then north, then south again of the parallel, all in six miles, and this is likely to cause mistakes in the setting of offsets, thus doubling errors. The variation of lengths of the offsets is difficult to remember and the general problem is more involved than is necessary.

The variation of the tangent method here offered is practically the same as that used on railroad work. The only difference is that instead of using the ordinary functions of the railroad curve, the latitude and longitude radii and the normals of the earth are substituted. Basically, the parallel of latitude is taken. A 6-mile tangent is assumed for a given latitude and the intersection angle calculated by the following equation:

$$\tan \phi \times 5'.2,$$

or,

$$d\lambda \sin (m\phi) = \text{curvature or intersection angle}$$

The offsets, in links, from the tangent to parallel are obtained by the equation:

$$1.01 \times (d\lambda)^2 \times \tan \phi$$

in which, $d\lambda$ is difference of longitude, in miles, $m\phi$ is the mean latitude, and ϕ is the latitude.

Having thus secured the curvature or intersection angle for 6 miles and calculated the offsets for each ½ mile (the first 3 miles of offsets are all that are required as the offsets of the other 3 miles repeat in inverse ratio), obtain by an observation, preferably on Polaris, a true meridian. Then, commencing at *A*, as in Fig. 1, turn an angle of 90° east and run on a tangent with offsets as follows: ½ mile, no offset; 1 mile, 1 link north to parallel; 1½ miles,

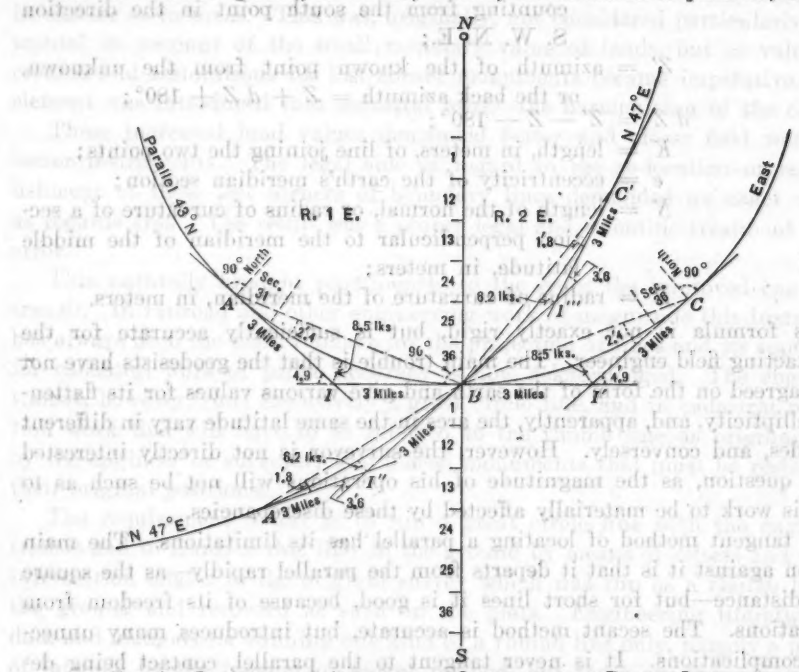

FIG. 1.—PROJECTING A PARALLEL OF LATITUDE OR A RHUMB LINE FROM A TRANSIT OR GREAT CIRCLE LINE.

2 links north; 2 miles, 4 links north; 2½ miles, 6 links north; and at 3 miles, 8½ links north to parallel. At this point, turn a deflection angle of 4'.9 to the left and run on a tangent 3 miles, setting the offsets in inverse order always north. The next 3-mile point will be tangent to the parallel and will be due east, the same as at the starting point, and a back-sight of one-half the intersection angle, or 2'.4, will intersect the starting point, 6 miles westward. Repeat this procedure indefinitely if it is required to prolong the parallel, or, as in Fig. 1, 6 miles farther to *C*. A great many of the old lines to be relocated on the ground, given on the plats as east or west, vary in reality considerably from these cardinals. Manifestly, the intersection angles and offsets of such a line will differ from those given for a parallel.

In Fig. 1, assume the parallel to be revolved about B as an axis, to occupy the position, $A'\,B'\,C'$, and assume the course of $A'\,B'\,C'$ to be N. 47° E. Its mean latitude will still be 43° N., and, without involving any excessive irregularities, the intersection angles and offsets may be written:

$$\sin\theta\,(\tan\phi \times 5'.2) = \text{intersection angle, 6-mile tangent}$$

or,

$$(d\lambda \sin\phi_m)\sin\theta = \text{intersection angle}$$

$$\sin\theta\,(1.01 \times D^2 \times \tan\phi) = \text{offsets in links from tangent}$$

in which, θ = course of line, ϕ = latitude, D = distance, in miles, $m\,\phi$ = mean latitude, and $d\,\lambda$ = difference in longitude.

Table 1 for Latitudes 33° to 49°, giving offsets in links, deflection angles, tangential angles, and difference of longitude for 6 miles for the given latitudes, will cover the latitude of the United States.

TABLE 1.—Offsets in Links, Deflection Angles, Etc., Latitude 33° to 49°.*

Latitude.	Miles.						Deflection angle.	Tangential angle.	6 Miles difference in longitude for:
	½	1	1½	2	2½	3			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
33°	0	1	1	3	4	6	0°3′ 23″	0°1′ 41″	0°6′ 12″
35°	0	1	2	3	4	6	0°3′ 38″	0°1′ 49″	0°6′ 21″
37°	0	1	2	3	5	7	0°3′ 55″	0°1′ 57″	0°6′ 31″
39°	0	1	2	3	5	7	0°4′ 13″	2′ 06″	0°6′ 41″
41°	0	1	2	4	5	8	0°4′ 31″	2′ 15″	0°6′ 53″
43°	0	1	2	4	6	8½	0°4′ 51″	2′ 25″	0°7′ 06″
45°	0	1	2	4	6	9	0°5′ 12″	2′ 36″	0°7′ 21″
47°	0	1	2	4	7	10	0°5′ 34″	2′ 47″	0°7′ 37″
49°	0	1	3	5	7	10	0°5′ 59″	2′ 59″	0°7′ 55″
	5½	5	4½	4	3½	3			
	Miles.								

* Multiply offsets and deflection angles by sine of course for rhumb line.

Interpolation can easily be made for any of the functions given, as they vary only slightly and are smaller than the least reading of the verniers of the field instruments. These functions, multiplied by the sine of the course of a rhumb line of the same mean latitude, will give the corresponding functions of that line. For a long line the total curvature is equal to the sum of the intersection angles, which will restore at once the desired meridian and will enable the engineer to check accidental or accumulated errors by check observations.

Back-sights or back azimuths are one-half the sum of the intersection angles, or the sum of the tangential angles. A 6-mile tangent is assumed in this example for convenience, as its apex distance is small and is kept near its objective—the parallel or rhumb line—but a tangent of any greater length

could be assumed and calculated by the formulas, as well as the new intersection angle and offsets.

Column (10) in Table 1, "Difference of Longitude", can be rigidly calculated by an equation in use by the Naval Observatory. Referring to Fig. 1:

Let, $A\ B\ C$ = parallel of latitude, in 43° N., for 12 miles.
$A'\ B'\ C'$ = rhumb line (N. 47° E.), in mean latitude 43° N., for 12 miles.
$N\ S$ = meridian through Point B.
I, I', I'', I''' = intersection angles, tangents, 3 miles.
C, C' = tangential angles, one-half intersection angles.
Offsets from tangent to parallel, in links = $1.01 \times D^2 \times \tan \phi$.
D = distance, in miles.
ϕ = latitude.
λ and λ' = longitudes.
Intersection angle, parallel = $\tan \phi \times 5'.2$.

Referring to the rhumb line:

Offsets from tangent to curve = $\sin \theta\ (1.01 \times d^2 \times \tan \phi)$.
θ = course of line.
Intersection angle = $\sin \theta\ (\tan \phi \times 5'.2)$.
Total curvature of line = Σ intersection angles.
Back-sights = Σ tangential angles.
Curvature of meridians = $(\lambda - \lambda') \sin \frac{1}{2} (\phi + \phi')$.
Length of 1° longitude = $69.3 \times \cos \phi \pm$ (statute miles).*
1° Lat. (statute miles) = $69.0569 - 0.3494 \cos 2\phi + 0.0007 \cos 4\phi$.
1° Long. (statute miles) = $69.2316 \cos \phi - 0.0584 \cos 3\phi + 0.0001 \cos 5\phi$.

This is very close for $\phi = 49°$; for $\phi = 30°$, use the factor, 69.23.

This formula is also true of the difference for latitude, namely, 1° longitude, in statute miles, is,

$$69.2316 \cos \phi - 0.0584 \cos 3\phi + 0.0001 \cos 5\phi.$$

In using this formula, care should be taken with the signs of the latitudes. It can be written for the ordinary field practice: Length of a degree in longitude = $69.30514 \cos \phi$ = statute miles, for Latitude 49° N. and $69.23125 \cos \phi = 1°$ (statute miles) for Latitude 30° N. For intermediate latitudes, by interpolation for a new factor between 69.30514 and 69.23125, a close approximation can be secured, close enough for any ordinary field use and far within the least count of the field instrument. It will be wise to check the longer formula by the shorter one, as it is more or less involved. The latitude is not so important as the values do not change rapidly; it can be taken from any good map or obtained by an observation on Polaris at the upper or lower culmination.

In presenting this problem, the main idea has been to simplify complicated mathematical formulas and involved solutions. This is exemplified in the equation for the intersection angle, $\tan \phi \times 5'.2$, which eliminates the necessity

* Interpolate.

for solving the difference of longitude. This study is confined to a tangent of 6 miles, which is naturally the most convenient one to use and is rigid up to 70° latitude, where it varies only 2″ in arc.

The main appeal of this method is to the field engineer who will probably recognize something for which he has been looking. A search through engineering or surveying literature reveals apparently nothing bearing distinctly on this subject, except as previously mentioned, nor, of course, on the reductions for latitude, longitude, azimuth, and back azimuth in geodetic surveying, especially as it applies to primary and secondary traverses and reductions for triangulations. The methods here described are too involved for the ordinary field surveyor's use and for engineering work which requires rapid progress with a minimum of office calculations, at the same time bringing the field data to be placed on the ground well below the least count of the verniers. These calculations will do more than this, however, even if they are simplified as far as possible. It might be well for the field engineer to have a copy of Fig. 1 in his field book, as a glance at it will show him just how far he is departing on any transit line from his objective rhumb-line course, and consequently from his corner position. The course given as N. 47° E. at Latitude 43° N. in Fig. 1 will show (1) just how much the offsets and intersection angles vary from the parallel to the meridian; (2) where they will be at a minimum and finally disappear on the meridian; and (3) where they are at a maximum on the parallel. It should be remembered that the offsets vary nearly as the square of the distance on the tangent and that the intersection angles are proportional to the distance in any given latitude, and, therefore, for the longer tangents that these two functions will materially increase and depart from the objective rhumb line. This is especially important from a cadastral standpoint, as the rhumb line is the legal line on which the corners or monuments are situated.

In regard to the maximum error involved by the use of these short formulas for field purposes in preference to the more extended and laborious methods, it is well to remember that the rigid geodetic calculations are based on a theoretic ellipsoid of revolution which, of course, closely approximates the real form of the earth. The writer's object is to project a rhumb line which will represent, as nearly as field conditions render possible, the actual curve on the earth's surface within the limits of the latitude selected for operations, and to furnish the field parties with amply accurate and simplified formulas. These limits are from the Equator to 60° latitude. A comparison of the short formulas given herein with those developed by the rigid theoretic ellipsoid of revolution, shows a maximum error well within 1″ of arc for a rhumb line of 24 miles. This is a fairly long line for re-survey work between known original corners, for original work between monuments established by astronomical determination, or, again, for other work before check azimuth determinations are made. At 70° latitude this error is about 2″ of arc. As the engineer's transit seldom reads to less than single minutes of arc and as it is known that few lines of any length projected on the earth's surface actually are kept within the limit of even 1′ of arc on account of physical conditions, the calculations are far more exact than actual field operations can ordinarily achieve.

DEVELOPMENT OF INDUSTRIAL SITES AND TRACKAGE LAYOUTS AT KANSAS CITY, KANSAS, AND LOS ANGELES, CALIFORNIA, BY THE UNION PACIFIC SYSTEM*

WALTER R. ARMSTRONG,† M. AM. SOC. C. E.

SYNOPSIS

In this paper the writer has given brief descriptions of the Fairfax Industrial District at Kansas City, Kans., one of the eastern terminals of the Union Pacific System, and of the industrial districts at Los Angeles, Calif., one of the western terminals of the System.

INTRODUCTION

With the keen competition which enters into the business of the railway transportation systems of the country generally, and, more particularly, with the recent competition of motor vehicles, the railroad companies of the United States have recognized the necessity of providing suitable trackage for various industries, warehouses, and manufactories at their terminals and larger stations, to enable the industries to handle commodities transported in carloads with the least possible delay and at the least expense.

In line with this progressive policy, the Union Pacific System has encouraged the establishment of industrial trackage. At Kansas City and Los Angeles, as here described, it has invested in large areas of land contiguous to its railroad, and has planned thereon a system of trackage to accommodate most conveniently industries seeking locations.

KANSAS CITY, KANS.

With the development of Kansas City it has been apparent for some time that an industrial district should be added to the city, as well as districts for residences, retail business, and manufacturing of certain classes. With this end in view, the Union Pacific System, in line with its policy to aid in the development of worthy institutions within its jurisdiction, in co-operation with other enterprises, has established the Fairfax Industrial District, at Kansas City, Kans. The map (Fig. 1) shows its location north and east of the city, on a tract of land which many years ago was built up by the action of the Missouri River in depositing sands in what was formerly the old channel. Fig. 1 indicates how well the various tracts that have been laid out are to be served by a railroad.

NOTE.—Written discussion on this paper will be closed with the September, 1926, *Proceedings*. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

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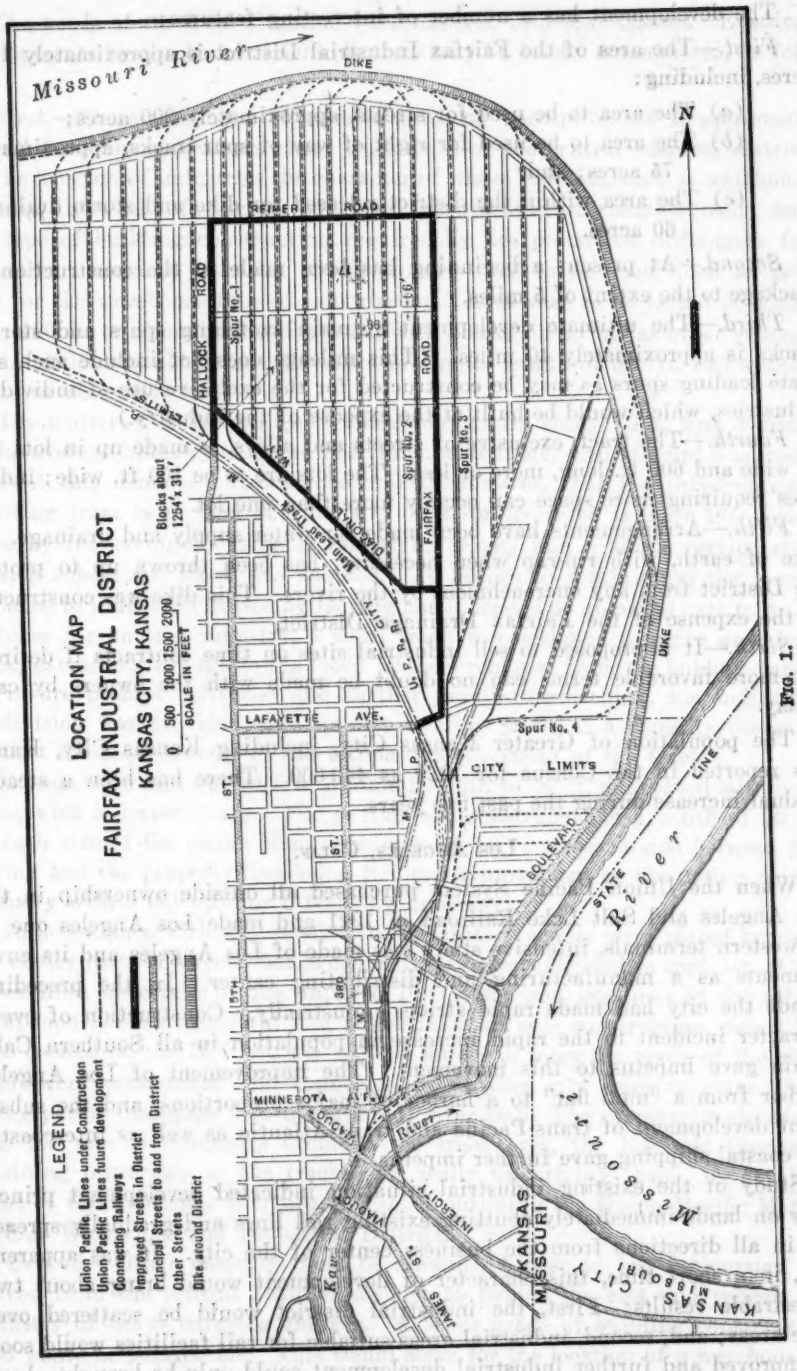
Missouri River
DIKE
N
REIMER ROAD
HALLOCK ROAD
Spur No. 1
Spur No. 2
Spur No. 3
FAIRFAX ROAD
Blocks about 1254' x 311'
Main Lead Track
Chicago
Union Pacific R.R.
LOCATION MAP
FAIRFAX INDUSTRIAL DISTRICT
KANSAS CITY, KANSAS
SCALE IN FEET
0 500 1000 1500 2000
LAFAYETTE AVE.
Spur No. 4
CITY LIMITS
STATE LINE
BOULEVARD
River
Missouri
MINNEOTA AVE.
Kaw River
WYANDOTTE
INTERCITY
KANSAS CITY MISSOURI
KANSAS MISSOURI
LEGEND
Union Pacific Lines under Construction
Union Pacific Lines future development
Connecting Railways
Improved Streets in District
Principal Streets to and from District
Other Streets
Dike around District

FIG. 1.

The development has a number of interesting features:

First.—The area of the Fairfax Industrial District is approximately 1 300 acres, including:

(*a*) The area to be used for streets, approximately 200 acres;
(*b*) The area to be used for right of way of spur-tracks, approximately 75 acres; and
(*c*) The area within the district released for dike and storm drainage, 60 acres.

Second.—At present a beginning has been made in the construction of trackage to the extent of 5 miles.

Third.—The ultimate development of main switching spurs and storage tracks is approximately 40 miles. (This mileage does not include such separate loading spurs as may be constructed for the exclusive use of individual industries, which would be built at the expense of the industry.)

Fourth.—The tract, exclusive of streets and alleys, is made up in lots 100 ft. wide and 600 ft. long, more or less. The lots are to be 100 ft. wide; industries requiring more space can occupy more than one lot.

Fifth.—Arrangements have been made for water supply and drainage. A dike of earth, with rip-rap when necessary, has been thrown up to protect the District from any encroachment by the rivers. This dike was constructed at the expense of the Fairfax Drainage District.

Sixth.—It is proposed to sell industrial sites on time contracts if desired, but more favorable terms can no doubt be made with the owners by cash outlay.

The population of Greater Kansas City, including Kansas City, Kans., was reported in the Census for 1920 as 440 000. There has been a steady, gradual increase during the past five years.

Los Angeles, Calif.

When the Union Pacific System purchased all outside ownership in the Los Angeles and Salt Lake Railroad in 1921 and made Los Angeles one of its western terminals, intensive study was made of Los Angeles and its environments as a manufacturing and distributing center. In the preceding decade the city had made rapid strides industrially. Construction of every character incident to the rapid increase in population in all Southern California gave impetus to this movement. The improvement of Los Angeles Harbor from a "mud flat" to a harbor of major proportions, and the subsequent development of trans-Pacific and trans-Atlantic as well as intercoastal and coastal shipping gave further impetus.

Study of the existing industrial situation indicated development principally on lands immediately abutting existing rail lines and gradually spreading in all directions from the business center of the city. It was apparent that, in a short time, this character of development would bring about two undesirable results: First, the industrial district would be scattered over large areas; and, second, industrial areas suitable for rail facilities would soon be improved and further industrial development could only be brought about at excessive cost.

The needs of the local situation were resolved into four classes. Ensuing developments by the Union Pacific System may be briefly summarized, as follows:

First.—There was need of close-in property susceptible of improvement by rail facilities and within short truck haul of the central business district for the location of industries the character of whose business, such as warehousing and light manufacturing, will bear the overhead of valuable realty and the type of building construction required by fire protection ordinances for down-town business districts. Such property was to be subdivided to provide for the warehouse type of construction. Lots were to be varied in width from 100 ft. to a maximum of 150 ft., with blocks, in so far as other governing conditions would permit, having a minimum length of 600 ft. and a maximum of about 800 ft.

The matter of industrial spurs was given careful study. The first demand of the average industry is for an individual spur-track. An attempt was made to meet this demand. It was determined, however, that the inconvenience resulting from more than one industry on a service track is more than overcome by the saving in valuable underlying realty and a single service spur to each subdivided block was adopted. To reduce to a minimum the inconvenience to industries caused by moving partly loaded or unloaded cars to make switches for another industry, a third track was designed to serve as a lead track, extending through to the block farthest removed from the holding yards. To insure prompt deliveries and further facilitate switching, the industrial subdivision was provided with its own holding yards of a capacity designed to meet the requirements of the ultimate development of the tract.

Streets were laid out with a minimum width of 60 ft. and paved with concrete, with a center drain. The paving was constructed to a width of 20 ft. on each side of the center line, an additional 10 ft. on each side between the paving and the property line being left for improvement at the option of the industry with a sidewalk or a trucking driveway.

In accordance with city requirements, sanitary sewers were placed beneath the center lines of streets, house connections being extended to each lot. City water lines were laid in one unpaved 10-ft. strip and gas lines in the other. Telephone and electric light and power lines on pole construction were likewise located on the 10-ft. strips. Storm sewers to meet the needs of the situation were constructed and connected to existing city lines or given their own separate discharge into the adjacent river bed. The property made available for sale to industries excluded the track area, the Railroad Company retaining ownership to the tracks and realty underlying them. A development of this character is shown on Fig. 2, outlining the Union Pacific Metropolitan Warehouse and Industrial District.

Second.—Other property must be reasonably close in but far enough removed from the central business district to reduce the cost and relieve the industry of heavy carrying charges. A lower grade of building construction also would be permissible. This would serve for the location of a warehousing or manufacturing business requiring convenient close contact with the central

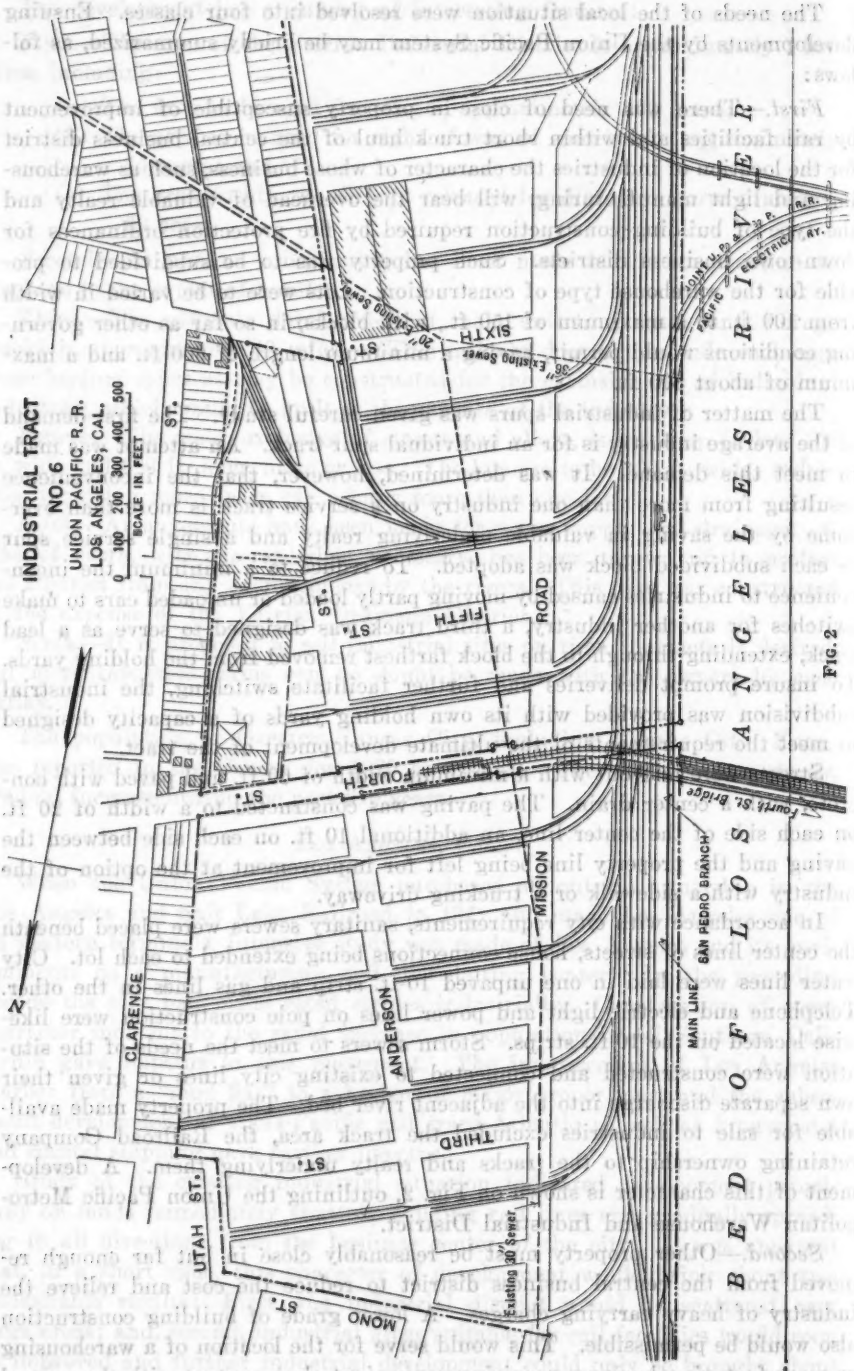
INDUSTRIAL TRACT
NO. 6
UNION PACIFIC R.R.
LOS ANGELES, CAL.
0 100 200 300 400 500
SCALE IN FEET
N
CLARENCE
ST.
UTAH ST.
ANDERSON
THIRD
ST.
MONO
ST.
Existing 30" Sewer.
MISSION
FOURTH
ST.
FIFTH
ST.
ROAD
SIXTH
ST.
20" Existing Sewer
36" Existing Sewer
BEDDO
LOS ANGELES RIVER
JOINT U. P. & P. R. R.
PACIFIC ELECTRIC RY.
SAN PEDRO BRANCH
MAIN LINE
Fourth St. Bridge

FIG. 2

business district, but unable to support the greater overhead of high land values and expensive building construction, as well as for industries requiring somewhat larger areas and open storage.

In planning the subdivision of this property, an effort was made to vary the width from about 210 ft. maximum to a minimum of 120 ft., with blocks having a minimum length of 600 ft., and subject to requirements of the city in conforming with existing dedicated streets. The layout was so made, as shown on Fig. 3. This subdivision is laid out with 60-ft. streets, paved with concrete as described in the preceding class, and is also furnished with sanitary sewers, storm sewers, high-pressure gas, city water, telephone, electric light, and power service. Service tracks, intermediate lead tracks, and holding yards are designed substantially as described for the Metropolitan Warehouse and Industrial District (Fig. 2), except that for this development the third, or intermediate, track was extended throughout the entire subdivision. This tract covers an area of 70 acres. An additional undeveloped area of about 170 acres, immediately adjacent to the tracks on the east, is held to protect future needs.

Third.—Another class comprised property carrying a still lower land value, well removed from the wholesale and retail business center, suited for the development of a class of industry requiring acreage, or for embryo industries covering small areas but in their early stages unable to sustain the overhead of the more expensive property in the preceding classes.

In selecting these locations, attention was given to direct and adequate highway access to the city and to surrounding towns, but of necessity, in order to secure cheap land, the locations were 5 or 6 miles removed from the heart of the business district. In this character of development, in order to hold the selling price as low as possible, improvements were held down to bare necessities. The properties selected, being located without the city limits, are at this time without sanitary sewers. It is contemplated that cesspools will take care of this feature until the development of projected county facilities affords better disposition. Water will be furnished in adequate quantities from a local private system.

The subdivision as designed will provide, on about one-half the area, for lots varying in width from 150 to 300 ft.; the lengths of blocks, in so far as conforming with orderly development, varying from a minimum of 800 ft. to a maximum of 1 200 ft. Streets were designed with a minimum width of 60 ft. Where a central street may be used as the main artery of trucking, for entrance and departure, an 80-ft. width is assigned. About one-half of each subdivision is allowed to remain without projected street and trackage development. This will later accommodate the large manufacturing or assembling plant requiring acreage, it being contemplated that development would be built around such requirements.

For trackage, a 17-ft. right of way is withheld between each subdivided street and a lead track constructed thereon. It is expected that owners will furnish their own spur-tracks to be located on their own right of way. Street paving, as provided, consists of a 24-ft. strip of concrete paving in the center

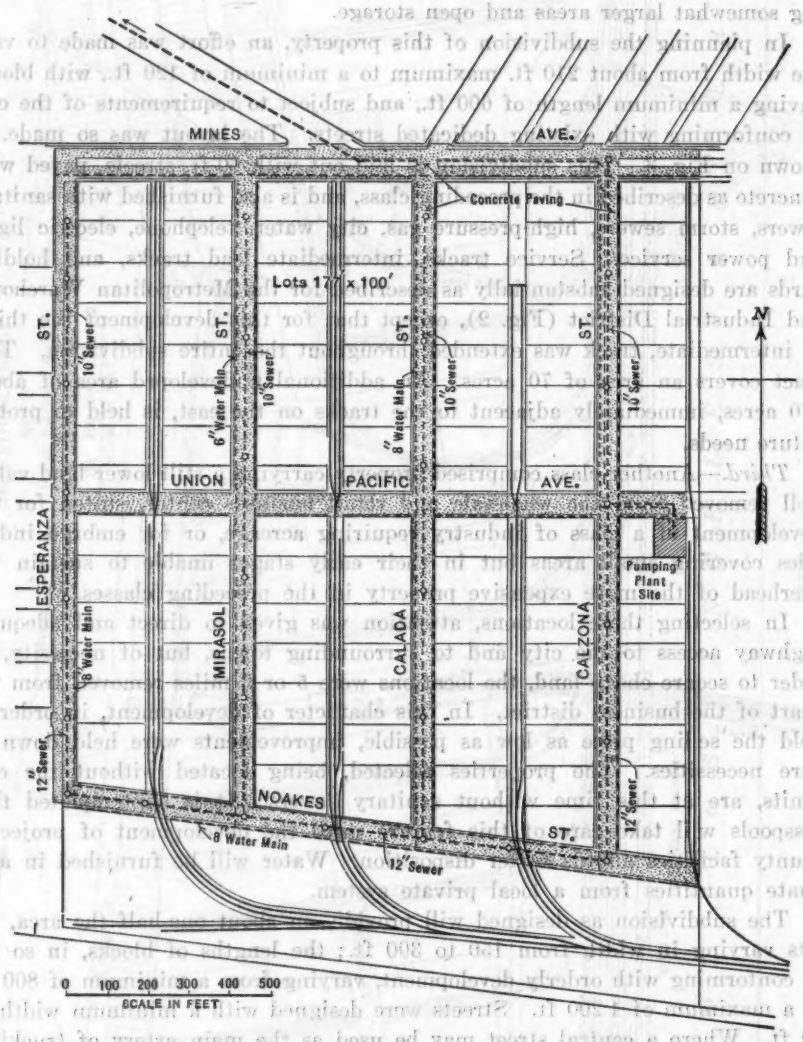

FIG. 3.—IMPROVEMENT OF INDUSTRIAL TRACT NO. 1, UNION PACIFIC SYSTEM, LOS ANGELES, CALIF.

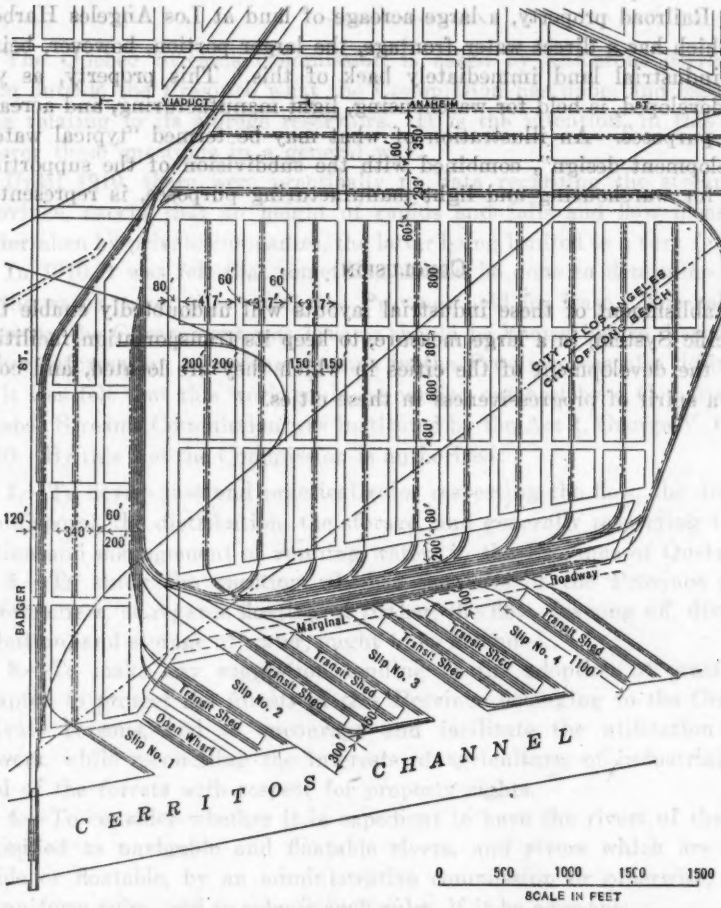

FIG. 4.—UNION PACIFIC WATER-FRONT AND INDUSTRIAL SUB-DIVISION, LOS ANGELES HARBOR.

of the street reservation. Industries may, at their option, extend the paving to their property lines. In each instance, gas mains are available within reasonable distance. It may become necessary later to construct service mains on each street. In order to hold the price down, this facility is not now so conveniently accessible to the owner. Electric power and light, and telephone service are provided.

Fourth.—The Union Pacific System acquired with the Los Angeles and Salt Lake Railroad property, a large acreage of land at Los Angeles Harbor, some of which has a direct water frontage, the larger portion, however, being potential industrial land immediately back of this. This property, as yet almost undeveloped, is held for warehousing, light manufacturing, and acreage industrial purposes. An illustration of what may be termed "typical waterfront development design", combined with the subdivision of the supporting back area for warehousing and light manufacturing purposes, is represented in Fig. 4.

Conclusion

The establishment of these industrial layouts will undoubtedly enable the Union Pacific System, in a large measure, to keep its transportation facilities abreast of the development of the cities in which they are located, and contribute to a spirit of progressiveness in these cities.

THE WORK OF THE QUEBEC STREAMS COMMISSION*

By O. Lefebvre,† Esq.

The Quebec Streams Commission is asked by numerous correspondents from outside the Province what the Commission has done, and especially for data relating to its storage reservoirs. It is the intention, in this paper, to answer these questions in a general way.

Until 1910, there were practically no data regarding the streams of the Province, except that on height of rapids and falls and flow measurements undertaken by private companies, the latter being limited to a very few streams.

In 1910 it was felt that something should be done to determine the value of the water power sites which the Province held for lease. The value of the property the Government was leasing should be known before disposing of it. Other information regarding streams in the Province was also desirable, and as it was felt that this work could be carried out best by a Commission, The Quebec Streams Commission was instituted by the Act 1, George V, Chapter 5, 1910. By this Act the Commission is authorized:

1.—To devise just and practical rules respecting the flow, the drawing off, the disposal, the distribution, the storage, and generally respecting the preservation and management of running waters in the Province of Quebec.

2.—To study the question whether the laws of the Province governing watercourses, as regards floating of timber, the flow, drawing off, disposal, distribution, and storage of water, ought to be amended.

3.—To make any suggestion tending to the adoption of practical rules adapted to protect the forests of this Province belonging to the Crown or to private persons, and to encourage and facilitate the utilization of water powers, while reconciling the interests of agriculture, of industrial pursuits, and of the forests with respect for property rights.

4.—To consider whether it is expedient to have the rivers of the Province classified as navigable and floatable rivers, and rivers which are not navigable or floatable, by an administrative commission or otherwise, according to uniform rules, and to submit such rules, if it be advisable.

5.—To make any inspection and examinations of the lakes, rivers, ponds, creeks, and watercourses in this Province, which it may think necessary.

6.—The Commission shall proceed with all possible diligence to carry out the purposes for which it is constituted. In matters not provided for by this Act, it shall be guided by instructions from the Lieutenant-Governor in Council.

Note.—Written discussion on this paper will be closed with the September, 1926, *Proceedings*. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

* Presented at the Fall Meeting, Montreal, Que., Canada, October 14, 1925.

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7.—The Commission shall report to the Lieutenant-Governor in Council as to its researches, as well as its recommendations and suggestions. This report shall be laid before the Legislature during the first ten days of the next session.

Members of the Commission, however, were not appointed until December, 1911. It was the good fortune of the Government to appoint as the first Chairman of the Commission, the Hon. S. N. Parent, who had previously been Prime Minister of the Province of Quebec, a man of great business ability, who realized fully the responsibility of the task which was entrusted to him and his associates. He was ably assisted in his work by two engineers, W. I. Bishop, M. Am. Soc. C. E., and the late Mr. Ernest Bélanger.

The present Commission has at its head the Hon. J. A. Tessier, Arthur Amos, Chief of the Hydraulic Service, Department of Lands and Forests, and S. F. Rutherford, Engineer. The Commission commenced its work in 1912, and has issued annual reports giving details of its activities.

In 1912, the Commission was in the organization stage and made numerous investigations as to conditions of different streams, more particularly flood conditions, log-driving difficulties, and the storage possibility on the St. Maurice River.

In 1913, the Commission's work was becoming quite important, and an Engineering Staff was appointed, with the writer as its Chief Engineer. Immediately, systematic flow measurements of the principal streams of the Province were organized, together with the collection of more meteorological data, as well as numerous investigations of storage possibilities here and there.

Stream Measurements

In 1913, a systematic organization for flow measurements was initiated. On the principal streams gauges were established and read daily, sometimes three or four times a day. The flow of the streams was measured and related to the gauge height, wherever possible. The number of stations, and the number of streams investigated, have increased from year to year and data are now available for nearly all the principal streams of the Province. Since 1922, the work of stream measurement has been carried out by the Dominion Water Power Branch, Department of the Interior, Léo. G. Denis, District Engineer, with headquarters in Montreal. This was made possible through an agreement with the Commission, whereby the two services co-operate with each other, to the advantage of the Province.

Data on gauge heights are received monthly at the office of the Commission, tabulated, and made available for distribution to those who may be interested. A copy of this information is furnished to the Dominion Water Power Branch, which supplies the Commission with the results of the meterings.

Generally speaking, it may be stated that the average yearly run-off from the streams is about 1.6 sec-ft. per sq. mile of drainage area. The ordinary maximum run-off usually occurs in the spring and varies from 6 to 10 sec-ft. on the larger streams to 20 and 25 sec-ft. on the smaller streams, the ratio between minimum and maximum flow being about 1 to 25. The minimum

flow takes place in winter, usually in March, and reaches 0.25 to 0.40 sec-ft. per sq. mile of drainage area, depending on the characteristics of the streams.

Flood flows much larger than the maximum just given have been recorded, especially during 1924, and the figures which prevailed during this period should be of interest. On the Ste. Anne de la Pérade River, a discharge of 43 000 sec-ft. was recorded at St. Alban; the drainage area is 672 sq. miles, which is a rate of 64 sec-ft. per sq. mile. On the Ste. Anne de Beaupré River, at the Laurentian Power Company's plant, where the drainage area is 400 sq. miles, the discharge was 32 000 sec-ft., or 80 sec-ft. per sq. mile. At Lake Kenogami, in the Saguenay District, the run-off was 20 sec-ft. per sq. mile for a water-shed of 1 400 sq. miles.

Meteorological Data

The collection of data regarding rainfall and temperature has been followed for a number of years. For the Cities of Montreal and Quebec, data are available from 1871 to date.

The work of collecting meteorological data has been initiated by the Dominion Meteorological Service of Canada, a branch of the Department of Marine and Fisheries of Ottawa, with headquarters at Toronto, under the direction of Sir Frederick Stupart, who is known as the Director of Meteorological Service of Canada.

The Quebec Streams Commission took a great deal of interest in this work, and suggested a large number of new stations. It has offered its full co-operation to the Federal Service. There are now 78 stations throughout the Province, where rainfall and temperature are recorded. The data are sent monthly to the Commission and transmitted to the Federal Office. This information is tabulated and put in proper form for distribution to engineers. The Provincial Government, through the Department of Lands and Forests, pays the observers, and many stations are in charge of power companies and manufacturing firms, the co-operation of which is greatly appreciated. The Streams Commission supervises the collection of this information and follows closely the work of the observers who are being impressed with its value and the importance of recording it properly.

It may be of interest to state that the average annual rainfall in the Province is about 36 in., including snow. The average snowfall is about 100 in., which is considered to be equivalent to 10 in. of rainfall. The run-off may be assumed as averaging 50% of the rainfall. This percentage may reach 70 in very wet years and 35 to 40 in dry years.

Great difficulty is experienced in having the snowfall properly recorded. The best method is to have a small platform laid horizontally in the bush, protected against the action of the wind. This condition is one that is not always possible, and, in many instances, the figures given are not reliable, when the snowfall is accompanied by wind.

Power Sites

The Quebec Streams Commission has investigated, for the Department of Lands and Forests, a great number of water power sites covering all districts in the Province, from Abitibi in the northwest, to the Gaspé Peninsula

in the Gulf of St. Lawrence, including the lower sections of the streams on the north shore of the River St. Lawrence. Numerous surveys and plans have been made of the power sites on the principal streams, namely, the Quinze River in the Ottawa District, the St. Maurice and St. Francis Rivers in the central section, the streams tributary to Lake St. John, and the principal streams of the north shore. In other cases, only sketch plans have been made.

In every instance, a plan for the proper development was submitted with an estimate of the power available and the probable cost. This information is given to the Department of Lands and Forests, at Quebec.

One point should be made clear: A great many people have an idea that the Quebec Streams Commission has the control of water powers in the Province. That is an error. The Commission investigates the value of power sites; it does not negotiate for the lease of such sites. Its report is placed in the hands of the Minister of Lands and Forests, and the Hydraulic Service is the body through which one must negotiate for the acquisition of any power site from the Crown.

To July 1, 1925, the Commission has spent $646 942.11 for these investigations and studies. A large part of this sum will eventually be refunded by the parties leasing power sites. A clause of the lease stipulates that the money spent by the Commission for surveys, plans, etc., must be refunded. All special problems regarding extraordinary conditions on any river have been referred to the Commission for investigation and report, such, for example, as the great floods on the Chaudière River, and their possible prevention.

The principal work of the Commission, however, is the construction of storage reservoirs, in order to regulate the flow of streams. A system of storage has been constructed on the following streams: the St. Maurice River; the St. Francis River on the south shore, in that section known as the Eastern Township; the Ste. Anne de Beaupré River; the Mitis River; and Lake Kenogami. Storage propositions have been investigated and authorized on other streams, such as the North River, north of Montreal, River du Loup (upper), and the Ste. Anne de la Pérade River, but no work has been commenced.

Legislation

Although the Act appointing the Commission gives that body authority to investigate conditions on any of the streams, a Special Act of the Legislature has to be passed giving the Commission the authority to construct dams and regulate the flow of certain streams and tributaries. A separate Act has to be passed for each individual plan. Such an Act gives the Commission authority, subject to approval by the Lieutenant-Governor in Council, to construct dams, acquire existing dams necessary for its work, buy all lands to be flooded, either by mutual agreement or by expropriation, to build new roads, if required, and to replace flooded highways. To pay the cost of all these works, the Provincial Treasurer is authorized to supply the Commission with the necessary funds, the total amount not to exceed a figure mentioned in the Act. He may borrow money for the purpose, or he may take it out of

ordinary revenue. These Acts are practically all worded in the same way, and a copy of the latest, that for Lake Kenogami, is attached hereto. (See Appendix I.)

Before any storage project is authorized by the Lieutenant-Governor in Council, the Commission has to supply the Government with a complete report relating to the proposed works, their estimated cost, the companies likely to benefit by the works, and, if possible, the extent to which each company will benefit.

The Act gives the Commission authority to collect annual dues from the benefiting companies. It may impose a tariff. However, it is preferred that the annual charge be agreed to between the parties who bind themselves by long-term contract.

No storage plan is undertaken until the Commission is assured of a yearly revenue sufficient to pay the interest charges, the sinking fund in thirty years, and the cost of operating and maintaining the dams, plus a profit whenever possible, which introduces the question of the determination of rates.

As previously stated, all companies, associations, or persons benefiting from the storage works of the Commission are called upon to pay an annual charge proportionate to the amount of benefits derived. In many instances, the determination of this amount is quite a problem owing to lack of information on conditions prevailing before any storage work is undertaken.

In the case of the St. Maurice River, complete flow data were available for fourteen years, thanks to the initiative of the Shawinigan Water and Power Company, and the benefits could be calculated exactly at each power site. The usual procedure of the frequency curve was followed, and the additional power reduced to a basis of the horse-power year.

For the St. Francis River, no flow data were available and it was not possible, therefore, to establish with certainty the number of additional horse-power years each company could generate under the controlled conditions. It was then thought that each company should be charged according to the head under which its plant was operated. However, under the storage plan comprising two separate reservoirs, all companies do not receive the same quantity of water, one of them being located between the two reservoirs and getting water from the upper reservoir only.

Another method, therefore, had to be adopted. It is called the "second-feet-head" method, which has been in use on the Wisconsin River. Multiplying the number of cubic feet per second supplied at each plant by the head under which the water is used, gives the "second-feet-head". The annual charge which is intended to be levied is divided proportionately to the number of "second-feet-head". On the St. Francis River, the rate is $0.50 per sec-ft.-head. On the St. Maurice River, when new power sites are developed, the owners have to pay for the storage, and this means added revenue to the Commission. In the case of the St. Francis River, the rate may be revised every five years, if new developments are carried out, and the companies should then enjoy a reduction.

On the other streams, the revenues of the Commission are limited to the percentage of the capital cost, and any new development means a reduction

in the amount paid by the present companies. This condition is to prevail for thirty years.

Storage Dams

General information will be given on the different storage dams which were built by the Commission, as follows:

Gouin Dam.—Gouin Dam, shown Fig. 1, is on the St. Maurice River, 240 miles from the mouth. The dam permits a total control of the water supplied by the drainage area of 3 650 sq. miles. A series of large lakes, at different levels, have been combined into one lake forming a reservoir of a total capacity of 160 000 000 000 cu. ft., or 5 725 sq. mile-ft. The dam, which is of the gravity section type, was completed in December, 1917, and has been operated since the beginning of 1918. The stored water is used to regulate the minimum flow of the St. Maurice River, at Shawinigan. When the dam was built, the minimum guaranteed to the power companies was 12 000 sec-ft. For the last three years, the minimum flow has been held to 16 000 sec-ft.

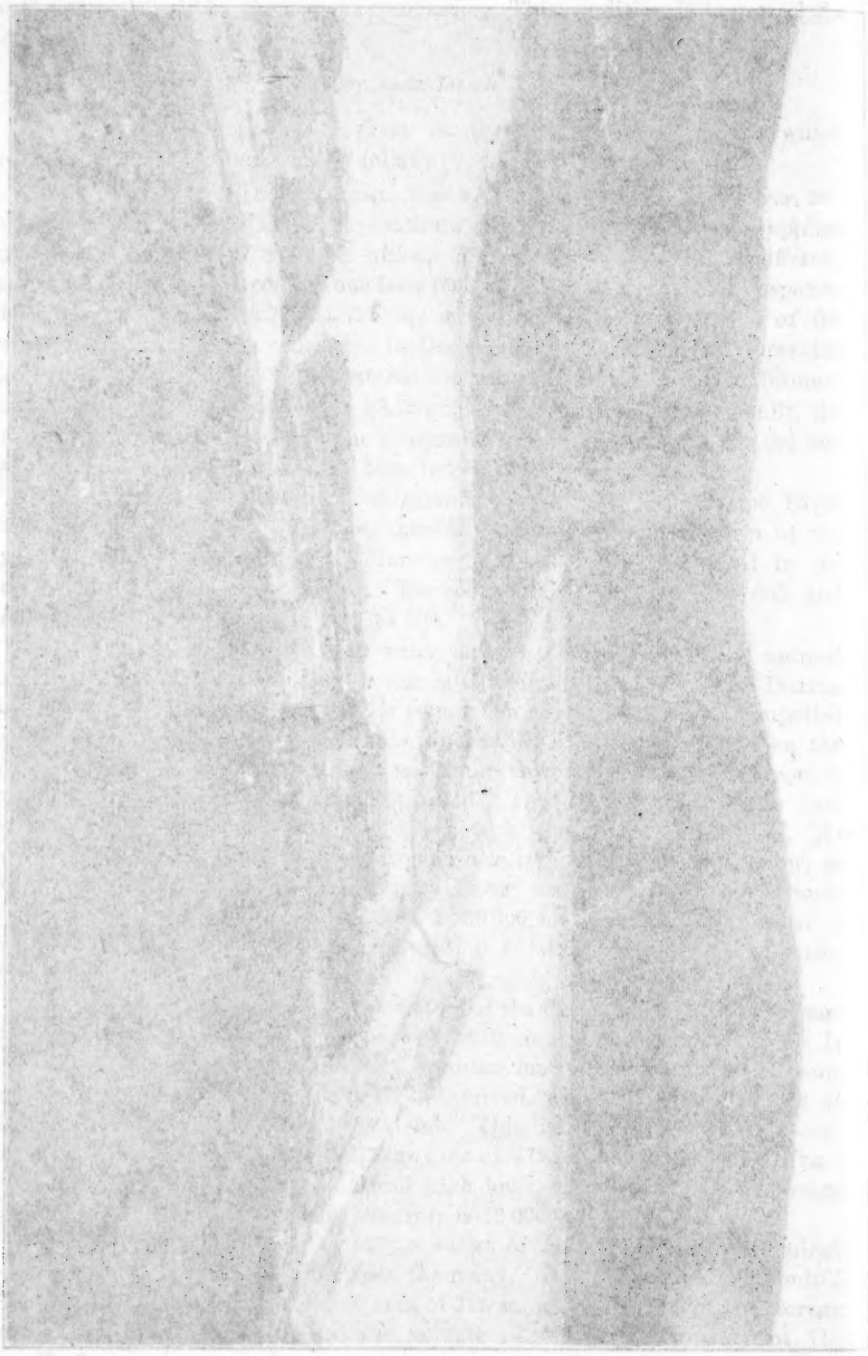
Under natural conditions the minimum flow of the St. Maurice River was about 6 000 sec-ft. It is seen, therefore, that the primary power of the hydro-electric plants on the St. Maurice River has been increased in the ratio of 6 to 16, or by 166 per cent. The cost of this plan was $2 500 000, and the annual revenue therefrom is $234 000.

The ratio of low-water to high-water flow at Shawinigan, under natural conditions, was 1 to 26, and under regulated conditions, is 1 to 10. During high-water periods the storage dam is completely closed, and water is supplied to hold the minimum required at Shawinigan. All power sites between the dam and the regulating point have a minimum flow which is variable, depending on the storage required at Shawinigan. The power site near the dam loses its practical value, as no water is available when the dam is closed. At that point, what was high water at flood times is now practically no flow, as illustrated by Fig. 2. The St. Maurice River, under regulated conditions, affords power possibilities of more than 1 000 000 h.p., which is twice what it was under natural conditions. The duration curves for flow at Shawinigan are shown on Fig. 3.

St. Francis Storage.—At the head-waters of the St. Francis River, a storage dam, called the "Allard Dam", has been built and operated since 1918. It is a hollow concrete dam, built on a hardpan foundation with cut-off walls beneath a concrete floor-slab properly reinforced, which supports a series of piers, all connected by a sloping deck slab. This dam affords a complete control of the water supplied by a drainage area of 472 sq. miles. Lake St. Francis was raised 18 ft. above its natural high level, the total depth of storage being 27 ft. The capacity of the reservoir is 12 000 000 000 cu. ft.

Another storage dam located at the outlet of Lake Aylmer was acquired from the Brompton Pulp and Paper Company. It affords a partial control of the water supplied by a drainage area of 135 sq. miles. The depth of storage on Lake Aylmer is 12 ft., and the volume of water stored is about 110 sq.-mile-ft.

FIG. 1.—VIEW OF GOUIN DAM, UPPER ST. MAURICE RIVER, CANADA.



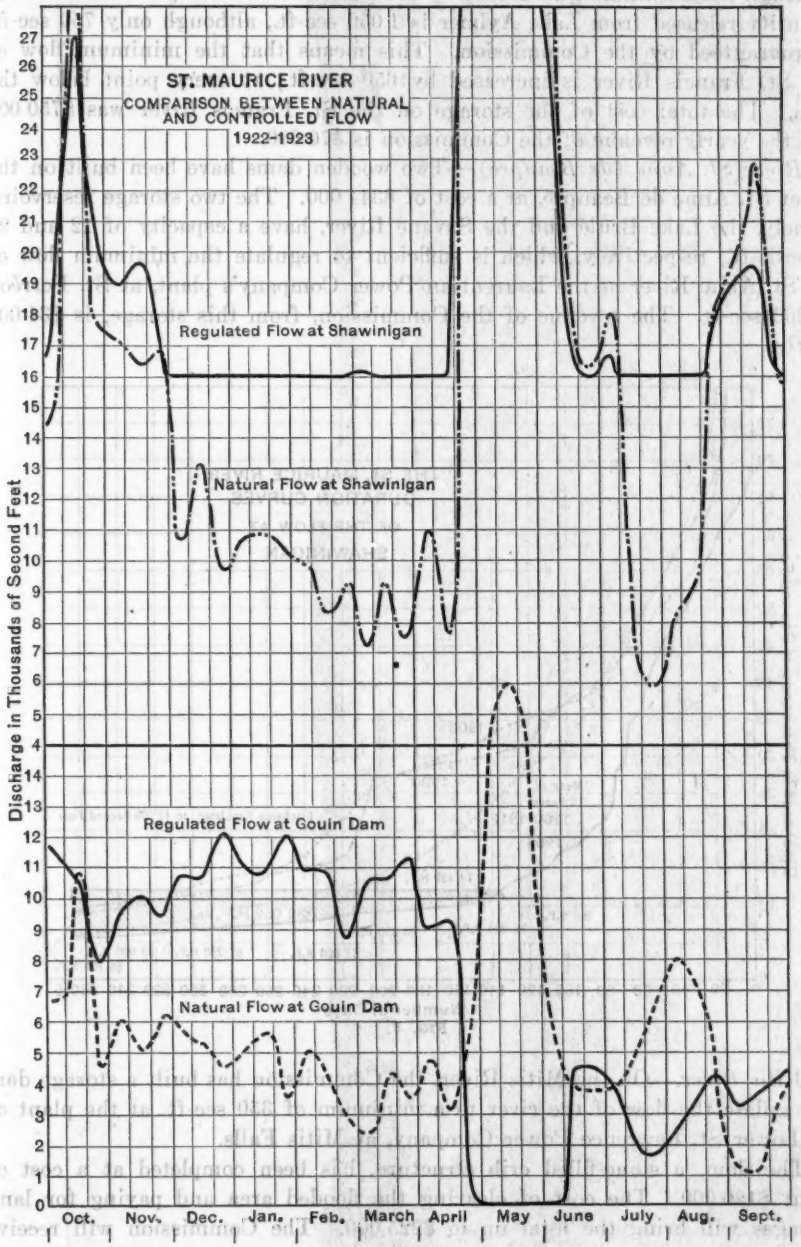

ST. MAURICE RIVER
COMPARISON BETWEEN NATURAL AND CONTROLLED FLOW
1922-1923
Regulated Flow at Shawinigan
Natural Flow at Shawinigan
Regulated Flow at Gouin Dam
Natural Flow at Gouin Dam
Discharge in Thousands of Second Feet
Oct.
Nov.
Dec.
Jan.
Feb.
March
April
May
June
July
Aug.
Sept.

FIG. 2.

The quantity of water discharged from the St. Francis Dam is 900 sec-ft., although the minimum guaranteed by the Commission is only 600 sec-ft. The quantity released from Lake Aylmer is 1 050 sec-ft., although only 750 sec-ft. is guaranteed by the Commission. This means that the minimum flow of the St. Francis River is increased by 650 sec-ft., at every point below the dam. The total cost of the storage on the St. Francis River was $750 000 and the yearly revenue of the Commission is $70 000.

River St. Anne (de Beaupré).—Two wooden dams have been built on the River St. Anne de Beaupré, at a cost of $344 000. The two storage reservoirs, namely, the Lake Brûlé and the Savane River, have a capacity of 12 and 20 sq.-mile-ft., respectively, which is sufficient to regulate the minimum flow of the St. Anne River at the Laurentian Power Company's plant, at St. Ferréol, to 260 sec-ft. The revenue of the Commission, from this storage, is $34 000 yearly.

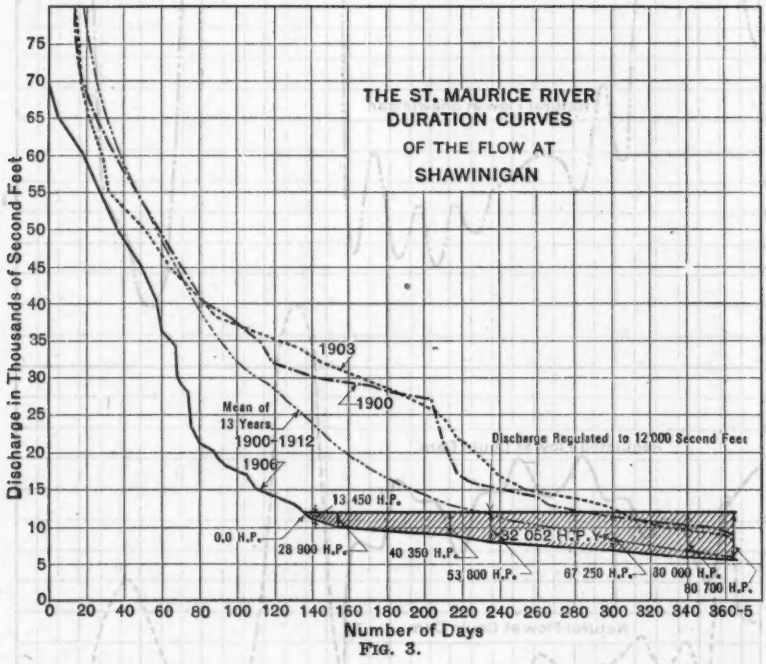


FIG. 3.

Mitis River.—On the Mitis River, the Commission has built a storage dam to regulate the flow of the river to a minimum of 350 sec-ft. at the plant of the Lower St. Lawrence Power Company, at Mitis Falls.

The dam, a stone-filled crib structure, has been completed at a cost of about $120 000. The cost of clearing the flooded area and paying for land damages will bring the total up to $225 000. The Commission will receive $22 500 yearly.

Lake Kenogami Dams.—Lake Kenogami, in the Saguenay District, has a drainage area of 1 400 sq. miles. The lake itself has an area of 12 sq. miles,

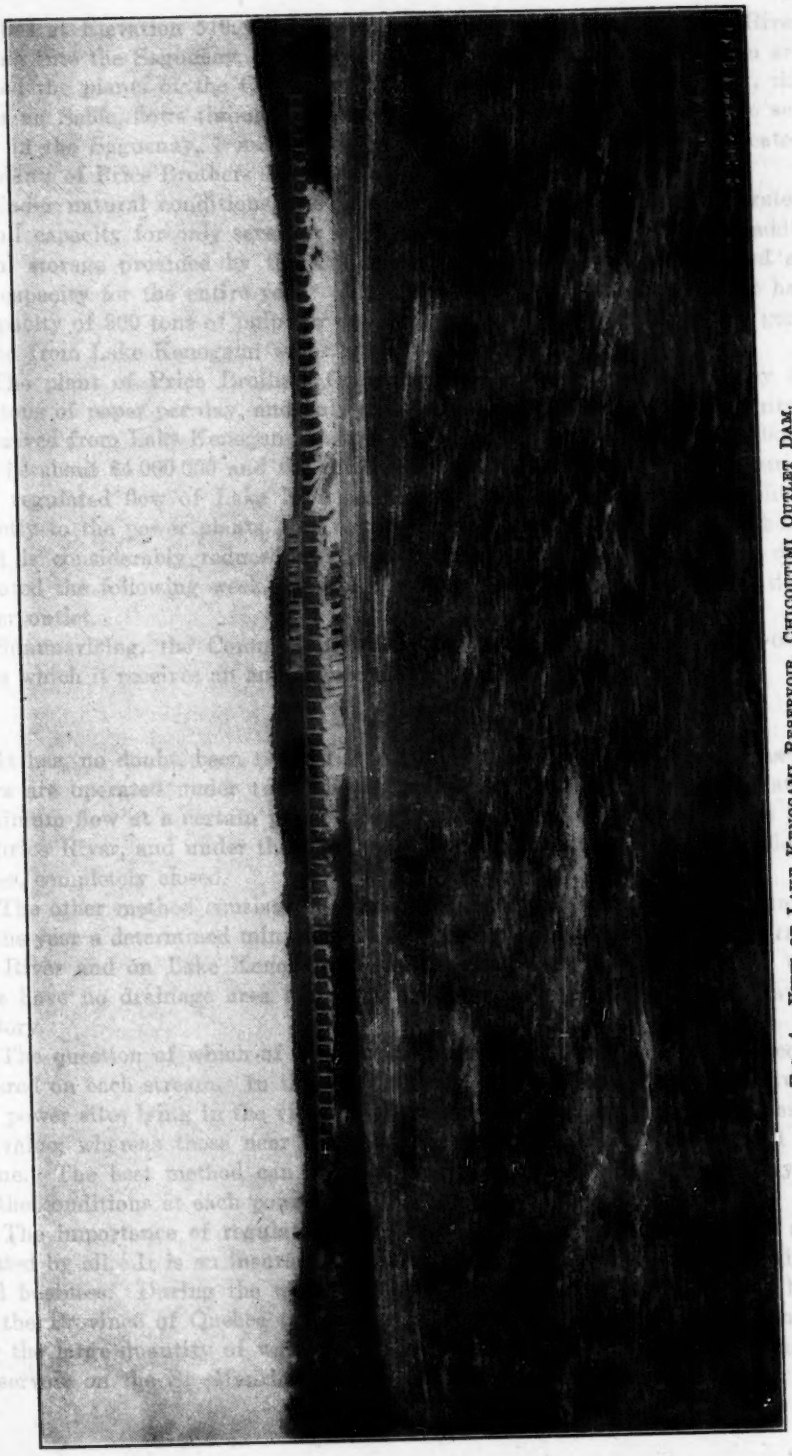
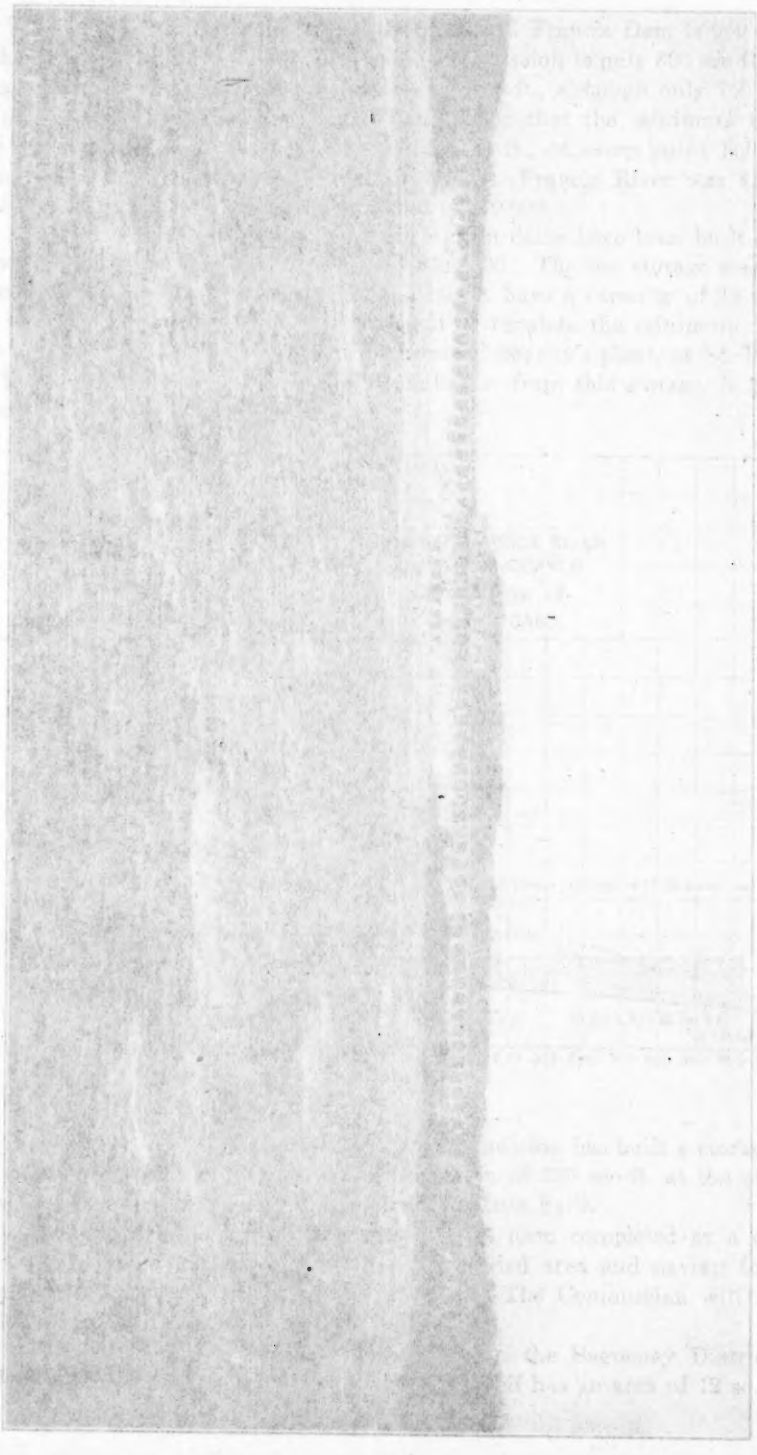


FIG. 4.—VIEW OF LAKE KENOGAMI RESERVOIR, CHICOUTIMI OUTLET DAM.



and lies at Elevation 510. It has two outlets; one at the Chicoutimi River flowing into the Saguenay, at sea level, 15 miles distant. On this stream are located the plants of the Chicoutimi Pulp Company. The other outlet, the River au Sable, flows through the Towns of Jonquière and Kenogami to sea level in the Saguenay, 7 miles from the outlet. On this stream are located the plants of Price Brothers and Company, Limited.

Under natural conditions, the plants on both streams could be operated at full capacity for only seven or eight months of the year. With the additional storage provided by the Commission, the plants can be operated at full capacity for the entire year. The Chicoutimi Pulp Company's plant has a capacity of 300 tons of pulp per day, and all its power (32 000 h.p.) is generated from Lake Kenogami water.

The plant of Price Brothers Company, at Kenogami, has a capacity of 500 tons of paper per day, and only a small proportion of the power required is derived from Lake Kenogami waters. The cost of the Lake Kenogami dams will be about $4 000 000 and the revenue to the Commission, $320 000 yearly. The regulated flow of Lake Kenogami is 1 800 sec-ft., which is distributed directly to the power plants. At no time is the dam closed. The flow, however, is considerably reduced on Sundays, the water thus saved being distributed the following week. Fig. 4 is a view of the dam at the Chicoutimi River outlet.

Summarizing, the Commission has spent to date for storage $7 500 000, from which it receives an annual income of about $685 000.

Operation

It has, no doubt, been noticed from the foregoing that the storage reservoirs are operated under two different methods. One calls for a regulated minimum flow at a certain point, such as Shawinigan, in the case of the St. Maurice River, and under that method of control the storage dam is, at flood times, completely closed.

The other method consists in releasing from the storage dam at all times of the year a determined minimum flow. This is the method on the St. Francis River and on Lake Kenogami. In the latter case, the outlets from the lake have no drainage area and no other method of control could be satisfactory.

The question of which of the two methods is the better is one to be considered on each stream. In the first instance, such as the St. Maurice River, the power sites lying in the vicinity of the storage dams are greatly decreased in value, whereas those near the controlling point are greatly enhanced in value. The best method can only be determined after a complete analysis of the conditions at each power site.

The importance of regulating the flow of rivers is fully realized and admitted by all. It is an insurance against factory shut-downs and loss of time and business. During the winter of 1923, low-water conditions were so bad in the Province of Quebec that industry would have suffered greatly except for the large quantity of water which could be drawn from the Government Reservoir on the St. Maurice River. As it was, the districts served by St.

Maurice power did not suffer from the lack of water; neither did the St. Francis District. In other districts, however, plants were closed and trade and labor suffered a heavy loss.

Although the revenue derived by the Commission is important, it is insignificant when compared with the advantages which the Province at large derives from these works. This policy will be followed also for other streams which the Commission is now investigating.

APPENDIX I

ACT 8, GEORGE V, CHAPTER 13:

AN ACT TO GRANT CERTAIN POWERS TO THE QUEBEC STREAMS COMMISSION, RELATING TO THE STORING OF THE WATERS OF LAKE KENOGAMI

Whereas the Quebec Streams Commission recommends the construction of works required for the storing of the waters of lake Kenogami and its tributary lakes and rivers in order to better regulate the flow thereof; and

Whereas it is in the public interest to carry out the said recommendation, and to grant the Commission the powers necessary for that purpose;

Therefore His Majesty, with the advice and consent of the Legislative Council and of the Legislative Assembly of Quebec, enacts as follows:

I.—The Quebec Streams Commission, after having deposited in the Department of Lands and Forests all the plans and specifications necessary to precisely indicate the work which it wishes to undertake under the authority of this act, and the probable cost thereof, may be authorized by the Lieutenant-Governor in Council to do the work necessary for the erection of storage dams or other works in lake Kenogami and in the Chicoutimi and au Sable rivers and of their tributary lakes and rivers, and of regulating their flow, both from the point of view of the lessening of floods, and for the better utilization of the water powers to be obtained from such lakes and rivers and their tributaries.

II.—The Commission shall give out the works authorized by this act, by means of tenders and contracts, after public advertisements and notice that the plans and specifications have been deposited in the office of the Commission for examination.

The giving out of the work shall be evidenced by a contract which shall be awarded to the contractor making the lowest tender, and who also, in the opinion of the Commission, has sufficient ability, experience and means to do the work properly.

Nevertheless, no contract shall be entered into by the Commission except with the approval of the Lieutenant-Governor in Council.

III.—

1.—The Commission may, with the approval of the Lieutenant-Governor in Council, acquire, either by mutual agreement or by expropriation:

(*a*). The existing dams and works at lake Kenogami in the Chicoutimi river and the river au Sable;

(*b*). All the immovable property required for the carrying out and upkeep of the work it is authorized by this act to execute;

(*c*). All the immovable property that may be flooded or otherwise affected through the execution and upkeep of the said works;

(*d*). All the immovable property required for the making of public or private roads to give access to the said works or to replace roads flooded or otherwise destroyed or damaged;

(*e*). All the immovable property required for establishing the necessary servitudes;

(*f*). All the servitudes, real rights, rights conferred by the Legislature, rights resulting from contracts, and all other rights of any kind whatsoever.

2.—Every expropriation under this act, shall be effected in accordance with the Quebec Railway Act, and the Superior Court of the district or a judge of that Court may grant immediate possession on such conditions as it or he may deem advisable.

3.—When it is necessary to expropriate only a piece of a lot, whether cadastral or not, the Commission, with the consent of the Lieutenant-Governor in Council, may acquire the whole of such lot and afterwards sell such portions thereof as it does not require.

IV.—So soon as the plans and specifications mentioned in Section 1 are deposited, it shall be the duty of the Commission, after hearing the parties interested, to submit to the Minister of Lands and Forests:

(*a*). All forms of contracts to be entered into between the Commission and any person, company or association, who may benefit by the works for the storing and regulating of the waters coming within the scope of this act;

(*b*). A general tariff fixing the rates, prices and conditions which may be demanded from or imposed upon any person, company or association, as regards any matter not governed by the contract mentioned in Paragraph (*a*), for the use of the said waters.

No contract passed in virtue of Paragraph (*a*), shall have any force or effect until approved by the Lieutenant-Governor in Council.

The tariff to be established in virtue of Paragraph (*b*), after having been approved by the Lieutenant-Governor in Council, shall not come into force until published in the *Quebec Official Gazette*.

V.—To ensure the construction of the works and the acquisition of the immovables coming within the scope of this act, the Lieutenant-Governor in Council may authorize the Provincial Treasurer to contract, from time to time, such loan or loans as he thinks necessary, but the loan or loans so contracted shall not exceed the sum of one million eight hundred thousand dollars.

The said loan or loans may be effected by means of debentures or inscribed stock issued for a term of not more than fifty years, and at a rate of interest of not more than five per cent. per annum.

The said debentures or inscribed stock shall be in the form and for the amount that the Lieutenant-Governor in Council may determine, and shall be payable, in principal and interest, annually or semi-annually, at the place mentioned by him.

The debentures or inscribed stock issued under this act shall not be subject to the duties imposed by the Quebec Succession Duties Acts.

VI.—Sections 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 of the Act 3, George V, Chapter 6, shall apply, *mutatis mutandis*, to the present act, as if they had been specially enacted therefor.

VII.—Nothing in this act shall authorize the Commission to acquire by expropriation any of the property or rights mentioned in Section 3 of this act, belonging to the companies called "The Jonquière Pulp Company" and "Price Brothers & Company, Limited".

VIII.—This act shall come into force on the day of its sanction.

SECTIONS 11, 12, ETC., OF ACT 3, GEORGE V, CHAPTER 6

11.—The Lieutenant-Governor in Council, pending the negotiation of the said loan or loans may also authorize the Provincial Treasurer to advance

from time to time out of any unappropriated public monies, the sums necessary to carry out this act on complying with the requirements of Section 12.

Special accounts of such advances shall be kept in the Treasury Department, and the proceeds of the loans shall first of all be applied to pay them off.

12.—The sums required for the doing of the works authorized by this act shall be paid from time to time by the Provincial Treasurer upon a certificate of the Minister of Lands and Forests stating that such sums have been applied for by the Commission, that they are necessary and that the payment thereof may be made in accordance with this act.

13.—A sinking fund sufficient to pay off the loan or loans authorized by this act shall be established. The annual instalments to form part of this fund as well as the interest accrued thereon shall be invested or deposited by the Provincial Treasurer under the direction of the Lieutenant-Governor in Council.

14.—The monies received for rent or other dues in respect of works constructed under the authority of this act shall be paid through the Minister of Lands and Forests into the Consolidated Revenue Fund to form part of the sinking fund until the loan or loans have been paid off in full.

15.—The Commission shall require from each tenderer a deposit in money or an accepted cheque, sufficient to ensure, to the satisfaction of the Commission, the due execution of the work. The contracts shall also contain a stipulation that a part of the contract price shall be retained for such time and upon such conditions as may seem necessary for the due carrying out of the work.

16.—No money shall be paid to a contractor until the engineer in chief has certified that the work in respect of which the monies are claimed has been properly done and that the sum claimed is due and payable in virtue of this act; and until such payment has been recommended by the Commission.

17.—The properties which the Commission may hold in virtue of this act are exempt from taxation, and shall be held in trust for the Government exclusively.

18.—The members of the Commission shall not be liable to be sued for acts done, carried out, or omitted in the performance of their duties under this act.

The Commission can be sued only with the consent of the Attorney-General.

19.—The Commission shall within the first fifteen days of each session, make a report to the Legislative Assembly of the work done under its control during the year next preceding.

20.—The Lieutenant-Governor in Council may make, amend, or repeal such regulations as he thinks necessary to carry out this act and as he thinks proper to make to provide for cases which may arise and for which no special provision has been made.

DEVELOPMENTS OF THE HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO*

By Frederick A. Gaby,† M. Am. Soc. C. E.

Synopsis

This brief description of the power developments of the Hydro-Electric Power Commission of Ontario presents its general historical and economic relationship to the program of public service as expressed by the work of the Commission. This will lead to a more correct understanding of that work and will explain the continuity of policy that has actuated the Commission in obtaining and supplying electrical energy to meet the rapidly growing municipal, industrial, commercial, and domestic needs of the citizens of the Province of Ontario. Ontario's great hydro-electric undertaking is a co-operative less than 1 000 h.p. and now, fifteen years later, distributing about 800 000 h.p. and operating twenty-three plants, which, when fully developed, will have a potentiality of more than 1 000 000 h.p., could not have progressed to its present magnitude and stability without following some well-defined policy.

The hydro-electric enterprise of the Province of Ontario has sometimes been referred to as a Government-owned undertaking; but the phrase, "Government ownership", as frequently used elsewhere—and so often in the United States—has a significance radically different from any conception that should be attached to the operations of the Hydro-Electric Power Commission of Ontario. Ontario's great hydro-electric undertaking is a co-operative municipal-ownership enterprise, operating through the agency of independent Commission control and administration, and is free from political influence.

Such public ownership, administered with honesty and for the circumstances existent in the Province of Ontario, is eminently satisfactory, and on behalf of the citizens of the Province renders a service that is unequalled in any other comparable territory.

Administration

Broadly speaking, the Hydro-Electric Power Commission is entrusted with the duty of supplying the electrical needs of the citizens of Ontario municipalities at the lowest possible cost. At first, this end was achieved most efficiently by purchasing the power from existing sources of supply. Later, however, the source of supply itself was, in some instances, purchased outright

Note.—Written discussion on this paper will be closed with the September, 1926, *Proceedings*. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

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by the Commission; while, sometimes, a number of generating plants have been purchased which together with an existing electric network have constituted the basis from which extensions have been made. Finally, the Commission has made its own hydro-electric developments. At present, the Commission is considering the desirability of obtaining a substantial quantity of electrical power by means of steam.

Financial Structure.—Certain principles which the inaugurators of this undertaking believed sound, and which have since proved to be so, were laid as a basis for administering the various assets in which the municipalities are concerned. The general system of administration may briefly be summarized as follows:

First.—The generation and transmission of power on a wholesale scale is managed by a Commission which, although appointed by the Government of the Province, acts independently in the capacity of trustee for the partnership of municipalities.

Second.—The local distribution of electrical energy within the borders of each municipality is, in general, under the administration of a Public Utilities Commission appointed under the provisions of the Public Utilities Act.

Third.—Capital required for plant to generate and transmit power is loaned by the Government on receipt of formal requisition from the Commission. Contracts are executed between the Commission and the municipalities under the terms of which the municipalities undertake to repay, over a period of thirty years, the monies thus loaned by the Government, with interest in full.

Fourth.—The local distribution system is financed by the issue of municipal debentures. Provision is made in the rates charged to the ultimate consumers for revenue with which to retire these bonds in from twenty to thirty years.

Fifth.—The Commission supplies power to the municipalities, charging each municipality the actual cost. To do this, an interim charge, based on the estimated cost, is made monthly and, at the end of each year, credit or debit adjustment is made of the amount charged in order to make up the actual total cost. The "cost of power" includes all the usual costs of operation and maintenance of the generating, transforming, and transmission plant and equipment, and, in addition, the annual interest charges on the monies borrowed for the initial cost of installation, also provision for renewal (depreciation) and sinking fund reserves, as well as a special reserve fund for contingencies.

Sixth.—Each municipality sells electrical energy to its own local consumers at rates and under conditions approved by the Commission. The rates charged to its consumers by a municipality are made sufficient to take care both of the cost of distribution within the municipality, and of the estimated cost of power to be paid to the Commission by the municipality. The cost of distribution is ascertained in a manner identical with that used by the Commission in arriving at its wholesale costs.

Seventh.—Under the Power Commission Act, the Commission is required to determine, annually, the actual cost of service supplied to the municipal corporations by the local commission for such strictly municipal purposes as street lighting and the operation of water-works' pumps and street railways, and if any profit has accrued through the charging of the rate used throughout the year, this surplus is returned to the municipality.

The basic conception of the whole municipally-owned, electrical undertaking as administered by the Hydro-Electric Power Commission of Ontario is a partnership of municipalities formed to obtain power at cost, each municipality paying its proportion of the cost for the service received. The Commission, acting as agent and trustee for the municipalities, exercises both administrative and constructional functions, and by application of the principles adopted, has evolved a well-defined and successful working policy for the development, transmission, and distribution of hydro-electric power under municipal ownership.

Sources of Power Supply

In 1908 the Commission, in view of the vested interests in power developments at Niagara, secured its first electrical energy for supplying the municipalities of the Niagara System by contracting with the Ontario Power Company for a supply of electrical energy up to a maximum of 100 000 h.p., at a price of $9.40 per h.p.-year, until a load of 25 000 h.p. should be reached, after which the price would be $9.00 per h.p.-year. This 100 000 h.p. was completely utilized by 1915. The Commission arranged for additional power from other companies, and in 1917 purchased outright the Ontario Power Company with its plant capacity of more than 160 000 h.p. In 1920, the plant of the Toronto Power Company with a capacity of more than 125 000 h.p. was purchased. At present (1925), the Commission is completing its own Queenston-Chippawa plant with a capacity of about 550 000 h.p. All these provisions were made to supply power to the Niagara System.

From time to time various municipalities in districts adjacent to other sources of hydro-electrical energy requested the Commission to take action with a view to supplying their power needs. Thus, separate systems were formed in localities in general proximity to various sources of power supply.

Concurrently with its efforts to keep pace with the continually increasing demand for power on the Niagara System, the Commission found it necessary to construct power developments for many of the other systems, which were experiencing corresponding growth in demand for electrical energy. These developments differ not only in physical characteristics, but also in their economic relationships to the various systems which they supply, as well as to the Commission's undertaking viewed in the large.

At present, there is a tendency to reduce the number of separate systems by a process of consolidation, brought about by their expansion and the extension of their transmission networks to a point where they interlock with those of other systems. When this stage in their growth is reached, it becomes advantageous to connect the adjacent systems by tie-lines.

In some cases, special tie-lines have been run to transfer energy from one generating station to another. Thus, two or more systems became physically one from an operating standpoint. Finally, when this stage was reached, it was found advantageous to extend the consolidation to include the financial and administrative features. Both from the operating and administrative viewpoint the consolidations have effected substantial economies.

In other cases the Commission purchased the properties of companies operating within the confines of a system already established. For a time these properties were operated as separate entities, but as it became convenient they were absorbed into, and co-ordinated with, the larger systems. These consolidations also effected economies in operation and administration.

Systems Operated by the Commission

The systems at present operated by the Commission (Fig. 1) will be described briefly.

The Niagara System.—This embraces all the territory between Niagara Falls, Hamilton, and Toronto on the east, and Essex and Goderich on the west, as served with electrical energy generated at the several plants on the Niagara River. These include the plants formerly owned by the Ontario Power Company and the Toronto Power Company, also the new Queenston-Chippawa development. The area served by this system is about 13 500 sq. miles, with a population of 1 800 000. The load supplied is about 700 000 h.p.

Georgian Bay System.—This system, which is a consolidation of four systems previously existing—the Severn, Eugenia, Wasdells, and Muskoka Systems—serves that part of the Province of Ontario surrounding the southern end of Georgian Bay and to the north of the territory served by the Niagara System. It includes the district surrounding Lake Simcoe and extends as far north as Huntsville in the Lake of Bays District. It is served by five generating stations, the Wasdells and the Big Chute, on the Severn River; the Eugenia, on the Beaver River; and the South Falls and Hanna Chutes developments, on the Muskoka River. Of these, the Big Chute plant was purchased from the Simcoe Railway and Power Company and subsequently enlarged by the Commission. The South Falls plant was purchased from the Town of Gravenhurst and is also being enlarged, while the other three plants were constructed by the Commission. In addition, power is purchased from the Corporation of Orillia and from the Niagara System. The area served by the Georgian Bay System is about 8 900 sq. miles, with a population of 115 000. The load supplied is about 20 000 h.p.

St. Lawrence System.—This serves the district immediately to the north of the St. Lawrence River between Brockville and Cornwall. The supply of power for the System is purchased under agreement from the Cedar Rapids Power Company which generates its power at Cedar Rapids on the St. Lawrence River, delivery being made at a point near Cornwall. The area served by this System is about 2 100 sq. miles, with a population of 28 000. The load supplied is about 6 000 h.p.

Rideau System.—This System serves the district in the vicinity of Smiths Falls, Perth, and Carleton Place. Power is available from two generating

HYDRO-ELECTRIC POWER COMMISSION OF ONTARIO
TRANSMISSION LINES AND STATIONS
SCALE IN MILES
0 10 20 30 40 50 60
LEGEND
High-Tension Lines — Generating Stations
Proposed H-T. Lines — High-Tension Stations
Low-Tension Lines — Low-Tension Stations
Proposed L-T. Lines — Proposed L-T. Stations
PROVINCE OF QUEBEC
PROVINCE OF ONTARIO
NORTHWESTERLY PART OF ONTARIO
MANITOBA
Lake Winnipeg
Keewatin
Lake Nipigon
Lake of the Woods
Port Arthur
Thunder Bay System
Nipigon
Abitibi R.
Ottawa R.
North Bay
Nipissing
Ottawa
Mattawa
Chaudière
Ottawa System
Ottawa-Gatineau
Ottawa-Lachute
Ottawa-Lachine
Ottawa-Hull
Ottawa-Carillon
Ottawa-Chute à Blondeau
Ottawa-Chute à Blondeau
Ottawa-Hawkesbury
Ottawa-Hawkesbury
Ottawa-Chute à Blondeau

plants, one at Carleton Place and the other installed by the Commission at High Falls. Both plants are situated on the Mississippi River. The Commission also purchases power from the Rideau Power Company, of Merrickville. The area served by this system is about 1 500 sq. miles, with a population of 16 350. The load supplied is about 3 000 h.p.

Thunder Bay System.—This System serves the district at the head of the Great Lakes, including the Twin Cities of Port Arthur and Fort William, with power from the development at Cameron Falls on the Nipigon River. The area served by this System is about 4 000 sq. miles, with a population of 17 000. The load supplied is about 45 000 h.p.

Ottawa System.—This System serves the City of Ottawa, and an extensive rural power district adjacent thereto. Power is purchased from the Ottawa and Hull Power and Manufacturing Company under an agreement whereby 20 000 h.p. is provided. The present load is about 16 500 h.p. The population served is about 134 650.

Central Ontario and Trent System.—This System serves the district bordering the north shore of Lake Ontario between the territory on the west served by the Niagara and Georgian Bay Systems and that on the east served by the St. Lawrence and Rideau Systems. The nucleus of the Central Ontario and Trent System was the group of properties formerly controlled by the Electric Power Company, Limited, and operated by it through the agency of twenty-two subsidiary companies. All these properties were purchased by the Province of Ontario on March 1, 1916. Since June 1, 1916, they have been operated by the Commission as Trustee for the Province and the System has been greatly enlarged to meet the constantly growing needs of the district.

The power supply for the Central Ontario and Trent System is obtained from nine power developments situated on the Trent and Otonabee Rivers, which have been made in conjunction with dams required for navigation purposes. Two of these are new developments now (1925) under construction at Dams Nos. 8 and 9. The area served by this System is about 4 700 sq. miles, with a population of 145 000. The load supplied is about 45 000 h.p.

Nipissing System.—The Nipissing System comprises the Town of North Bay and certain small municipalities south of Lake Nipissing. It was purchased together with the Central Ontario System by the Province in 1916 and has since been operated by the Commission. It is supplied with power from two hydro-electric developments on the South River at Nipissing and Bingham Chute. The area served by this System is about 700 sq. miles, with a population of 13 500. The load supplied is about 2 750 h.p.

Plants Constructed by the Commission

In addition to the systems purchased several noteworthy hydro-electric developments have actually been constructed by the Commission. Early in 1912, on the application of a number of municipalities in Ontario County situated on the east side of Lake Simcoe, the Commission made a thorough survey of the available water powers in the district, and recommended a site at Wasdells Falls (Fig. 2), on the Severn River, about three miles below Lake Couchiching, as being the most economical for power development. This

marked a new step in the progress of the policy of the Commission in that it was the first development constructed by it on behalf of municipalities. Work was commenced in July, 1913, and the project was completed in September, 1914.

Wasdells Development

The detailed hydraulic investigations showed the available head to vary considerably, owing principally to back-water. However, taking into consideration the effect of forebay levels, the maximum and minimum heads available after construction were determined to be approximately 15 and 9 ft.,

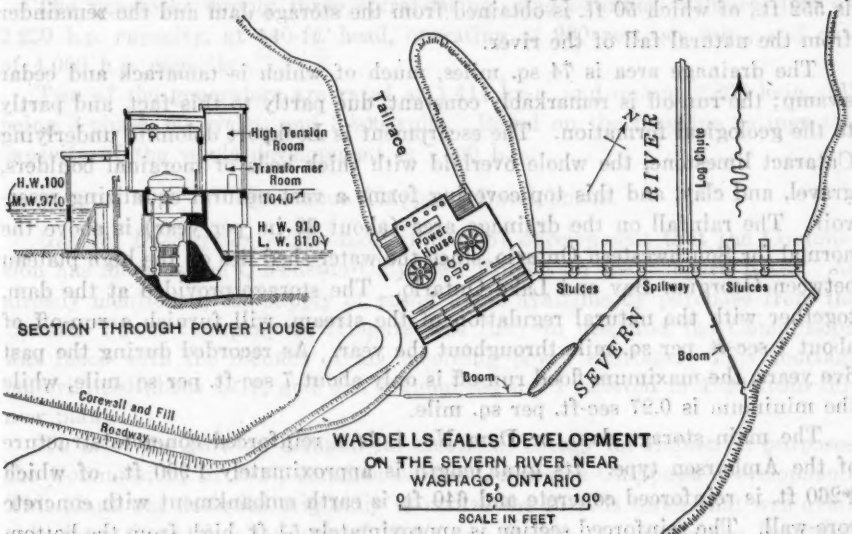


FIG. 2.

respectively. Except for difficulties encountered due to this extremely low head, all the inherent features of the site were favorable for development. The natural formation is solid granite, affording an excellent foundation for the superstructure.

The dam substructure and superstructure are constructed of mass and reinforced concrete, and offer no special features of design. The two main turbines are of the vertical double-runner type in open-flume settings, and each runner is provided with a separate draft-tube. The units are designed to operate normally at a speed of 90 rev. per min. under a head of 12 ft., with a guaranteed capacity at three-quarters gate of 600 h.p. When operating under a 9-ft. head at full gate, the guaranteed capacity is 500 h.p. The guaranteed efficiencies vary from 75 to 85%, for gate openings of from 50 to 100 per cent. The main generators are of the vertical, 3-phase, 60-cycle type, and each is of 400 kv-a. capacity with the usual guaranties and specifications.

Eugenia Development

The second development undertaken by the Commission, situated on the escarpment near Georgian Bay, in the County of Gray, and designed to serve

the municipalities in the Owen Sound-Durham District, was commenced in July, 1914, the initial development being completed in November, 1915. The unique features of this plant, known as the Eugenia Development, are the arrangements for maximum economic utilization of the run-off from the drainage area, and the high head under which reaction turbines are used.

The development consists essentially of a storage dam on the Beaver River, about ½ mile above the Eugenia Falls, forming a reservoir, a canal 5 000 ft. long, a forebay or settling basin, two wood-stave pipes and steel penstocks, and the power-house. The gross head under which the plant operates is 552 ft., of which 50 ft. is obtained from the storage dam and the remainder from the natural fall of the river.

The drainage area is 74 sq. miles, much of which is tamarack and cedar swamp; the run-off is remarkably constant, due partly to this fact, and partly to the geological formation. The escarpment is Lockport dolomite underlying Cataract limestone, the whole overlaid with thick beds of morainal boulders, gravel, and clay, and this top covering forms a vast natural equalizing reservoir. The rainfall on the drainage area (about 39 in. per year) is above the normal for Southwestern Ontario, since the water-shed lies on the high plateau between Georgian Bay and Lake Ontario. The storage provided at the dam, together with the natural regulation of the stream, will furnish a run-off of about 1 sec-ft. per sq. mile throughout the year. As recorded during the past five years, the maximum flood run-off is only about 7 sec-ft. per sq. mile, while the minimum is 0.27 sec-ft. per sq. mile.

The main storage dam, or Dam No. 1, is a reinforced concrete structure of the Ambursen type. Its total length is approximately 1 900 ft., of which 1 260 ft. is reinforced concrete and 640 ft. is earth embankment with concrete core-wall. The reinforced section is approximately 51 ft. high from the bottom of the cut-off to the top of the crest at the highest point. The crest of the spillway is at Elevation 646, and its length is 103 ft. The maximum to which the water is raised is Elevation 649, at which point an area of about 1 650 acres is flooded. This upper 3 ft. is controlled by flash-boards, and the capacity gained by this means is 190 000 000 cu. ft., while the total storage capacity of the dam is 780 000 000 cu. ft., or about one-third of the total flow of the river in a normal year.

The canal connecting the reservoir with the head-works is constructed as an open trench protected by a wooden box-flume at points subject to earth slides. At its lower end a natural basin was made by placing an earth-fill dam across an opening in the valley. The length of this dam is about 800 ft., its maximum height, 30 ft., and its width, 10 ft. at the top, with slopes of 1 on 3 on the water side and 1 on 2 on the lower side. The two wood-stave pipe lines are 46 in. in diameter and about 3 350 ft. in length, ending in surge tanks located at the brow of the escarpment above the power-house.

The wood-stave and steel pipe lines are joined by head-blocks, from which risers, 40 in. in diameter, connect with surge tanks, which are designed to handle without spilling a complete shut-down of the plant when all units are operating.

The steel penstocks are 52 in. in diameter, 1 557 ft. in length, and vary in thickness from $\frac{5}{16}$ in. at the upper end to $\frac{27}{32}$ in. at the lower end. The girth seams at the top are single-riveted and at the lower end double-riveted; the longitudinal seams are triple-riveted, double-butt strap joints, with rivets varying from $\frac{3}{4}$ in. to $1\frac{1}{4}$ in. The penstock rests on concrete saddles 20 ft. apart, and rides on steel channels bent to the outer radius of the pipe. Four concrete anchors are placed at changes in grade below the head-blocks; at the ends of the first three anchors are expansion joints of the stuffing-box type. The penstock is housed for about one-half its length as protection against frost.

The penstocks supply three spiral-casing, single-runner turbines, two of 2 250 h.p. capacity, at 540-ft. head, operating at 900 rev. per min., and one of 4 000 h.p. capacity.

Two of the generators are rated at 1 411 kv-a. and one at 2 800 kv-a., all being 3-phase, 60 cycles, and 3 800 volts. Based on the machine ratings the capacity of the whole development is 7 500 h.p..

Queenston-Chippawa Development

General Description.—The most important development that the Commission has made is the Queenston-Chippawa Project on the Niagara River. As already mentioned, the supply of power made available by purchase from the Ontario Power Company at Niagara was fully in use, and the Commission was faced with the menace of an oncoming power shortage. This shortage materialized about 1918, and was pronounced until relieved in 1922 from this new plant.

To meet the growing demand for electrical energy for industrial purposes, the Commission, in 1917, commenced the Queenston-Chippawa Development. This plant was designed originally to utilize the surplus water that was available from Canada's allotment of 36 000 cu. ft. per sec. under the Boundary Waters Treaty, but when the Commission purchased the Niagara plants of the Ontario Power Company and the Electrical Development Company the size of the projected new Queenston-Chippawa plant was increased from less than 200 000 h.p. to about 450 000 h.p., and eventually to about 550 000 h.p. This enlargement was possible because a greater quantity of water became available for use under much more efficient conditions; that is, in the Queenston-Chippawa plant the available water could be used at nearly double the head.

The general scheme of development (Fig. 3) comprises an intake structure in the Niagara River at Chippawa; the deepening of the Welland River for a distance of $4\frac{1}{2}$ miles; the construction of a canal, $8\frac{1}{2}$ miles long, from Montrose to the forebay and screen-house, which are located on the cliff about 1 mile south of the Village of Queenston, and the construction of the power-house in the Niagara Gorge.

Throughout the period of preliminary study and, later, as the design progressed, continuous use was made of models of the various structures in order that the mathematical analyses might be augmented by actual demonstrations under the assumed conditions. Such models were made for studying

the intake, the bends in the canal, the transitions (that is, points where change of section occurred in the canal), the diffuser at the mouth of the forebay, and the draft-tubes and station substructure. It is believed that the beneficial results of such studies and of the care taken in the design of what are often considered minor elements of a power development, have been demonstrated by the results obtained. The tests already made indicate conclusively that the over-all efficiency secured is at least as high as has ever before been obtained.

True conservation of the waters of the Niagara River for power purposes demands that practically the whole fall of about 327 ft. between Lake Erie and Lake Ontario be utilized. The various power plants now operating at Niagara Falls utilize heads of from 130 to 210 ft. only, and with widely different degrees of efficiency. The Queenston-Chippawa power development, the first unit of which was placed in operation during December, 1921, will have a normal operating head varying from 294 to 305 ft. when the installation is complete. The conservation of head effected by the reduction of hydraulic losses to a minimum (Fig. 3), and by refinements in the design of the various essential elements of the project as a whole, has resulted in a power development which is believed to represent the best in modern engineering practice.

The present installation will consist of nine 55 000 to 65 000 h.p. turbine units operating at 187.5 rev. per min. The generators are of 45 000 to 55 000 kv-a. capacity at 12 kv., 3 phase, 25 cycles, with an ultimate aggregate capacity of from 450 000 to 550 000 h.p., depending on the water levels. The electrical energy is stepped up from 12 to 110 kv. for transmission.

Intake.—One of the great obstacles to the continuity of service is the entry of ice into the canal. Great fields of ice, formed in Lake Erie with its shallow bays and sloping shores, are discharged down the Niagara River every spring, and at frequent intervals during the winter. The river itself never freezes over, but considerable anchor and frazil ice are formed at times of low temperature.

The site of the intake of the Queenston-Chippawa power development, at the mouth of the Welland River, is favorable in that floating ice does not ordinarily follow the shore lines at this point; but the smooth gradient of the river surface, and the comparatively shallow water with its low velocity, are unfavorable to the proper handling of ice.

The removal of water in large quantities from a river heavily charged with ice is always a difficult problem, but is greatly simplified when a natural break in the river surface, accompanied by a sudden drop, gives a source of power for the separation of floating ice, and for its continuous disposal. The use of a horizontal diaphragm to skim the surface water, with its burden of ice, from the lower strata, thus permitting the upper water layer to be accelerated and removed clear of the intake without objectionable eddies, while the lower layer, free of ice, is changed in direction and flows through the intake into its new channel, gives a positive and satisfactory solution.

When, as in the present case at Niagara, the natural conditions do not permit such an arrangement, radically different measures must be taken. To confirm certain ideas developed as a result of many years of experience and

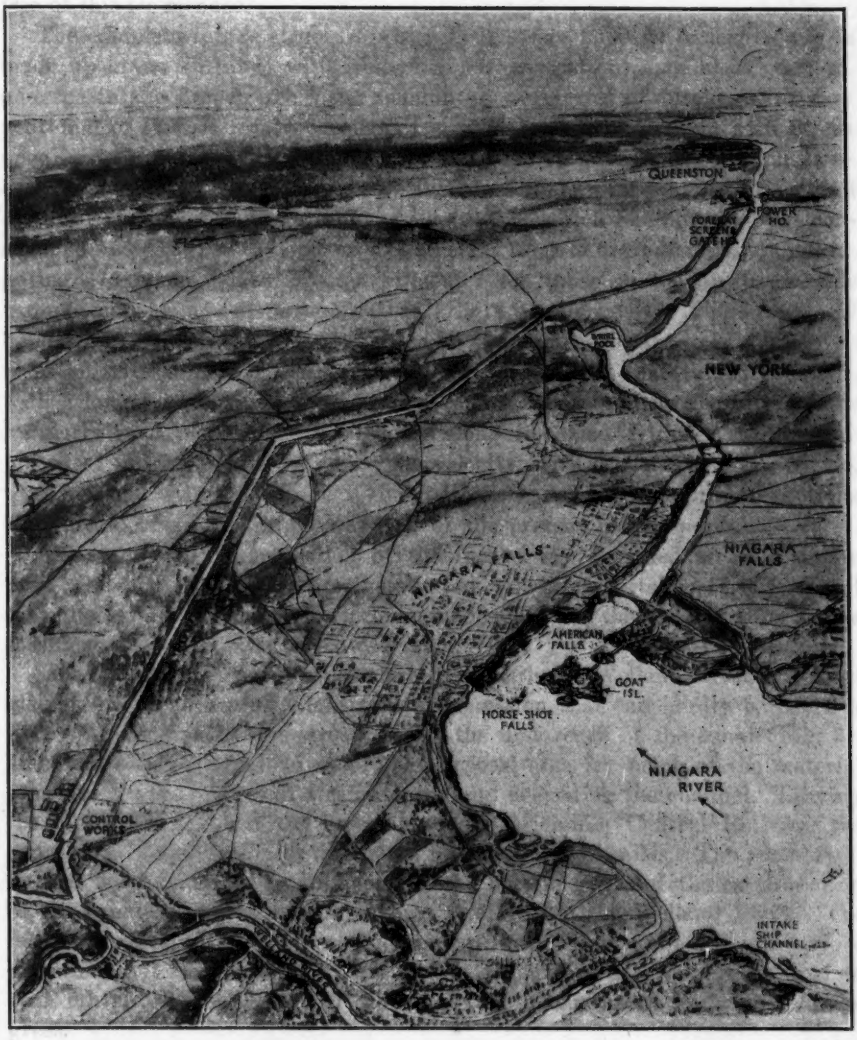
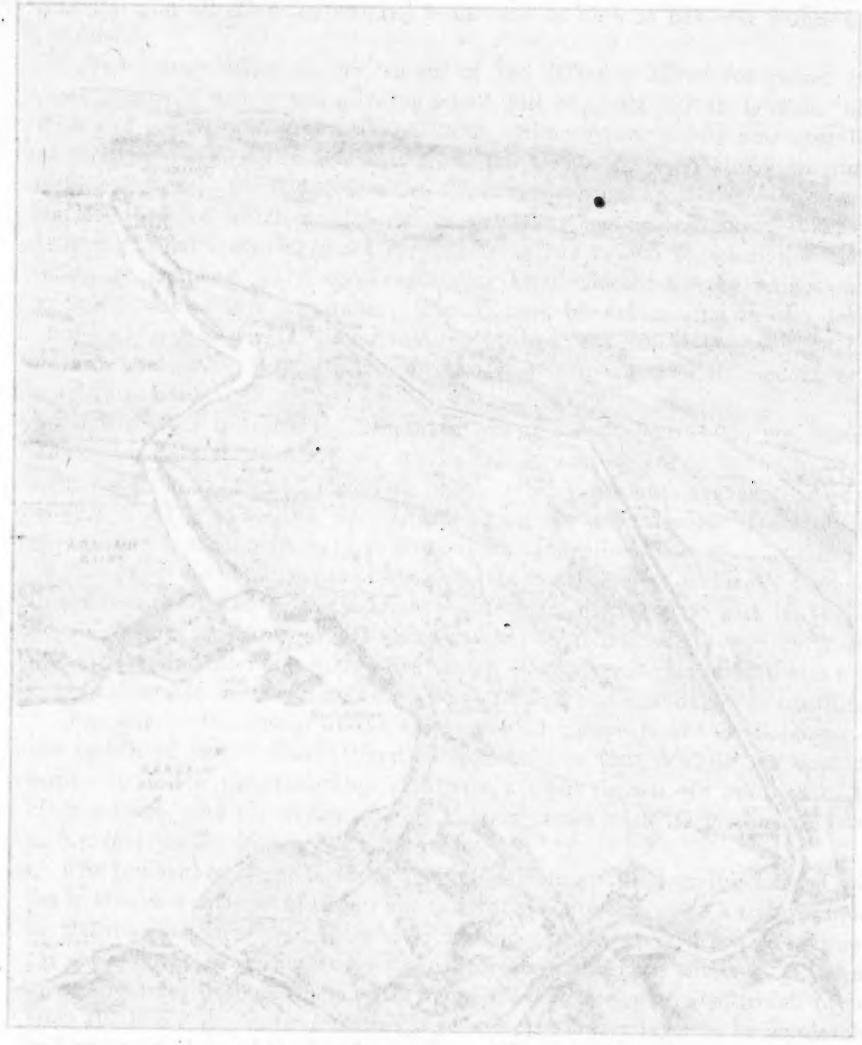


FIG. 3.—BIRD'S-EYE VIEW OF THE QUEENSTON-CHIPPAWA CANAL.



observation by the engineers of the Commission in connection with the present plant operating on the Niagara River, an extended series of tests and experiments on large-sized models was made. These models duplicated to scale the topographical conditions at the site of the intake and resulted in a design which, it is expected, will operate in such a manner as to keep the plant free of this ice menace.

The complete intake structure (Fig. 4) is about 1 100 ft. in length and is made up of an entrance with lock-gates for navigation, a bulkhead section, and the intake proper, the latter combining two forms of intake. The conventional or surface intake consists of a concrete barrier or boom with fifteen openings each 18 ft. wide, having normally 8 ft. of submergence, which, however, by means of drop-gates can be increased to any amount up to the full depth of water, or 35 ft. A submerged intake—which has not yet been constructed—as designed consists of six gathering tubes or draft distributors, aggregating 675 ft. in length. Water will enter the tubes on the up-stream side through slots along a distance of 500 ft. (Fig. 4, Section *A-A*). These tubes will be controlled by gates similar to those on the surface intake—air tubes comprise an outer tapering section wherein the velocity is maintained constant, with a longer inner section 20 ft. in diameter, wherein the velocity increases regularly with respect to the distance along its axis. Diffuser sections are placed at the inner end to reduce the velocity, with as little loss as possible, to correspond with that in the Welland River section. These diffuser sections have already been constructed as part of the present intake works.

The submerged tube section of the intake was designed as an ultimate protection against ice, and its installation will be undertaken if it is found that the existing construction proves inadequate; to the present time it has satisfactorily taken care of ice diversion.

The Canal.—For many miles above its mouth the Welland River is a sluggish stream, meandering slightly in a depression that can hardly be called a valley. For 4½ miles this stream forms the first reach of the canal (Fig. 3), and its low banks provided a suitable disposal area for much of the material excavated in the process of straightening and deepening the channel. Leaving the river channel near the crossing of the Michigan Central Railway, the canal takes a course almost due north for more than 3 miles. The earth overburden is quite heavy on this stretch, the bottom grade of the earth section of the canal meeting the rock surface 1 mile from the Welland River. The maximum rock elevation is not coincident with the maximum earth surface elevation, but the profiles of rock and earth surface are roughly parallel to each other. The five bends in the canal vary in curvature from 27 to 51 degrees.

One depression that is crossed (Bowman's Ravine) is apparently an old river channel through which, in pre-glacial times, the Niagara River flowed toward Lake Ontario. At the time the construction started this old channel was the course of a small stream having its outlet at the Whirlpool. The ravine crossing is made on a fill and the valley itself proved a convenient disposal area for about 1 500 000 cu. yd. of excavated material.

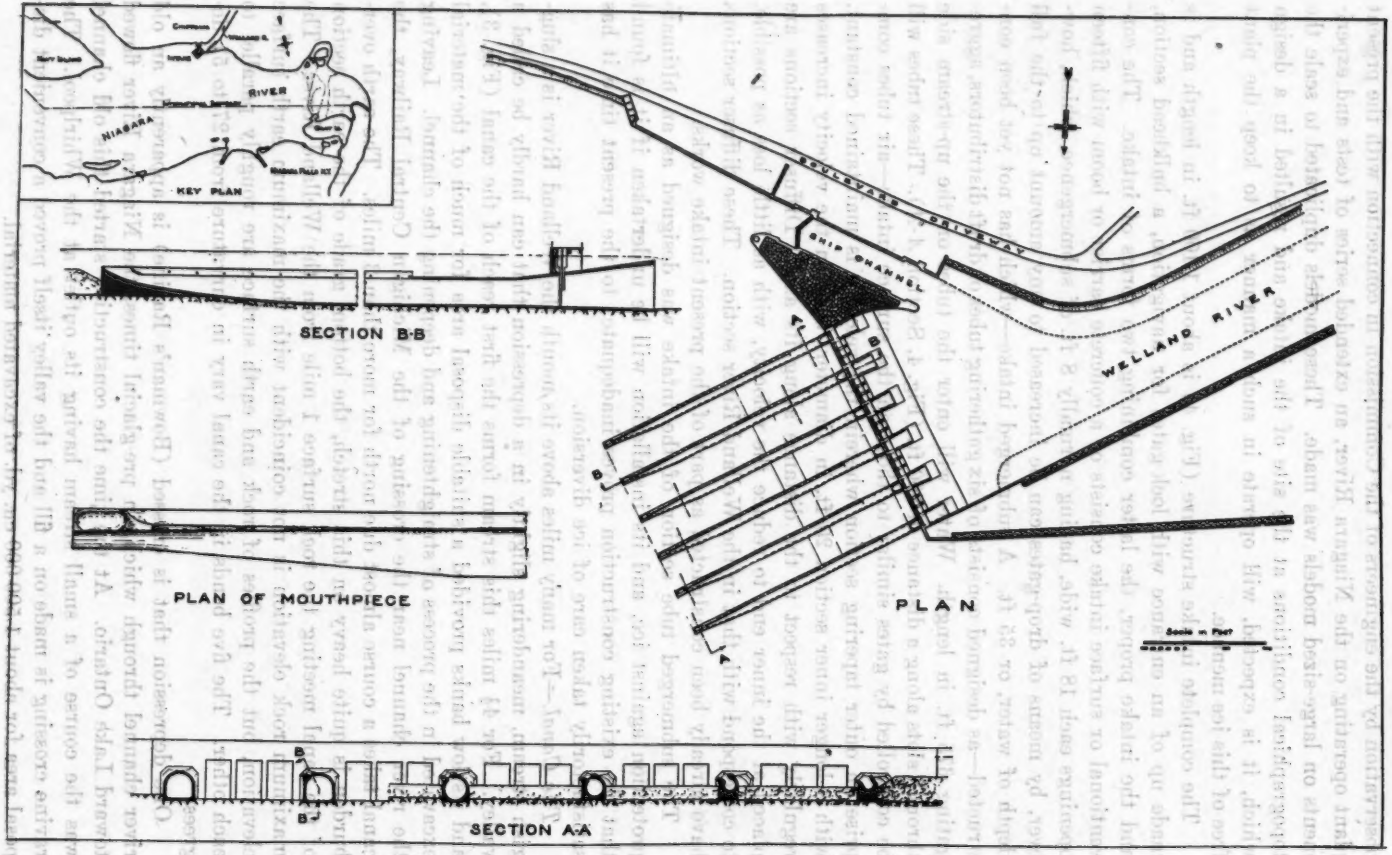


FIG. 4.—PLANS OF INTAKE STRUCTURE, QUEENSTON-CHIPPAWA DEVELOPMENT.

FIG. 4.—PLANS OF INTAKE STRUCTURE, QUEENSTON-CHIPPAWA DEVELOPMENT.

FIG. 5.—VIEW OF ELECTRICALLY-OPERATED ROLLER SLUICE-GATE, QUEENSTON PLANT.

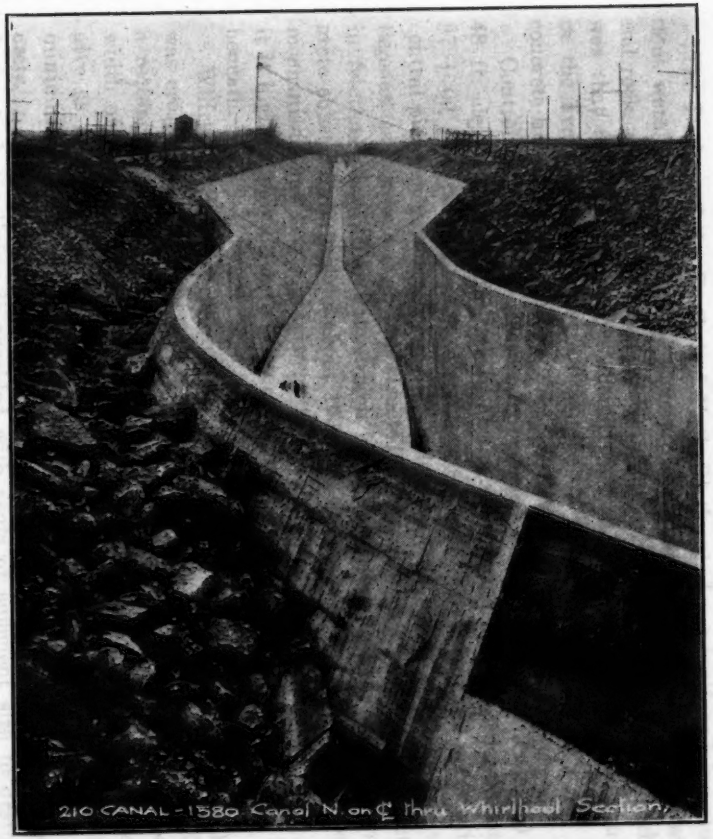


FIG. 6.—CROSS-SECTION OF CANAL, WHIRLPOOL SECTION, QUEENSTON-CHIPPAWA DEVELOPMENT.

The Design of the Earth and Rock Sections.—Long-continued investigations were made to determine factors of roughness for large canals in earth and rock, with and without concrete lining. One of the conclusions reached was that the Kutter formula should be used. The roughness factors used in the hydraulic studies were 0.035 for the river section and 0.012 for the concrete-lined rock section.

Control Works.—An electrically-operated roller sluice-gate (Fig. 5) of 48 ft. clear span—the full width of the rock section—is located at Station 97 + 00, near the beginning of the rock section. The use of two gates with a central pier was considered, but the single gate was found to be the more advantageous, as it provided an unobstructed waterway with a consequent reduction in friction losses. The gate, which is supported on steel towers with a concrete substructure, weighs about 100 tons, and is provided with two hoisting mechanisms and two counterweights. When the gate is at the top of its run it is high enough above the water surface in the canal to permit a tug to pass beneath.

Whirlpool Section.—Bowman's Ravine, situated west of the Whirlpool, was crossed on a rock-fill, the cross-section of the canal being changed from a rectangular section 48 ft. in width to a trapezoidal section with a bottom width of 10 ft. and side slopes of 1 on 1½ (Fig. 6). This was designed to give as great a cross-sectional area as the rock section has at the extreme minimum water level. The Whirlpool Section is lined with reinforced concrete. In order to withstand the pressure of the back-water when the canal is emptied, the lining is provided with vents of sufficient size to drain the fill as quickly as the water can be drawn down in the canal.

Concrete Lining.—Economic considerations prompted the lining of the rock section of the canal with concrete (see Figs. 5 and 6). The height of the lining was fixed slightly lower than the profile of the water surface existing when the load conditions on the plant are at a maximum and the flow in the Niagara River is at a minimum. Thus at all times the lining will be protected by submergence against the action of frost. The thickness of the lining varies according to the rock over-break, but averages about 20 in. Where necessary, steel dowels are used to anchor the concrete lining to the rock.

Extreme smoothness of surface is not the only determining factor, precise alignment being also a most important element in the reduction of hydraulic losses. Great care was taken to obtain a smooth surface by the use of steel forms and for this purpose a positive and rigid method of form-setting was devised. This insured almost perfect alignment.

Screen House.—At the lower end of the forebay, serving as a dam, is the screen house (Figs. 7, 8, and 9). This structure forms the entrance, and the control, for the penstocks. The entrance to each of the main penstocks is a modified bellmouth consisting of three openings, each 12 ft. 8 in. wide and 29 ft. high, at the rack supports. These three openings gradually converge into one opening, 16 ft. in diameter, at the point of connection to the penstocks. In designing these water passages, particular care was taken to secure smooth stream lines and consistent changes in velocity. The bellmouth en-

trances are sealed by a concrete curtain-wall extending down to Elevation 542.0, which gives a depth of 28 ft. above the floor of the forebay. Immediately behind the curtain-wall, steel-lined gate-checks are provided to support structural steel gates. These gates provide a means of unwatering in case it is necessary to inspect the lower sections of the racks, or the penstocks. The intake is divided into three waterways to reduce the spans for the gates to a convenient size and also to permit the use of racks of a somewhat new design. The entire rack structure is removable and is split horizontally into two sections for convenience in handling. A specially designed rack follower with an automatic latch arrangement facilitates the removal of the racks.

The screen house, as constructed, provides for nine main units, a service unit, and an ice chute, and is arranged so that a further unit entrance may be added at the north end. It is near the edge of the escarpment, only a narrow ledge of rock being left between it and the gorge. Owing to the disastrous results that would follow a failure, the screen house substructure was designed to resist the full head exerted by water in the forebay.

Penstocks.—From the screen house, the water is carried to the turbines in steel penstocks (Fig. 9). The first five main unit penstocks are 16 ft. in diameter for approximately two-thirds of their length, and then reduce by a taper section to a diameter of 14 ft. The remaining penstocks are 16 ft. in diameter for their complete length. The penstocks have bends only at the top and at the bottom. These elbows are held in massive concrete anchor-blocks, the one at the upper bend forming a foundation for the piers supporting the sidewalk and roadway extending along the edge of the escarpment (Figs. 8 and 9).

Each penstock ring consists of two plates with longitudinal joints on the horizontal center line. The joints are all double butt, varying from double riveted at the top to quadruple riveted at the lower end. The circumferential joints are single butt, double riveted, with the butt strap on the outside. The longitudinal joints are caulked on the inside, but the circumferential joints are made water-tight by electric welding. This type of circumferential joint gives a much better alignment to the inside of the pipe than can be obtained with the usual outside and inside course with lap joints. The plates vary in thickness from ½ in. at the top section to 1¼ in. at the lower section. In the erection of the penstock, an innovation was tried, namely, the use of electric rivet heaters. Each penstock is covered throughout its length with a concrete envelope having a minimum thickness of 24 in., a protection which will prolong the life of the steel.

Generating and Transformer Station.—The generating and transformer station is situated below the escarpment and close to the river's edge, extending about one-half the distance to the top of the escarpment (Figs. 7, 8, and 9). The structure required to house nine main units and the service equipment is 650 ft. long. The substructure is of massive concrete carried down to rock foundation. The superstructure consists of a structural steel framework with reinforced concrete floors and roofs, and concrete, brick, and tile walls and partitions.

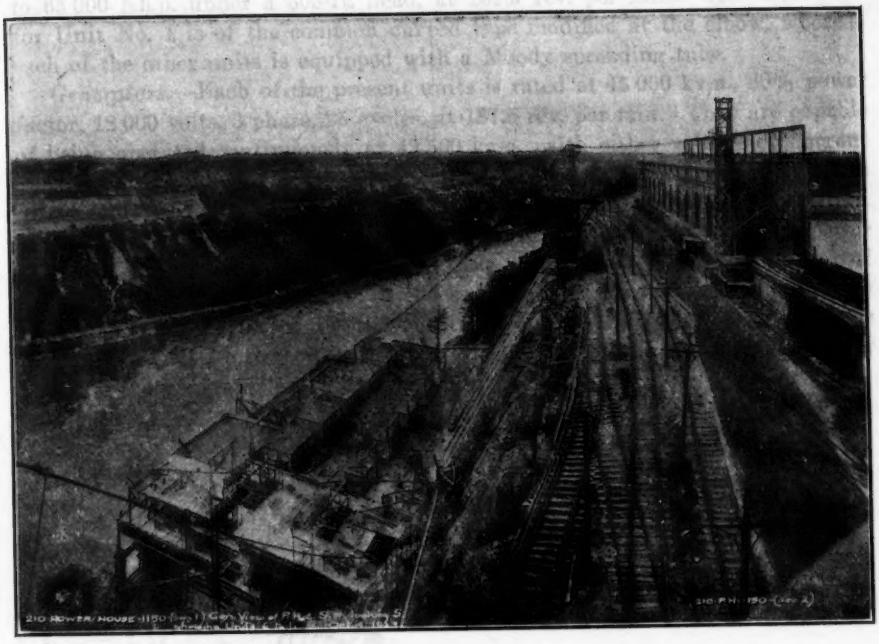


FIG. 7.—POWER PLANT AT QUEENSTON FROM LEVEL OF HEAD-HOUSE.

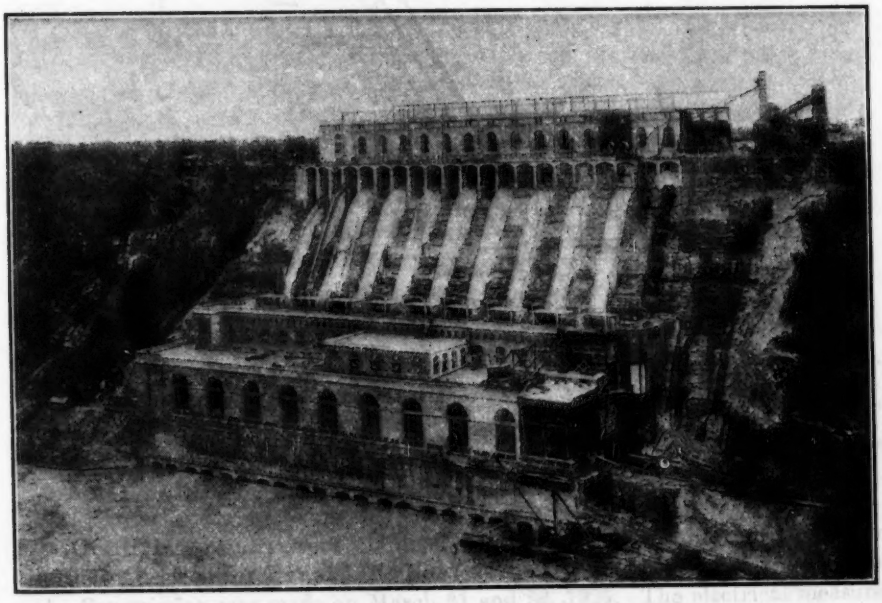
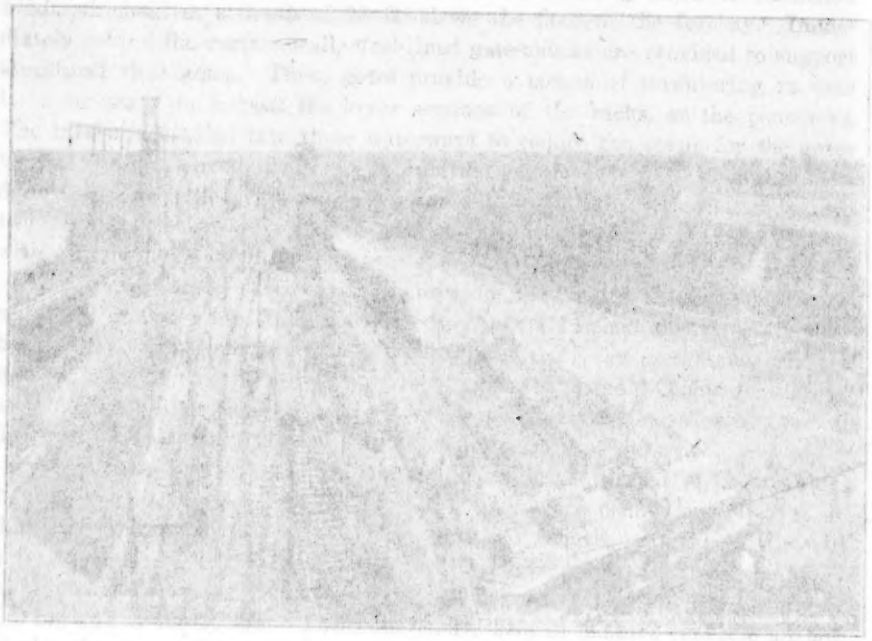
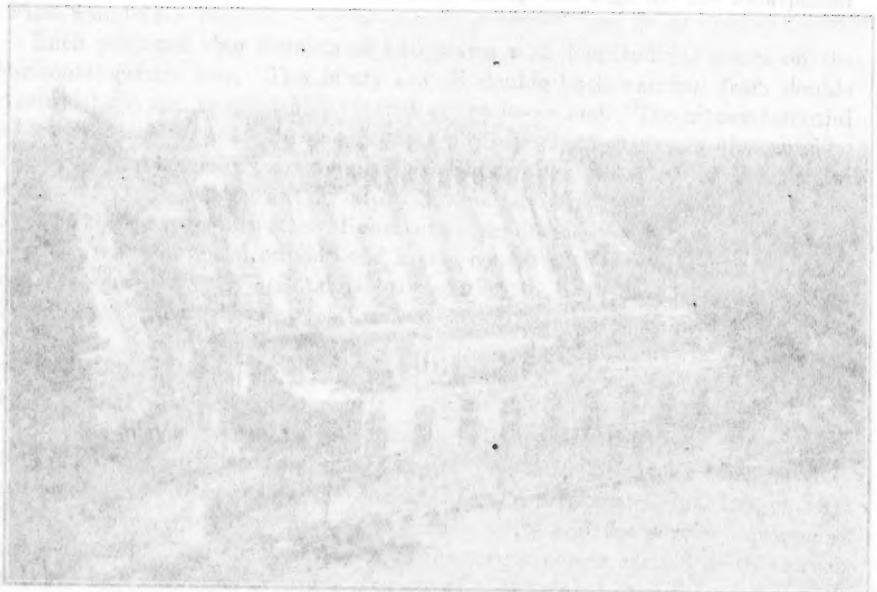


FIG. 8.—VIEW OF PLANT AT QUEENSTON FROM CLIFF ON EAST SIDE OF GORGE.





Turbines and Generators.—Each turbine has a capacity of from 55 000 to 65 000 b.h.p. under a 305-ft. head, at 187.5 rev. per min. The draft-tube for Unit No. 1 is of the common curved type modified at the elbow, whereas each of the other units is equipped with a Moody spreading tube.

Generators.—Each of the present units is rated at 45 000 kv-a., 80% power factor, 12 000 volts, 3 phase, 25 cycles, at 187.5 rev. per min. They are capable of being operated continuously at 49 500 kv-a., with either voltage or current 10% in excess of the rated values. The units are vertical (Fig. 9), with direct-connected shunt-field commutating-poles, and 250-volt, 150-kw. exciters. The over-all efficiency of the generators is slightly in excess of 97% at a power factor of 80 per cent. The thrust bearing is designed to support a load of 1 000 000 lb., which is slightly in excess of the weight of the rotor plus the hydraulic thrust imposed by the turbine. Upper and lower guide-bearings are provided, the latter on account of the length of the shaft and in order to make the generator a self-contained unit.

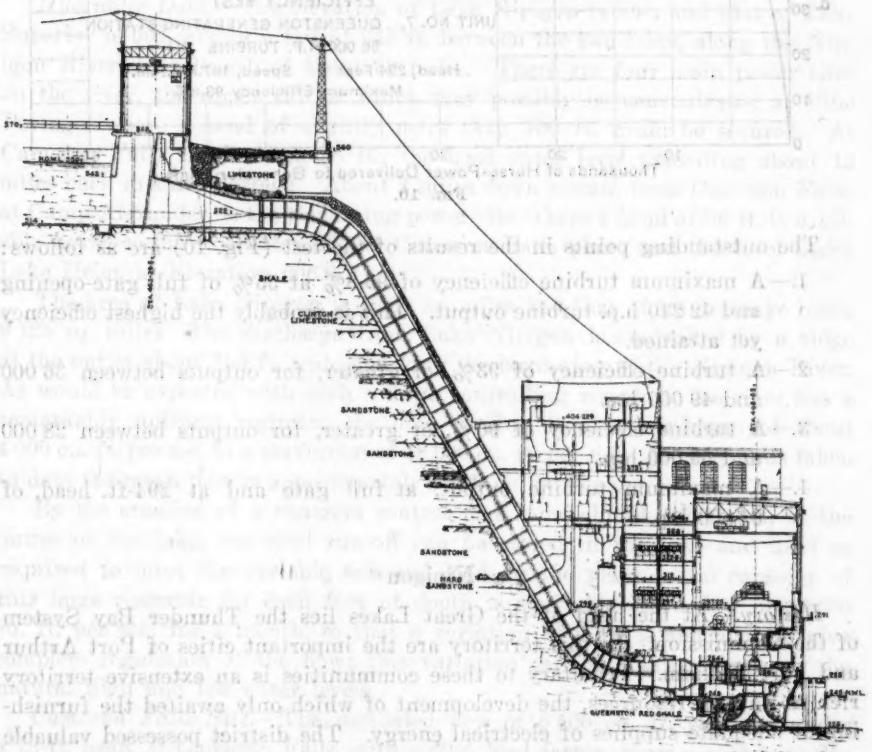


FIG. 9.—CROSS-SECTION OF STEEL PENSTOCKS, QUEENSTON-CHIPPAWA PLANT.

Gratifying results were obtained in testing the turbine and generator units. An efficiency test of Unit No. 7 at the Queenston generating station of the Commission was made on March 21 and 22, 1925. The electrical measurements were made by carefully calibrated electrical instruments. Sufficient

readings were taken to compute the generator losses and the turbine output was taken as the sum of the generator output plus generator losses. The measurements of water consumption were made by the Gibson pressure-time process, a detailed description of which has been published.*

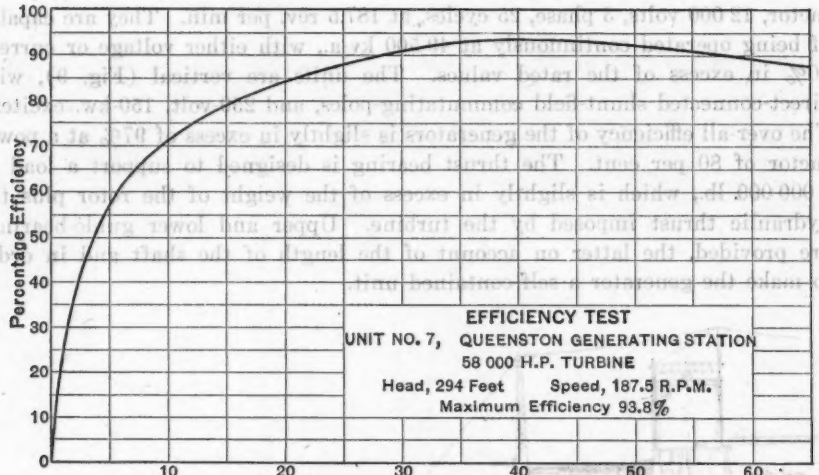


FIG. 10.

The outstanding points in the results of the test (Fig. 10) are as follows:

1.—A maximum turbine efficiency of 93.8% at 55% of full gate-opening and 42 210 h.p. turbine output. This is probably the highest efficiency yet attained.

2.—A turbine efficiency of 93%, or greater, for outputs between 36 000 and 49 000 h.p.

3.—A turbine efficiency of 90%, or greater, for outputs between 28 000 and 59 000 h.p.

4.—A maximum turbine output, at full gate and at 294-ft. head, of 64 800 h.p.

Nipigon

History.—At the head of the Great Lakes lies the Thunder Bay System of the Commission. In this territory are the important cities of Port Arthur and Fort William. Tributary to these communities is an extensive territory rich in natural resources, the development of which only awaited the furnishing of adequate supplies of electrical energy. The district possessed valuable undeveloped water powers.

As early as 1901, the City of Port Arthur had embarked in the business of municipal power development by constructing a plant on the Current River. The growing demand for power, however, soon made it necessary to provide for further supply. The results of the activities of the various munic-

* *Transactions*, Am. Soc. Mech. Engrs., Vol. 45, 1923.

ipalities in the Niagara District prior to 1906 and the creation of the Commission inspired the city officials of Port Arthur to seek a solution of its power problems along lines similar to those adopted by Niagara municipalities. Subsequently, the Commission purchased power from a power company operating in the district and, later, when this source of supply was beginning to prove inadequate, the municipalities of Fort William and Port Arthur entered into agreements with the Commission for additional power, with the result that the Commission made a thorough canvass of possible new sources of power supply.

The large quantities of power available from the Nipigon River, if developed, would permit the conversion of great supplies of pulpwood into paper at satisfactory costs and the opening up of the mineral and other resources of the Thunder Bay District. In view of this, and as a result of the investigation, Cameron Falls on the Nipigon River was selected as the site of operations, and construction was started in 1918.

Hydraulic Data.—The elevation of Lake Nipigon is 850, and that of Lake Superior is 602, giving a fall of 248 ft. between the two lakes, along the Nipigon River, in a length of about 32 miles. There are four main power sites on the river, the upper two of which may possibly be concentrated at Pine Portage, where a head of slightly more than 100 ft. could be secured. At Cameron Falls the head is 78 ft., the head-water level extending about 12 miles back to Pine Portage. About 2 miles down stream from Cameron Falls, at Camp Alexander, is the remaining power site, where a head of 60 ft. is available between the tail-water of the Cameron Falls plant and the level above Lake Helen at Elevation 606.0.

The area of Lake Nipigon is 1 530 sq. miles and that of its drainage basin 9 125 sq. miles The discharge from Lake Nipigon is controlled by a ridge at the outlet about 150 ft. wide, marking the beginning of the Nipigon River. As would be expected with such a large controlling reservoir, the river has a remarkably uniform regimen, the flow varying from a minimum of about 4 000 cu. ft. per sec. to a maximum of 18 000 cu. ft. per sec. From records taken to date the mean flow is approximately 6 500 cu. ft. per sec.

By the erection of a concrete control dam, now almost completed, at the outlet of the lake, the total run-off can be stored in the lake and used as required to meet the variable seasonal load on the plant. The capacity of this huge reservoir for each foot of depth is equivalent to a flow of 16 000 cu. ft. per sec. for 1 month, so that a variation of 4 ft. is sufficient to give complete regulation of the flow; this variation can be obtained between the natural high and low-water levels.

Cameron Falls Site.—The regulated flow of 6 500 cu. ft. per sec., under a 78-ft. head, at Cameron Falls, with a 60% load factor, gives a peak capacity of 75 000 h.p. The plant has been laid out for this installation. As it is situated at the lower end of Lake Jessie, which is merely an expansion of the Nipigon River that floods back through Lake Maria to Pine Portage, a distance of 12 miles, a large still reservoir is created. This area freezes over in the early winter, and, by thus providing an ice cover until the spring break-up, prevents the formation of the troublesome frazil and anchor ice.

This head pond has an area of more than 4 sq. miles, and gives sufficient storage to provide for the weekly load factor on the plant, with a variation in the head-water level of less than 3 ft.

At the location chosen, Nature has provided a convenient site where the power works may be concentrated within a small area. The power plant is made possible by the construction of a dam across the Nipigon River, and the excavation of short head-race and tail-race channels across the point of land formed by a bend in the river.

Power-House.—The main dam is a concrete spillway structure, provided with eight sluice-ways. Regulation is obtained by stop-logs, and wing-walls on either end extend the crest to the required contour level. The stop-logs in the sluice-ways are raised or lowered by an electrically-operated spud-winch, which moves along an operating deck over the top of the spillways. The total length of the dam is approximately 300 ft., and the maximum height in the center is 67 ft. In addition to the main dam, five auxiliary earth-fill dams and one timber crib dam were required.

The forebay occupies an opening in the natural rock walls of the river approximately 300 ft. long by 263 ft. wide. The power-house, containing six 12 500-h.p. units, is of the concentrated type of reinforced concrete construction in which the head-works, supply pipes, and power-house all form part of the same structure. The inlet for each unit consists of three sections 35 ft. high to the under side of the curtain-wall, by 10 ft. wide. Integral steel racks in sections to permit easy handling are installed in these openings, inside the head-works; through them the maximum velocity is approximately 2 ft. per sec. Behind the racks are the head-gates, three gates for each unit, the two outside ones being of the sliding type and the middle one a free roller gate which acts as a by-pass in filling or emptying the supply pipes. Each gate is 11 ft. wide by 18 ft. high. The sliding gates have bronze bearing strips, while the roller gates are of the Stoney type and can be opened against full water pressure. Ahead of the racks and head-gates, stop-log checks are provided for convenience in unwatering the racks or gates, should this be found necessary. It was considered essential in this climate to house the racks and gates in a building that could be heated in cold weather. It is due to this feature, in combination with the large ice-covered forebay, and with the low velocity, which latter not only prevents ice being drawn down under the curtain-wall, but also gives a negligible loss through the racks, that freedom from the operating difficulties usually experienced by power plants in the more northern latitudes has been found, even under severe climatic conditions.

The supply pipe for each unit is composed of three rectangular sections each 10 by 13 ft. The distance from the head-gates to the center line of the units is 86 ft. The supply pipes, which are of concrete, are reinforced for internal water pressure, and for loads from above when empty. The turbine scroll cases are also of reinforced concrete. Although the reinforcement required for this head was necessarily heavy, no difficulty was experienced either in the design or the construction, and by proper care at construction joints, the pipes are remarkably free from leakage. Ample drainage is provided in the turbine case to handle the leakage from the head-gates. The units are spaced

at 45-ft. centers. The draft-tubes are of the elbow type, the upper section having a steel-plate liner.

The tail-race, about 1 000 ft. long by 125 ft. wide at the water line, is an open cut across to the smooth water in the river below the falls. The upper section near the power-house is in rock and the lower section in earth, a rock-filled timber crib protecting the down-stream end from débris carried down by flood water.

Turbines.—The turbines are of the Francis, single-runner, vertical type, rated at 12 500 h.p. under a 72-ft. head, and operate at a speed of 120 rev. per min. The guide-bearings for Units Nos. 1 to 4 are of the Lignum-vitæ, water-lubricated type, and those for Units Nos. 5 and 6 of the oil-pressure, babbitted type. The governors are supplied with pressure from a central pumping plant having duplicate centrifugal pumping units with capacities of 650 gal. per min. The water used for the operating fluid, containing a small quantity of potassium bichromate, has proven quite satisfactory.

Construction.—Due to the location of the development considerable work of a preliminary nature was found necessary. This included the construction of a temporary power plant, of about 1 mile of railway, including a bridge across the Nipigon River, as well as of various camp buildings. The temporary power plant consists of two turbines obtained from an abandoned plant, which were utilized to drive one 350-kw. generator and three 300-h.p. air compressors.

The Nipigon River is justly famous for its trout fishing, and to help preserve this sport a fish-ladder has been provided around the west end of the power-house. This is of timber construction, about 400 ft. long. The design of the ladder was approved by the Provincial Game and Fisheries Department, and, it is believed, provides a satisfactory passage for the fish in their annual up-stream migration.

Central Ontario System

The Commission, in compliance with applications from municipalities in what is known as the Central Ontario System, negotiated the purchase of the Electric Power Company's system operating on the Trent River, including six generating plants with a total capacity of about 27 000 h.p. To take care of the continued increase in demand for electrical energy in this district the Commission has since constructed additional developments.

The first of these new plants was located at what is called Dam No. 10, or Ranney Falls, on the Trent Canal, near Campbellford (Fig. 11). In the construction of the Trent Canal, below Dam No. 10, the Dominion Government installed intake sluices in the canal wall immediately above Locks 11 and 12. It is through these sluices that the water is obtained for operating the Ranney Falls plant.

An open forebay, 124 ft. long, connects the sluices with the power-house. The regulated canal level at this point is 477.0 (sea-level datum). From the power-house, the tail-race, 30 ft. wide, extends 275 ft. to the Trent River, the level of which at its junction with the tail-race is 429.0. The forebay is 116 ft. wide by 12 ft. deep, at the intake. The side-walls converge, and the forebay

increases in depth, until at the gate-house the dimensions are 71 ft. wide by 24 ft. deep. This design is conducive to low velocity at the racks.

The gate-house substructure (Fig. 12) is a block of reinforced concrete, 84 ft. by 30 ft. in plan. There are four entry chambers, two for each unit, 14 ft. wide. The racks are at the head of these chambers, and are built of structural steel in two sections. In the center pier of the gate-house a 5 by 4-ft. chute is provided to carry off any ice or other floating material from the forebay.

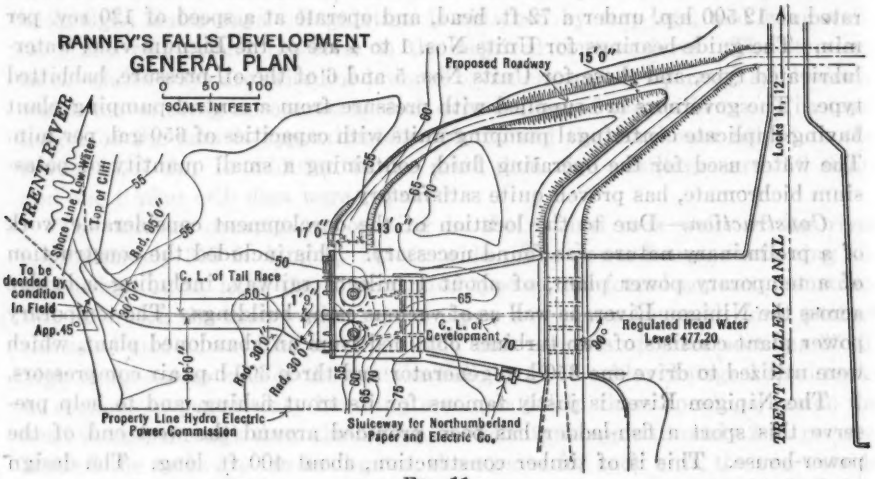


FIG. 11.

The double supply pipes connecting the gate-house to the turbine cases are each provided with a Stoney gate, one gate for each unit being of the sliding type, and the other of the rolling type. The supply pipes are of reinforced concrete. The double supply pipes converge at the bottom, leading the water to the turbine scroll cases in the proper direction, with minimum eddy loss.

The two turbines are of the vertical single-runner type, each direct-connected to a 4 500-kv-a. generator, and have a rating of 5 000 h.p. at 120 rev. per min., when operating under 47-ft. head. The two generators are each normally rated at 4 500 kv-a., 3 phase, 60 cycle, 6 600 volt, 120 rev. per min., at 80% power factor; but are capable of carrying 5 300 kv-a. at 80% power factor, with cooling air at an ambient temperature of 15° cent.

Developments at Dams No. 8 and No. 9

General.—The second and third developments recently placed in operation by the Commission at Dams No. 8 and No. 9 near Campbellford, contain some interesting features, in that they are automatic and operated by remote-control equipment. This enables their operation to be carried out from the central station at Ranney Falls, which is located about 1 mile from Dam No. 9 and 3 miles from Dam No. 8. It is felt that this remote-control feature will effect a considerable saving in the cost of operation for these two plants, which cost, in the case of smaller developments, is an appreciable factor in the cost of power.

The Trent River is a canalized stream forming part of the inland waterway between Georgian Bay and Lake Ontario by way of the Kawartha Lakes. The three main interests making use of the river—navigation, lumbering, and power—conflict considerably in their flow requirements, but a regimen for the control of the flow has been established, which serves as a basis for the installation of generating equipment. The regulating dams were constructed several years ago by the Federal Department of Public Works and the various power developments, of which there are a number, are installed in the neighborhood of these several dams and take advantage of the fall at the canal locks.

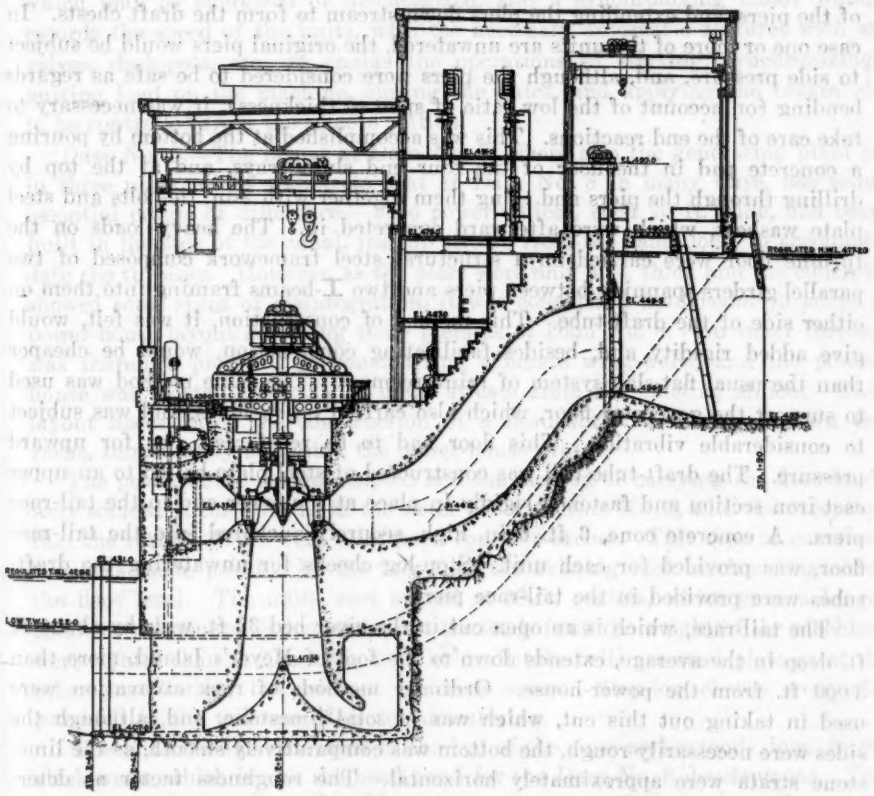


FIG. 12.—CROSS-SECTION, POWER-HOUSE, RANNEYS FALLS DEVELOPMENT.

A short distance below Campbellford the river is divided by Meyer's Island into two branches, known as the East and West Branches. Dam No. 9 extends across the river immediately up stream from the Island, being connected thereto by a concrete core-wall. Dam No. 8 is situated on the East Branch about 2 miles down stream from Dam No. 9 and a short distance from the lower end of the Island.

The minimum regulated flow down the East Branch at present available for power is 1 200 cu. ft. per sec., about 200 cu. ft. per sec. being allowed down the West Branch to satisfy riparian owners, but it would be possible to increase

this flow by means of storage, to 1 650 cu. ft. per sec. The net head at Dam No. 8 is 32 ft. and at Dam No. 9, 22 ft., and a flow of 1 650 cu. ft. per sec., at the load factor under which it is intended to operate these plants, gives a capacity of 6 600 h.p. at Dam No. 8 and of 4 800 h.p. at Dam No. 9.

Dam No. 8.—The generating plant at Dam No. 8 consists of three 2 200-h.p. units situated in the three most easterly sluice-ways. These sluice-ways are 20 ft. wide, the piers being 6 ft. thick, thus fixing the spacing of the units 26 ft. center to center. A considerable saving was effected by using the existing dam structure as a head-works, cutting slots for the racks in the sides of the piers, and extending the piers down stream to form the draft chests. In case one or more of the units are unwatered, the original piers would be subject to side pressure, and, although the piers were considered to be safe as regards bending (on account of the low ratio of span to thickness), it was necessary to take care of the end reactions. This was accomplished at the bottom by pouring a concrete pad in the floor of the four end sluice-ways, and at the top by drilling through the piers and tying them together with 2-in. tie-bolts and steel plate washers, which were afterward concreted in. The heavy loads on the turbine floor were carried on a structural steel framework composed of two parallel girders spanning between piers and two I-beams framing into them on either side of the draft-tube. This method of construction, it was felt, would give added rigidity and, besides facilitating construction, would be cheaper than the usual flat-slab system of reinforcement. The same method was used to support the generator floor, which also carried heavy loads, and was subject to considerable vibration. This floor had to be reinforced also for upward pressure. The draft-tube bell was constructed of steel plate bolted to an upper cast-iron section and fastened rigidly in place at the lower end to the tail-race piers. A concrete cone, 6 ft. 6 in. high, securely anchored into the tail-race floor, was provided for each unit. Stop-log checks for unwatering the draft-tubes were provided in the tail-race piers.

The tail-race, which is an open cut in the river bed 33 ft. wide by about 17 ft. deep in the average, extends down to the foot of Meyer's Island, more than 3 000 ft. from the power-house. Ordinary methods of rock excavation were used in taking out this cut, which was in solid limestone, and, although the sides were necessarily rough, the bottom was comparatively smooth, as the limestone strata were approximately horizontal. The roughness factor as determined from tests was about 0.37, which was slightly less than that used in the preliminary computations for tail-race losses.

Surplus and flood waters are diverted through the westerly sluices of the dam across Meyer's Island to the West Branch of the river, instead of allowing them to flow down the East Branch, thereby reducing the deposit of material in the tail-race cut, as well as avoiding some loss of head. This diversion necessitated the construction of a gravity retaining wall from the west corner of the power-house over to the Island, a distance of 220 ft., and a rock-filled timber crib and rock-fill section from the end of this retaining wall along the east side of the Island for a distance of about 1 500 ft.

The hydraulic equipment consists of three vertical units, in open-flume setting, each unit is rated at 2 200 h.p. under 32-ft. head. These turbines are reaction runners, and operate at 150 rev. per min. The governors are of the oil-pressure type, and as the piping for the three units is interconnected, and as one pumping unit is of sufficient capacity to operate two units, it is felt that no trouble will be experienced from this source under operating conditions. The noteworthy feature about this station is the installation of automatic and remote-control equipment to enable the units to be operated from the Ranney Falls Station. The additional attachments to the governors include a solenoid which may be energized or de-energized, and a synchronizing motor which records the speed of the units, with the necessary automatic features such as valves, dash-pots, etc., to enable the operations of starting, synchronizing, putting load on the machine, closing the gates, and applying the brakes all to be controlled from the remote station.

Dam No. 9.—The layout at Dam No. 9, where also the generating plant is in three units, while similar to that at Dam No. 8 in many ways, has some essential points of difference. Five power sluices, each 20 ft. wide, had been built in the side of the canal, just up stream from the canal lock, to accommodate the turbines. However, as the piers were only 4 ft. thick and the concrete showed some signs of disintegration, it was considered inadvisable to adopt a power-house layout adjoining these piers as at Dam No. 8. An inner forebay was therefore provided by constructing concrete wing-walls, and the power-house was located about 25 ft. down stream from the existing sluices. This layout necessitated the construction of a head-works section to contain the racks, immediately adjoining the power-house proper.

The power-house design adopted and the method of carrying the floor loads were somewhat the same as in the sister plant at Dam No. 8, with the exception that the turbine floors were reinforced concrete arches. This type of construction was made possible by the fact that the existing rock surface was above the floor level. The units were spaced at 30-ft. centers, which gave a longer span for the generator floor slabs, but the loads from the weight of the machines were lighter than at Dam No. 8. The draft-tube bells are steel plate as at the other installation, but without concrete cones. Stop-log checks were also provided in the tail-race piers for unwatering.

The tail-race was excavated in the bed of the river and extends down to the slack-water which is also the head pond for the Dam No. 8 development. This cut is 1 400 ft. long by 40 ft. wide and averages about 12 ft. in depth, branching out near the end into two channels, one on each side of a small island.

The turbines are of the open-flume, single-runner, vertical propeller type, and are rated at 1 600 h.p. under 22-ft. head, operating at 180 rev. per min. The governor and piping layout is almost a duplicate of that at Dam No. 8, the governors themselves having the same automatic and remote-control features.

Possibly it is too soon to state that these two plants, which are unique in that they are the largest of the remote-controlled type yet constructed in America, have been an unqualified success, as they have been operating only a

comparatively short time. Nevertheless, the conclusion to be drawn from operating experience thus far is that the development of many power sites hitherto considered commercially impracticable, will probably be brought within the range of economic feasibility, due to the saving that can be effected by the use of remote-control equipment.

TABLE 1.—HYDRO-ELECTRIC POWER COMMISSION. DISTRIBUTION OF POWER IN PROVINCE OF ONTARIO, CANADA.

System.	Dimensions of District, in Miles.		Number of municipalities.	Population.	Estimated system demand, in horse-power, 1925.
	Area served, in square miles.	East and west.	North and south.		
Niagara	13 500	236	101	254	1 800 000	700 000
Georgian Bay:						
Eugenia Division	4 050	76	76	30	46 500	20 000
Severn Division	2 300	50	62	21	44 500	
Wasdells Division	1 300	25	59	14	19 100	
Muskoka Division	1 250	42	30	2	3 900	
St. Lawrence	2 100	85	30	18	28 000	6 000
Rideau	1 500	60	36	5	16 350	3 000
Ottawa	400	30	16	5	134 650	16 550
Thunder Bay	4 000	90	50	2	17 000	45 000
Central Ontario and Trent	4 700	126	67	41	145 000	45 000
Nipissing	700	27	30	4	13 500	2 750
Total	35 800	....	....	396	2 268 500	838 300

TABLE 2.—HYDRO-ELECTRIC POWER COMMISSION OF PROVINCE OF ONTARIO, CANADA. SUMMARY OF CAPACITY OF PLANTS, IN HORSE-POWER.

System.			Capacity.	Ultimate capacity, present plus undeveloped.
Niagara:				
Ontario Power plant	183 500		788 500	898 500
Toronto Power plant	145 000			
Queenston-Chippawa plant	440 000	(8 units)		
Power purchased from Canadian Niagara Power Co	20 000			
Georgian Bay:				
Severn Division	5 800		19 450	29 950
Eugenia "	7 400			
Wasdells "	1 000			
Muskoka "	5 250			
St. Lawrence:				
Purchased power			10 000	.....
Rideau			3 450	6 550
Ottawa:				
Purchased power			20 000	.....
Thunder Bay			50 000	75 000
Central Ontario and Trent			47 028	53 128
Nipissing			3 600	5 600
Total			942 028	1 068 728

Statistics

In addition to the salient features of the more important of the generating plants serving the systems of the Commission already described Tables 1 and 2 present further details. When completed, the plants under development by the Commission will aggregate 1 000 000 h.p. The whole undertaking has been carried forward on the basis of municipal co-operation. The municipalities are financially responsible for the undertaking; the Commission on behalf of the municipalities, and the municipalities in connection with their local utilities, have invested $275 000 000 in order to obtain electrical energy at cost.

THE ST. LAWRENCE WATERWAY TO THE SEA

Discussion*

By Francis C. Shenehon, M. Am. Soc. C. E.†

Francis C. Shenehon,‡ M. Am. Soc. C. E. (by letter).§—The writer is grateful for the discussions, which supplement and illuminate his paper. These are mostly fair-minded and helpful in evaluating the several elements entering into this great project.

Let it be set down that the writer is not promoting this Waterway to the Sea. Instead, he states its ultimate economic desirability and therefore its inevitability. And he does not fix an early date of completion, because too many political and regional elements arrest the free play of economic forces; but he does anticipate immediate action on the restoration, deepening, and maintenance of the right of way of the Great Lakes. And he does advocate immediate action on the International portion of the Rapids Section of the St. Lawrence River, extending from above the Galops Rapids to the head of Lake St. Francis, primarily as a water-power project, but so developed that the ultimate deep-draft waterway will be provided for adequately.

When 25-ft. drafts become available in the Great Lakes System, in the Welland Canal and through the International St. Lawrence River, the remaining developments, between Lake St. Francis and Lake St. Louis and through the Lachine Rapids to Montreal, will follow in due course, possibly depending on the time when additional electric energy is needed in Canada.

It appears unnecessary to answer any discussions bearing on the commercial feasibility of the navigable waterway. The paper itself is clear and explicit as to the writer's views, which are still unchanged. The blending of navigation and water power on the St. Lawrence River is obvious. To what extent the opulence of the energy element will subsidize the transportation element is a matter of opinion. The utilization of all the water power as a commercial enterprise is certain; the utilization of the navigable waterway to the limit of its capacity is still uncertain. The water which will create untold millions of kilowatt-hours and horse-power-years, comes in largest part from Lakes Superior and Michigan-Huron and represents the contributions of the States of Minnesota, Wisconsin, and Michigan, and the middle and western portions of the Province of Ontario. The writer proposes to utilize, in the interest of larger continuous flow in the Niagara and St. Lawrence

* Discussion on the paper by Francis C. Shenehon, M. Am. Soc. C. E., continued from February, 1926, *Proceedings*.

† Author's closure.

‡ Cons. Engr., Minneapolis, Minn.

§ Received by the Secretary, March 29, 1926.

Rivers, the storage capacity of these great reservoirs. Navigation interests do not view favorably any draw-down in these reservoirs for the benefit of water power; and water-power interests do not view favorably any subsidization of navigation.

It is significant that navigation is the vital economic element for the United States rather than water power; and that for Canada, water power is the vital economic element rather than navigation. More than 90% of the commerce of the Great Lakes is American. With equal division of water at Niagara Falls and on the International Section of the St. Lawrence, approximately 67% of the ultimate water power will be Canadian.

Notwithstanding lesser commerce and lesser population, the Canadians are showing high optimism and enterprise in extending deep drafts down to Lake Ontario and into the Upper St. Lawrence by the expenditure of more than $100 000 000 in the New Welland Canal. On the other hand the United States—with a lake commerce eight times greater, with a population in the States bordering the Great Lakes more than four times greater and in the country as a whole more than ten times greater—has had neither the optimism nor the enterprise for a quarter of a century to secure and maintain even reasonably good drafts in this vital internal waterway; and has added no link—aside from the Davis and Sabin Locks at the "Soo"—to the ultimate waterway to the sea. And the Canadians have outstripped the United States in the volume of developed water power in the Niagara and St. Lawrence Rivers.

In reply to the suggestions of Mr. Hogg,* reference is invited to the judicial Engineering Reports (1925) of the Board of Review of the Sanitary District of Chicago. This Board was the most eminent group of engineers ever called together to investigate a great sanitary and hydraulic problem. It is well to point out the significant fact that the diversion of 50 000 cu. ft. per sec. in St. Marys River has yielded some beneficial horse-power years; and after the obvious engineering device of regulating works became effective, has made Lake Superior better for navigation. The local conservation use of the water is therefore twofold; and, in addition, the reservoir value of Lake Superior will eventually help to stabilize the energy output of the St. Lawrence River and the navigational use of the St. Lawrence Waterway. In any use of the water of the Great Lakes true economic analysis must balance the value of working water against wasting water; or must compare the present worth of water serving several purposes with the present worth of water to serve dual purposes twenty or thirty years hence. Engineers may leave certain legal questions to lawyers and statesmen and evaluate the service of the water, after regulating works have restored losses of depth and created storage to conserve in the future the wastes characterizing the past and the present.

Perhaps the writer's proposal to stabilize the flow of the St. Lawrence River by utilizing the reservoir capacity of Lakes Superior, Michigan and Huron, with some sacrifice of navigable depths in these lakes, represents more cosmopolitanism than patriotism. In analyzing a physical problem it is not easy for the engineering mind to give due weight to such intangibles

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1703.

as political boundaries and regional psychology. It may be well, however, to retain a detached world citizenship, leaving to statesmen and diplomats the allocation of the economic benefits possible with the best physical solution. Perhaps optimism and enterprise should be accorded the little sacrifice suggested.

The writer's thesis might be entitled "Conserving the Water Wealth of the Great Lakes System"; because, in the paper itself and in the discussions evoked, the value of the water and the ways of storing and utilizing it are developed. The water is valuable for navigation within the lakes—to the Gulf of St. Lawrence and to the Gulf of Mexico—and for water power in the Niagara, St. Lawrence and Illinois Rivers. Hydrological events, the views of discussing engineers, and months of further study have modified in some respects the tentative conclusions of 1924.

The chief hydrological event is the unprecedented prolongation and accentuation of the period of meager supply in the Great Lakes System, beginning in 1921 and still (March, 1926) prevailing. The mean supply for 35 years—1891 to 1925, inclusive—is 231 500 cu. ft. per sec. The supply and the St. Lawrence River outflow for the past 5 years are:

Year.	Mean supply, in cubic feet per second.	Mean outflow, in cubic feet per second.
1921	198 000	229 000
1922	215 000	229 000
1923	204 000	214 000
1924	197 000	221 000
1925	168 000	209 000
Mean	195 200	220 400

Assuming that the levels of all the lakes were maintained without any accumulated lowering and that 9 000 cu. ft. per sec. were diverted into the Illinois River, the mean outflow in the St. Lawrence River would have been 186 200 cu. ft. per sec. Conversely, assuming that the St Lawrence flow had been maintained uniform with a volume of 220 000 cu. ft. per sec., the levels of all the lakes would have been lowered 2.0 ft. to supplement by storage the deficient supply; and in order to recover the 2.0 ft. of surface-level loss in these 5 meager years, it will be necessary—with fixed St. Lawrence River outflow—to have 5 years of opulent supply as much in excess of normal as the meager years are below normal. Such a compensation in hydrological fortunes is vastly improbable.

It is interesting to note the actual maximum and open-season minimum monthly outflow volumes of the St. Lawrence River in this period of comparative water famine as given in Table 7. The largest unregulated outflow comes in months when the Harbor of Montreal is deep because of the large Ottawa River discharge; and the smallest outflow comes in the fall months when navigation needs ample river volume.

In these fairly generous outflow volumes for the St. Lawrence River is an excellent illustration of the antithesis of uniformity of reservoir levels and

uniformity of outflow volumes. In 1925 the supply is only 73% of normal, but the outflow is more than 90% of normal. The reservoirs of the lakes automatically waste the waters of the lakes to the ocean. The utilization of regulated Lake Ontario alone for storage, with the natural automatic drop in the big upper reservoirs, would have sufficed to maintain a reasonably uniform outflow in the St. Lawrence River. In the years 1926 and 1927, however, the St. Lawrence outflow must be meager, unless extraordinary water abundance prevails; because the St. Clair River with the present low Michigan-Huron elevation cannot outflow its normal supply. This must cause a deficit equal to the volumes to be stored in Lakes Michigan-Huron. This deficit in the St. Clair River contribution to the St. Lawrence flow will lessen each year until Lakes Michigan-Huron reach some elevation such as 580.2 ft. in the beginning of the year 1936. The elevations of the various lakes are as given in Table 8.

TABLE 7.—MAXIMUM AND MINIMUM OPEN-SEASON DISCHARGES OF THE ST. LAWRENCE RIVER.

Year.	Month.	Outflow, in cubic feet per second.	Month.	Outflow, in cubic feet per second.
1921	May	253 000	November	213 000
1922	June	258 000	December	210 000
1923	July	237 000	November	203 000
1924	June	244 000	December	208 000
1925	May	230 000	October	202 000

The problem of the co-ordinated regulation of the Great Lakes is the adjustment between the maintenance of the best surface elevations and the largest practicable minimum outflow in the St. Lawrence River. The degree of uniformity in the Niagara River is of relatively lesser importance, because water power may use a fixed volume of, perhaps, 100 000 cu. ft. per sec., while the volume applied to the preservation of the scenic grandeur of the Falls may be diminished in times of restricted flow.

TABLE 8.—ELEVATIONS OF THE GREAT LAKES.

Lakes.	Elevations, in Feet.		Deficiency, in feet.
	January 1, 1926.	Desirable.	
Superior	600.2	602.5	2.3
Michigan-Huron	577.5	582.0	4.5
Erie	570.2	573.0	2.8
Ontario	244.4	247.0	2.6

If the volume of prime flow in the St. Lawrence had been limited to 220 000 cu. ft. per sec. instead of 230 000 cu. ft. per sec. computed in Table 2,* the elevations of the Great Lakes on January 1, 1926, would have been as follows:

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1277.

Superior, 601.0 ft.; Michigan-Huron, 580.5 ft.; Erie, 571.5 ft.; and Ontario, 245.5 ft. The present unregulated deficiency, spread uniformly over all the lakes, would be 3.4 ft. Under co-ordinated regulation with a minimum flow in the St. Lawrence of 220 000 cu. ft. per sec., the deficiency would have been only 1.5 ft. The difference of 1.9 ft. represents the gain in navigational depth possible with co-ordinated regulation. Its value in the year 1926 would be not less than $10 000 000.

It is illuminating, if hazardous, to forecast the regain of surface elevations in Lakes Michigan-Huron based on average supply conditions and unregulated outflow in the St. Clair River. In making this forecast (Table 9) it is assumed that the restoration of regulated Lake Superior will be secured at the rate of about 2 in. per year—representing storage at the rate of 5 000 cu. ft. per sec.

TABLE 9.—ASSUMED RETURN OF LAKE ELEVATIONS.

Year.	ELEVATIONS, IN FEET, JANUARY 1.		ST. CLAIR RIVER.	
	Superior.	Michigan-Huron	Outflow, in cubic feet per second.	Quantity below normal, in cubic feet per second
1926	600.2	577.5	140 000	45 000
1927	600.4	578.5	156 000	29 000
1928	600.6	579.1	166 000	19 000
1929	600.7	579.5	172 000	13 000
1930	600.9	579.7	176 000	9 000
1931	601.1	579.8	.....	.....

Forecasting the unregulated outflow in the St. Lawrence River on the basis of normal or average supply, the mean volume of discharge for 1926 should be about 185 000 cu. ft. per sec., and for 1927 about the same quantity. It is an even chance that it will be somewhat greater and an even chance that it will be somewhat less. A portion of this outflow must come from the further draining and lowering of Lakes Erie and Ontario.

The serious deficiency in water supply of the past five years (1921-25) with resultant low lake levels and crippled transportation capacity demonstrates clearly the need of engineering control to conserve in years of abundant yield the waters of the Great Lakes and to expend by budgeted release in lean years the accumulated savings. The present concern over the depleted condition of the lakes had its parallel in the low-water period of 1895-96. The engineering remedy proposed then—more than a quarter of a century ago—was regulating works in the Niagara River.

Will regulation work? Colonel Pillsbury* expresses doubts and Mr. Sabin† after more than thirty-five years spent in Great Lakes problems—much the same as the writer's years of experience with these problems—is not sure. It is certain that the natural automatic operation of the lakes has failed to maintain the best navigable waterway.

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 2000.
† *Loc. cit.*, October, 1925, Papers and Discussions, p. 1706.

Mr. Sabin refers to the years 1910 and 1911 as illustrating the complexity of regulation. As a matter of fact a detailed analysis of the hydrology of these years—and of 1912 as well—demonstrates the practicability and necessity of regulation. In 1910 the supply naturally tributary to the St. Clair River was 68 000 cu. ft. per sec. less than their normal. The reserve storage of Lake Superior contributed 25 000, and Michigan-Huron, 28 000 cu. ft. per sec. The supply from the Erie and Ontario Basins was a little greater than normal. The outflow to salt water was at the rate of 234 000 cu. ft. per sec. Lake Superior by January 1, 1911, had fallen to Elevation 601.24 and Michigan-Huron to Elevation 579.33. Had super-regulation been in effect, these elevations would have been 601.8 and 581.3 ft., respectively. The gain of 2 ft. for vessel drafts in the St. Marys River and Lakes Michigan-Huron is significant. During 1911 the lakes would have risen slightly; and complete level restoration would have been accomplished in the generous supply year of 1912.

Mr. Sabin clings to the thought of single-river regulation illustrated by the works in St. Marys River—necessitated by water diversions—and suggests a tryout of regulating works in the Niagara River. Such works will help, but the principle to be followed if works are to be built in sequence is to build the lowermost works first; because the operation of works in the outlet of Lake Ontario cannot affect adversely the upper lakes. The operation of the Lake Superior outflow works has at times damaged the lakes below; and works in the Niagara River alone may damage Lake Ontario and the St. Lawrence River. Super-regulation, with works in the St. Clair, Niagara, and St. Lawrence Rivers making effective the existing works in the St. Marys River, is the sound engineering solution for the Great Lakes System. The cost of such works and the cost of operating them is negligible, because the vast financial importance of water transportation warrants adequate maintenance of this internal waterway, even aside from its ultimate destiny in overseas commerce. Does the operation of the scheme of super-regulation require an intelligence transcending that required in the operation of a super-power system or other engineering work? Will "omniscience" be required, as suggested by Mr. Sabin?

Four elements enter the problem: First, the release of a reasonably constant minimum volume of outflow through the regulating gates in the St. Lawrence River; second, the utilization of the forebays of Lakes Ontario and Erie as equalizing basins; third, the release of water through the St. Clair River to supplement the Erie and Ontario supplies in maintaining the fixed minimum volume of outflow through the St. Lawrence River; and, fourth, the operation of the reservoirs of Lakes Superior and Michigan-Huron as a unit of storage, with restrictions or enlargements of flow in the St. Marys River to keep the two basins in step with each other.

A detailed demonstration of the operation of the super-regulation of the Great Lakes System is beyond the limitations of this paper. The works to be utilized for outflow control do not need to be precisely detailed now. The writer would add to the works at the head of the St. Clair River a fourth

expansion basin and navigable pass (see Fig. 5*) to lessen current velocities and secure larger control volume.

No menace of unduly high water will exist in Lakes Erie and Ontario, because the relatively small areas of these lakes permit rapid disposal of surplus water by excess capacity channels; and these channels may be easily secured. Nor will any menace of unduly high water exist in Lakes Superior and Michigan-Huron, provided the base plane of control is conservative. Lake Superior has ample excess outflow capacity in lateral water-power canals. If a volume of 40 000 cu. ft. per sec. flows continuously through these canals, by-passing the rapids in St. Marys River (and this volume is never diminished), the value of the excess outflow capacity is unquestionable. And this same principle of continuous supplemental outflow creating excess capacity applies to the Chicago Drainage Canal and to the Welland Canal. Mr. Sabin questions the specific application of this principle.

Colonel Pillsbury raises the question of legality in stabilizing the levels of the lakes, suggesting that a riparian owner may insist on only the frequency of high-water periods shown by the hydrograph since 1860. The rights of riparians to natural sequences in lake levels and in volumes of river flow affect many engineering undertakings. In cases where the interests of shore owners in submerged lands clash with the necessities of navigation, the right of the riparian to compensation disappears. (See the case of United States *vs.* The Chandler-Dunbar Company.) The governmental right to modify the natural outflow of the St. Lawrence River in the interest of navigation and water power must be supreme.

The possibilities of the St. Lawrence Waterway to the Sea will continue to kindle the imagination. Some of the vital engineering principles are recorded in this paper.

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1260.

MULTIPLE-ARCH DAM AT GEM LAKE ON RUSH CREEK, CALIFORNIA

Discussion*

By Fred O. Dolson and Walter L. Huber, Members, Am. Soc. C. E.†

Fred O. Dolson,‡ and Walter L. Huber,§ Members, Am. Soc. C. E. (by letter).‖—The writers' conclusion that a type of dam other than the multiple arch should be selected for a site such as that at Gem Lake is well demonstrated by the great diversity of opinion, as to causes of deterioration and as to remedies, displayed in the discussions. The wide interest shown by this full discussion is, to them, a source of satisfaction.

The positiveness and the wide diversity of opinion regarding the quality of concrete in the original structure would be amusing if it were not to some extent a reflection upon the profession. It is interesting to note that criticism of materials and workmanship of the original structure is offered by those who have not made a personal examination, while others, as, for instance, Mr. Comstock,¶ who made a careful inspection of the structure previous to its repair, defend both and state that the evidence indicates that the work done and the results obtained were as good as could be obtained with the materials available. Many points brought out in the discussion are worthy of further consideration.

Mr. Jorgensen** contributes some interesting notes from the laboratory which made the analyses and tests of materials used in the original construction, as well as some interesting facts relating to that construction. He refers to some similar dams in Northern Sweden where low temperatures are encountered, and states that no damage has occurred in two years, although he admits that these dams are partly back-filled with earth as a protection against low temperatures.

In personal correspondence Mr. Jorgensen states that for each of the dams the back-fill behind the arch section covers that part which remains below the water level during cold weather. The writers are in entire accord with his conclusion†† that under severe freezing conditions it can hardly be expected that a plaster face applied to a thin concrete section will have a long life.

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* Discussion on the paper by Fred O. Dolson and Walter L. Huber, Members, Am. Soc. C. E., continued from March, 1926, *Proceedings*.

† Authors' closure.

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§ Cons. Civ. Engr., San Francisco, Calif.

‖ Received by the Secretary, March 25, 1926.

¶ *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 467.

** *Loc. cit.*, December, 1925, Papers and Discussions, p. 2005.

†† *Loc. cit.*, p. 2007.

Mr. Jewett* has offered some of the results of the many tests he made for The Nevada California Power Company in an effort to find a coating which would afford a water-tight face and, at the same time, would offer some degree of permanence. The defects of many so-called "water-proofing" compounds are intimated if not actually expressed. It will be noted that Mr. Jewett has, wisely in the writers' opinion, avoided the use of any trade names.

As an example of incorrect conclusions, Mr. Wheeler† and Mr. Lane‡ attribute the disintegration of the concrete to the presence of limestone and limestone dust. It is interesting to note that, in the lower part of the dam where the limestone (mentioned by Mr. Jorgensen as coming from the outlet tunnel) was used, no disintegration has occurred, whereas in that part of the dam which has disintegrated this limestone was not used.

Mr. Turner§ mentions certain concrete dams, much thinner than the Gem Lake Dam, and exposed at times to temperatures of 50° Fahr. below zero, but he does not state where these structures are located and at what season the water stored behind them is drawn down; consequently, the writers have been unable to make a comparison.

Mr. Wiggin‖ credits the writers with calling attention for the first time in a pointed way to the fact that the immersed side of a thin exposed wall of concrete is in the same danger from the formation of ice crystals in its pores, under the influence of cold coming through from the dry side of the structure, as the side not immersed in a thick, leaky wall.

The writers note Mr. Howell's agreement¶ with their conclusions and his reference to another dam of thin section of the flat-slab type which is deteriorating in the same manner.

Mr. Waugh,** who is thoroughly familiar with all conditions at Gem Lake Dam, having assisted in its original construction as well as having had charge of its repair, attributes the primary cause of the disintegration to the failure of the Gunite coating to prevent moisture from getting in back of the top skin. His conclusions are substantially in accord with those of the writers, that the slightest penetration of water under such climatic conditions will be followed by deterioration. In the paper doubt was expressed as to whether, under conditions imposed for construction at such sites, impervious concrete can be obtained under present-day conditions. After several seasons of hard-earned experience in carrying the responsibility of construction in this difficult location, Mr. Waugh is even more positive that "it is inconceivable that any concrete will prevent the penetration of moisture."

Mr. Flinn's†† suggestion that Engineering Foundation should collect data concerning the behavior of other thin section dams is a valuable one, pro-

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1453.
† *Loc. cit.*, December, 1925, Papers and Discussions, p. 2008.
‡ *Loc. cit.*, p. 2011.
§ *Loc. cit.*, p. 2008.
‖ *Loc. cit.*, p. 2009.
¶ *Loc. cit.*, p. 2011.
** *Loc. cit.*, p. 2012.
†† *Loc. cit.*, p. 2015.

viding such data are complete and include temperature records, raising and lowering of water surfaces, etc. The writers do feel that they have not overemphasized one example of deterioration. They have placed a notable example before the profession as completely as they have been able to present it and, through the discussion which has followed, as well as from other sources, have learned that this is by no means an isolated instance. In their opinion the serious tendency has too often been to observe complete silence in the case of sad experiences. Such experience is a rightful heritage of the Engineering Profession. Examples of similar structures that have stood (and the writers do not believe such success will be found in similar locations) will not afford a sense of security to the designer contemplating a new structure of this type to withstand similar conditions. To use the terse expression of another discussor, "engineering is not a matter of majorities".

Senator Luiggi* gives some interesting observations on the Gleno Dam, a multiple-arch dam in Italy, which failed but not from causes such as the deterioration at Gem Lake Dam. The writers are glad to note his concurrence in their opinion that for such a locality as that at Gem Dam a rock-fill dam has many advantages over other types. Senator Luiggi questions the use of preparations such as "ironite", or neat cement applied with a cement gun, in which view the writers concur. The effectiveness of these coatings is impaired by the expansion and contraction of the arches of the dam. He suggests methods of water-proofing the up-stream face of the dam; but since the methods suggested would necessitate the loss of at least one season's storage the cost is thus rendered greater than that of the method chosen.

The contribution of Mr. Merriman† is, in the opinion of the writers, one of the most valuable and constructive of all those submitted because he points out the difficulty, not only of making good concrete up to the point of placing it, but the danger in not properly caring for concrete while it is curing. Neglect of concrete during curing occurs in the great majority of cases. Mr. Merriman is to be commended for bringing this out so pointedly.

Mr. O'Shaughnessy‡ makes inquiry concerning the resultant losses sustained by the failure of the dam to function during certain seasons. During 1923, when an effort was made to water-proof the face of the structure, the loss was 10 000 acre-ft. of storage—a heavy loss when it is realized that at this locality this quantity amounts to approximately 12 000 000 potential kw-hr. The probability of having to face a similar, or even greater, loss in 1924 was one of the deciding factors in favor of the method of repair chosen, which was carried out entirely from the down-stream side with no loss of storage. Mr. O'Shaughnessy states that in an arch dam it is necessary to have concrete of such density that it will be water-proof. In the opinion of the writers, concrete which will not absorb moisture to any degree whatever does not exist. Therefore, for climatic and operating conditions, such as obtain at Gem Lake Dam, it is absolutely necessary that the impervious surface be provided next to the water, as required by the principles of Mr. George L. Dillman which

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 90.
† *Loc. cit.*, p. 93.
‡ *Loc. cit.*, p. 95.

were referred to by Mr. O'Shaughnessy. If this face is provided and is permanent, it should be immaterial whether the concrete back of it is porous, so long as this concrete is able to withstand the stresses imposed upon it as a structural element of the dam. The writers are glad to note that Mr. O'Shaughnessy, with his extensive experience, commends a rock-fill type for a location such as that at Gem Lake Dam. It is also interesting to note that Mr. O'Shaughnessy plans ultimately to replace the buttressed arch dam at Lake Eleanor with one of a rock-fill type.

Mr. Lippincott* very clearly expresses an idea, which the writers have endeavored to make plain, in his statement:

"Although a thin shell of concrete may be made water-tight against percolation, it is not possible by any process known to the writer to prevent it from absorbing water when in contact with it."

Observations of diagonal cracks in the buttresses of several multiple-arch dams have been made by the writers and they are in entire accord with Mr. Lippincott's views that these buttresses should be reinforced. These diagonal cracks, although not an important consideration in Gem Lake Dam, did exist near the up-stream toes of three of the buttresses. They are described and comments are made on their prevalence in other multiple-arch dams under "History of Operation",† in the writers' paper.

Mr. Reed‡ makes inquiry concerning the conditions at Lake Agnew Dam, a smaller but similar multiple-arch dam on the same stream approximately 550 ft. lower in elevation. The concrete in Lake Agnew Dam has not disintegrated, but a small amount of leakage has developed. The Gunite surface was removed in 1925 and various types of water-proofing compounds are now being tried on the up-stream face. In the opinion of the writers, deterioration of the concrete in Lake Agnew Dam has not occurred because some protection from extremely low temperatures is afforded by the snow with which the entire down-stream face is usually covered before the extreme cold weather occurs. In fact, the arches are banked full of snow at such times. The protection by such banking is apparent from the fact that a valve enclosed in a light board shaft does not freeze. Mr. Reed cites instances of multiple-arch and flat-deck dams that have been successful in Norway and Sweden, but indicates that protection against low temperatures has been provided. In the opinion of the writers such protection is essential.

Mr. Jakobsen§ opens his discussion with a statement that it would be hazardous in the extreme to conclude anything from the behavior of a single structure and more especially since other similar structures, subject to equally severe conditions, have shown no sign of deterioration. His references to such structures are very meager indeed. It would be interesting to know the exact location of each and to have exact climatic and operating data for each. Mr. Jakobsen apparently has missed the statement in the "Introduction"‖ to the

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 98.
† *Loc. cit.*, September, 1925, Papers and Discussions, p. 1319.
‡ *Loc. cit.*, January, 1926, Papers and Discussions, p. 99.
§ *Loc. cit.*, February, 1926, Papers and Discussions, p. 293.
‖ *Loc. cit.*, September, 1925, Papers and Discussions, p. 1310.

writers' paper to the effect that a large amount of mathematical discussion concerning multiple-arch dams had already been presented, and that it was not intended to include any addition to such material. The paper was a discussion of a failure and a method of repair, not of the theory of design. He can rest assured, however, that thorough analyses of the new section and of its effect on the remaining original structure were made, not only by the engineers of The Nevada California Power Company, but by the several consulting engineers who were employed and that no important considerations were left to off-hand assumption as he seems to infer.

Mr. Oesterblom* seems to have missed some of the salient points of the paper. It is easy to understand why the middle belt of the arches was affected and why the section at the bottom, as well as at the top, was little affected. The up-stream faces of the lower parts of the arches are generally submerged to considerable depth in water, which insures somewhat warmer temperature. At the same time the backs of these arches are protected from extreme low temperature by the early snows drifting against them, thus forming a blanket; also, the arches themselves are much thicker at this point, thus affording less opportunity for the extreme cold to chill the material completely. These conditions, combined with the fact that the upper sections, from which water is always drawn before the advent of cold weather, do not show deterioration, seem to afford ample proof that frost action is the controlling cause.

Mr. Collier† offers a conjecture on what results would have been obtained had a thicker coating of Gunite with reinforcing been applied. He states, in closing, that with a thickness of approximately 2 in. of reinforced Gunite, experience has shown that positive water-proof conditions will be obtained. It would be interesting to know the number of years during which such conditions have obtained on any structure and what conditions of expansion and contraction, or of deflection, occur in the operation of the structure. The writers believe that a Gunite face of these specifications would have prevented percolation of water at first, but that over a period of years the Gunite would have absorbed water and would have disintegrated to such an extent that water would have been admitted to the concrete back of it, unless it was replaced before this occurred.

With much of the discussion of Mr. Noetzli‡ the writers disagree entirely. Mr. Noetzli refers to the Lake Eleanor Dam as being an entirely successful structure, but apparently he has not noted that Mr. O'Shaughnessy, from his discussion, plans ultimately to replace this structure with a rock-fill dam. He refers to the Bear Valley Dam as being a similar structure with similar climatic conditions. At the Bear Valley Dam the reservoir is not regulated for hydro-electric power development and is ordinarily not drawn down during winter. At this location temperatures do not range as low as at Gem Lake, nor do long sustained extreme cold spells occur. Climatic conditions are not comparable. After referring to the adopted method of repair as being as radical and expensive as could be devised, he launches off with confidence into

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 296.
† *Loc. cit.*, p. 298.
‡ *Loc. cit.*, p. 299.

advocacy of a method that was considered and discarded by the several engineers who studied this problem for the owners. His method is not believed to be feasible, even if considered only with relation to this single structure. However, Mr. Noetzli has completely missed an important practical point. Gem Lake Reservoir is now an important and necessary part of a great operating hydro-electric system. Any method of repairing Gem Lake Dam that would render the reservoir useless for a storage season, or even a part of such season, must prove expensive, due to loss of revenue; Mr. Noetzli's proposed method would cause such a loss. The method adopted avoided this great economic loss and thus it has a large differential in its favor. Mr. Noetzli does not consider that a loose rock-fill, which is relatively porous, will materially protect a concrete core wall or a concrete face slab against low temperatures. The writers know the facts to be otherwise. Proof can be secured by taking temperatures in tunnels through rock-fill dams where it will be found that at a distance of about 60 ft. in from the tunnel portal water will not freeze. For example, Mr. Dolson had occasion to construct a tunnel 157 ft. in length through the Skagway Dam near Victor, Colo.; later, he used this tunnel for the storage of foodstuffs and found that 60 ft. in from the tunnel portal no freezing occurred.

The several examples of hollow concrete dams described by Mr. Scheidenhelm* are interesting, but those particular structures which are enclosed by a deck on the down-stream face are not comparable.

The writers wish to express their appreciation of the very clear statement from Mr. Galloway,† interpreting the lessons from Gem Lake Dam which they have offered to the profession. He has most clearly and concisely pointed out the value which may be gained from a proper interpretation of an engineering failure.

A dam of the thin multiple-arch type, if it is to resist the conditions existing at sites such as at Gem Lake, must depend on a permanently impervious face; but, under present-day conditions, the writers know of no practical means by which such permanent imperviousness can be obtained nor has the discussion disclosed one. They, therefore, adhere to the opinion already expressed that for this or similar locations a dam of the rock-fill type has many advantages over other types.

In closing, the writers express their appreciation to the officers of The Nevada California Power Company for their courtesy and their frankness in granting permission to publish a full account of the investigation and repair of this structure. It has been a source of satisfaction to note expressions of appreciation of the Company's attitude from many of those who have contributed to the discussion.

* *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 457.
† *Loc. cit.*, February, 1926, Papers and Discussions, p. 294.

PERMISSIBLE CANAL VELOCITIES

Discussion*

By Samuel Fortier and Fred C. Scobey, Members, Am. Soc. C. E.†

Samuel Fortier‡ and Fred C. Scobey,§ Members, Am. Soc. C. E. (by letter).‖—The writers wish to express their appreciation of the additional constructive information contained in the discussion. From comments in the discussion there appear to be two main points that may be enlarged upon, namely,

1.—Kennedy's theory and formula and its relationship to "permissible canal velocities"; and

2.—Soil classification particularly as influenced by colloidal material in a dispersed state, contributing to plasticity.

The flow of water in canals may be said to consist of three phases of velocity in its relationship to the movement of soil particles:

(*a*) Non-silting velocities, or in the words of Kennedy¶ himself, "critical non-silting mean velocities", further explained by the statement:

"Thirty channels were selected, all known to be more or less in permanent régime, and all having self-silted and similar sand beds. Evidently each of them was fully charged with silt, since each had the option of picking up more sand from the bed or dropping any overcharge."

(*b*) Transporting velocities—those which will move down stream particles already detached.

(*c*) Scouring velocities—those which will erode a well-aged canal bed.

Phase (*a*) implies a condition of equilibrium in the flow of a silt-laden canal—a water completely loaded with silt for that particular velocity and depth of channel, and neither taking on more silt nor depositing what it has. Likewise, the implication exists in the Kennedy quotation that the bed silt which might be taken on as a load is quite free to join the silt being transported—that it is not firmly bedded down, as defined on page 1401.** It is readily seen that any particular soil unit from the finest of clay particles to cobble-stones, could be more easily picked up for further transportation within

* Discussion on the paper by Samuel Fortier and Fred C. Scobey, Members, Am. Soc. C. E., continued from April, 1926, *Proceedings*.

† Authors' closure.

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‖ Received by the Secretary, March 24, 1926.

¶ "Hydraulic Diagrams for Channels in Earth, Giving Discharge, Mean Velocities, and Silting Data," Second Edition, 1907, p. 1.

** *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions.

a short time after it last came to rest than after it had either adhered to adjoining bedded particles or had become interkeyed with adjacent units.

Phase (*b*) differs from Phase (*a*) in that it deals with a water that may not be already loaded. In other words soil particles that would not be moved under Kennedy's specifications would be transported under the same velocity conditions by a water not already loaded to capacity. This phase applies only to soil particles that are perfectly free to move—either particles journeying down stream that have never become part of the settled bed of the canal or particles that have been raveled off from the bed due to velocities under Phase (*c*). Gilbert differentiates between material moved by "stream suspension" and that moved by "stream traction".

Phase (*c*) covers velocities that will scour a canal bed which has become well-seasoned under lesser velocities. These are the velocities that "permissible canal velocities" may be allowed to approach, but not to equal. For other given conditions they are higher than the velocities in Phases (*a*) and (*b*).

The writers have the utmost respect for the work of Mr. Kennedy and feel that his theory and formula, adjusted by the proper coefficients for the waters and silts under consideration, should be the basis for all canal design where silted waters are encountered. Their objections are directed at various writers who misapply Kennedy's formula in discussing scouring velocities.

There is the implication that below a certain velocity for a given channel and given soil conditions silting takes place and above it scour occurs. This may sometimes be true for the traction of loose particles. The writers' contention is that the well-seasoned bed will not scour under a velocity well above the silting velocity limit.

In a similar manner transporting velocities have been used as criteria for scouring velocities. One of the most quoted authorities on hydraulics (Merriman), in discussing velocities to be used in the design of irrigation canals, states:*

"The following statements show approximately the velocities which are required to move different materials:

0.25	ft.	per	sec.	moves	fine	clay.
0.5	"	"	"	"	loam	and	earth.
1.0	"	"	"	"	sand.		
2.0	"	"	"	"	gravel.		
3.0	"	"	"	"	pebbles,	1	in. in size.
4.0	"	"	"	"	spalls	and	stones.
6.0	"	"	"	"	large	stones.	

"The mean velocity in a channel may be somewhat larger than these values before the materials will move, because the velocities along the wetted perimeter are smaller than the mean velecity."

It is doubtful whether "somewhat larger" could be construed to mean from fifteen to twenty times as great for clay as is indicated in Table 6.† Likewise, Merriman's list would indicate that a greater velocity is permissible in a canal with a sand bed than in one with either clay or loam or "earth", whereas sand alone will be eroded by lesser velocities than any other type of soil particles.

* "Treatise on Hydraulics," Eighth Edition, N. Y., 1908, p. 288.

† *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1411.

Even pure sand will bed and pack down until it requires a higher velocity to move it than the "transporting velocity" which could move the same sand down a flume of such material that the sand could not start to "bed down". The amount of the "bedding down" is indicated in the difference between 1.00 ft. per sec., in Merriman's list, and 1.5 ft. per sec. given in Table 6.

Considering velocities that will scour a well set canal bed, the following statements regarding soils must be taken as general only. The soil types are so varied and blend from one gradation to another and from "light" to "heavy", and through so many mineral and chemical modifications that there are cases where general statements do not hold. For instance, ordinarily sand is not regarded as having plastic properties such as are usually associated with clay and probably it seldom occurs in a cohesive state in a canal bed. However, in a previous condition such sand may have been mixed with colloidal material that would cause more or less cohesion between the sand particles. In its progress down a river or canal such a sand would probably have much of the colloidal material removed by attrition. It is suggested by Macy Lapham, Inspector for the Western Division, U. S. Bureau of Soils, that the colloidal matter so prevalent in the waters of the Colorado River may have come in part from the sand particles which are comparatively free from plastic material by the time they reach Yuma, Ariz., and the Imperial Valley, California.

In a paper of this nature, discussion of soils must be confined as nearly as possible to the phases as they appear to engineers and its lack of detail will allow any soil technician to point out exceptions to the statements made. From the viewpoint of the engineer—particularly the irrigation engineer—it is unfortunate that the soil technicians have chosen the word, "silt", for their grade in size of material next above that of clay. When engineers speak of a reservoir "silting up" they mean the reduction of storage by everything brought in by the river. The engineer and the layman nearly always mentally associate silt and water and this is borne out by Webster, yet some of the great areas of soil of high silt content as classified by soil technologists were not water-laid but derived from loessial deposits, probably wind-laid. On the other hand, "clay" is not immediately associated in thought with water, yet the predominating areas of arable clay soils of the Western United States are water-laid.

To enlarge on the soil materials listed in Table 6, Column (1): Fine sand, sandy loam, and silt loam are listed as non-colloidal. There seems to be little question as to sand. It is composed of hard distinct units that ordinarily have no cohesive attraction one for another. There are exceptions, as mentioned before. Any resistance to scour exerted by a canal bed of sand, other than the weight of the particles, lies in whatever interkeying has taken place since the last disturbing condition.

As to the various modifications of "loam" there may be more uncertainty. The word, "loam", whatever the modification, with the possible exception of "clay loam", conveys the conception of friability. It must be of a low plastic property or it would not be a loam. A loam does not "clod up" when plowed, and when a lump is moistened the tendency is for it to break down into a rather non-sticky mud. A dry lump of loam crushes freely under a boot-heel.

Clods of silts or clays give back a resounding resistance when struck with a shovel. However, such clods may rapidly break down in water if the colloids are in a flocculated state. The fact is to be accented that in soil-particle sizes there is no such gradation as "loam". It must exist in a mixture. Sandy loam is rather more sandy than clayey. Clay loam indicates the predominance of clay rather than sand, and so on.

The next category is "alluvial silts when non-colloidal". Alluvial silts may or may not yield a slick sticky mud when wetted by canal water flowing in an excavation in such silts. The criterion of identity is based largely on the state of the colloidal matter in the silts. If the silts were laid down or filtered out in seepage while containing "effective colloids" as defined on page 1400,* the particles are in a dispersed—extremely fine—status; while if they were deposited as aggregates or flocculated units, they do not have the plastic attributes as described by Hilgard, and quoted by the writer.† In an unpublished paper on the silts of the Colorado River, J. F. Breazeale, Associate Biochemist of the U. S. Department of Agriculture, points out that a silt may change back and forth many times in its progress from the upper benchlands of Colorado, Wyoming, or Utah, and from the "bad lands" of New Mexico and Arizona. This change is a physico-chemical reaction between the silt particles and the particular waters that are encountered as the silt travels down stream. This reaction is from a dispersed (effective) to a flocculated state, and *vice versa*.

"Volcanic ash" or "lava ash" is popularly applied to much of the finer grained soils of agricultural value east of the Cascade Range, in the States of Washington, Oregon, and Idaho. The term, "volcanic ash", is a misnomer as this soil contains little or no volcanic ash material associated as to its origin with volcanic activity. It is mainly a wind-laid accumulation and derived from dessicated sediments of old lake beds and from the finely textured weathered particles of the basaltic lava flows. When wetted it forms a slick mud but without the plastic characteristics of the clays. It is friable when the moisture is dried out of the mud. However, when wetted, as in a canal bed, this material—intermediate in size between the loamy soils and the plastic clays—will withstand velocities slightly greater than soils that are distinctly loamy and, therefore, friable in character.

The foregoing discussion on the characteristics of colloidal and non-colloidal materials will suffice to explain the terms, "very colloidal", as applied to stiff clays and, "when colloidal", or "when non-colloidal", as applied to graded material that may have fine silts or even particles of "clay" size or less that are effectively plastic if in a dispersed state, but are not plastic if in a flocculated state.

Taking up the individual discussions in the order in which they appeared: Mr. Muckleston‡ senses the difficulty in soil determination as already described. He also mentions the same problem later found in the discussion

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions.
† *Loc. cit.*, p. 1400.
‡ *Loc. cit.*, December, 1925, Papers and Discussions, p. 2035

of Mr. Weiss,* that is, the fact of rapid changes in soil types within such a short distance that no adjustment in velocities would be feasible. He suggests that alkali soils erode easily. This is true for most alkalis found in the West with the exception of black alkali (sodium carbonate). (See discussion by Mr. Hart.†) Other alkalis, being powerful electrolytes, flocculate colloidal matter so that it no longer has the property of forming sticky, extremely cohesive, canal beds.

Mr. Chandler‡ brings out a point that perhaps should have been emphasized in the paper. This paper deals only with maximum permissible velocities—velocities just short of those that will erode a canal bed. Of course, there may be other factors, such as the need for silting of porous channels, which will entirely change the problem so that the velocities desired are just less than Kennedy's critical velocity or may involve transporting velocities, as already mentioned under Phase (*b*).

As to aquatic growth that seemed to flourish in high velocities, the writers have noted moss in chute-drops where the velocities approached 40 ft. per sec., and, furthermore, this moss was literally alive with small worms, some sort of larval growth. It is suggested that the "swell" of water of such high velocity, carrying almost 30% by volume of entrained air, gives the additional oxygen that makes this flourishing growth possible.

The writers appreciate the additional data contributed by Mr. Stockton.§ Few engineers have had the operating experience under many conditions that has been his lot. For about twenty years he has operated canals in the Mountain States and the Province of Alberta. His comments are authoritative. Mr. Hart, makes an excellent point that has previously been overlooked although it involves most obvious hydraulics. If a canal loses by seepage there is a current available to carry silt and sand down into the original bed material, making it tough and well graded by filtration alone. If the water-table is such that the flow of ground-water is inward no such current is available and there is less tendency for a canal bed to become well set by the grading of material, especially by the introduction of the fine plastic clays.

Mr. Hill‖ misunderstands the writers when he states that they dismiss Kennedy's formula and theory as fallacious. Kennedy's theory is accepted. When misused the results are fallacious. Kennedy's first edition of "Hydraulic Diagrams for Channels in Earth, Giving Discharges, Mean Velocities and Silting Data, etc." appeared in 1895. Presumably the second edition, written in 1907, contains his retrospect after twelve years of applied use of his formula. In his title he mentions "silting data". A careful perusal of this second edition discloses that in no place does Kennedy confuse his critical velocity with a velocity that will scour a well set canal bed. The writers do dismiss as fallacious the findings of later writers who have misinterpreted Kennedy's silting data, applied it to scouring data, and offered limiting velocities for

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 115.
† *Loc. cit.*, December, 1925, Papers and Discussions, p. 2039.
‡ *Loc. cit.*, p. 2036.
§ *Loc. cit.*, p. 2037.
‖ *Loc. cit.*, January, 1926, Papers and Discussions, p. 111.

earthen canals based on such silting data. They regret exceedingly that any statements in the paper could be construed as disagreement with Kennedy's theory in general. Much of Mr. Hill's discussion applies more directly to a paper on the silting of canals and should be borne in mind by the Sub-Committee of the Society's Special Committee on Irrigation Hydraulics, when it makes its final report on "silt".

Thus far in this paper and discussion no mention has been made of canal depth as a function of the permissible velocity. As pointed out by Mr. Hill, it is certain that depth would enter the problem in any case where sand bottom exists. The scouring velocity and transporting velocity might approach the same figure, and fluctuating pressures would cause disruption of the canal bed by developing "boils".

It is rather difficult to reply to Mr. Lindley.* It is regretted that he was not more specific regarding the Kennedy theory. Evidently, he, like Mr. Hill, misinterpreted the paper as disparaging to Mr. Kennedy and his work. The citations given by Mr. Lindley, have not been located. It is to be regretted that the valuable reports and professional papers of the irrigation engineers of India, Egypt, and South Africa, are not more generally available in the United States.

The data in the contribution of Mr. Rohwer† is similar to that of Mr. Hill in that it applies to silting velocities or transporting velocities rather than to limiting canal velocities. His photograph (Fig. 3)‡ of cracked silt deposits along the Arkansas River, is typical of other rivers as well. Exactly the same condition exists in the Palo Verde and Imperial Valleys of California, the silt coming from the Colorado River,§ and in the valley of the Rio Grande.

Mr. Rohwer's Fig. 2‖ does not appear typical of an erosion condition. Nearly all true erosion, or more properly "corrosion," shows ragged, nearly vertical slopes, at least on one side of the excavation. The tables given to show the movement of sand as the velocity increases apply to the transportation of a sand prism typical of many canals served by water from the eastern side of the Colorado Rockies. The true beds of these earthen canals are seldom exposed to view. This condition also exists in many of the concrete lined canals of the citrus region of Southern California. Here the concrete bottom is often covered with a traveling load of sand ranging from isolated patches to dunes more than 1 ft. thick.

Mr. Houk¶ adds a great deal of pertinent data, particularly as to the fact that higher velocities may be permitted in deep channels, for the same type of bed material. It is a question whether some of his cited figures may be applied directly to irrigation and power canals. One of the primary assumptions in the paper was that the canal bed should be well aged. For newly made

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 114.

† *Loc. cit.*, April, 1926, Papers and Discussions, p. 634.

‡ *Loc. cit.*, p. 635.

§ See discussion and views of Colorado River silt, by Harry T. Cory, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), pp. 1223 *et seq.*

‖ *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 635.

¶ *Loc. cit.*, p. 638.

canals an entirely different set of velocities would apply. Such velocities would not enter into the calculations of the designer unless the canal were to be used immediately at or near peak load and there were no possibility of checking the water by proper grade correction. Some of the velocities cited by Mr. Houk as scouring the channel in alluvial material recently inundated by a severe flood would undoubtedly not have been so effective in a well bedded canal, gradually seasoned. On the other hand some of his velocities cited as without scouring effect, may have had too little time. It is probable that inundating floods, in their first stages, pick up a tremendous quantity of trash, too heavy for suspension but not too heavy for bottom traction. Such a condition would cause a great lag at the bottoms of the vertical velocity curves. It is conceivable that mean velocities of 10 to 15 ft. per sec. might have bottom velocities of only 4 or 5 ft. This would be particularly true over the usual inundated area, grown over with more or less of brush and weeds. Given sufficient time, this protecting cover would be worn off and effective scour would develop.

The reason for not including the element of depth is the lack of data as to an increase in permissible velocities for an increase in depth. Mr. Houk mentions the work of several of the best known Far Eastern irrigation engineers. It is true that their writings yield much light on the subject of canal velocities, but all of them do not differentiate between these velocities as has been done in this paper. Specifically: Kennedy, as already mentioned, is dealing with non-silting velocities over a bed of shifting material. He may have written on other phases of velocity. Willcocks mentions scouring and silting velocities in several places; not under a specific heading dealing with either of these subjects. Regarding scour, he says:

"The laws of scour are not quite the same as the laws of transport, but are analogous. Here the scour depends on the pressure which the water is able to effect on the particles, on the size and weight of the particles, on the surface exposed to the water's action, and on the tenacity with which one particle clings to the next. Smoothness or roundness in the particles will diminish the pressure, but will decrease the tenacity. A mixture of various sizes of silt will probably diminish the surface on which the water can bear, and will decrease scour."

Again, in writing of silts, he says:

"In November and December the water is comparatively clear, and if advantage is taken of this clear water it can be utilised to scour out the deposits of fine mud in the canals before the mud has had time to harden and consolidate."

Evidently, Willcocks was under no misapprehension as to the various types of velocities to be encountered. It is to be regretted that he did not devote a chapter exclusively to the subject of canal velocities, bringing together his scattered material on this general subject. It may be noted in his second quotation that he senses the writers' premise of aging in a canal bed.

Buckley does not confuse the various phases of velocity and he has a table of permissible canal velocities that agrees quite closely with a similar one developed years before by Colonel Clibborn and that also agrees within reason

with Table 4* (Etcheverry), and Table 6, Column (2). Bellasis writes of silting velocities and transporting velocities only. Parker† in a large volume that goes exhaustively into some phases of irrigation and other hydraulics, treats of silting velocities and transporting velocities but makes no mention of those classed under the title of this paper. He must have had the writings of Clibborn, Willcocks, and Buckley as well as those of Kennedy and Deacon (on transporting velocities). He may have been convinced that permissible velocities were synonymous with those of one of the other phases mentioned at the beginning of this discussion.

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1408.
† "Control of Water," Second Edition, Lond., 1925.

UTILIZABLE CAPACITY OF STEEL MEMBERS OF STRUCTURES

Discussion*

By Messrs. F. E. Schmitt and Henry S. Prichard.†

F. E. Schmitt,‡ M. Am. Soc. C. E.—Hitherto, the relation between the ultimate strength and the working strength of structural members has of necessity been a matter of judgment based on structural experience, illuminated and supplemented by the data of tests on parts and entire structures. It remains so to-day, and the art of testing, in spite of its high development, has not yet progressed to the point of allowing the engineer to utilize it for definite verification of his design procedure or selection of his working stresses. In large part, this shortcoming is due to the virtual impossibility of determining service requirements and translating them into quantities commensurate with test values. The line of progress is strongly in the direction of placing safety margins and working stresses on a rational basis, however, and the author's scholarly paper is of value in inviting closer attention to this objective.

Views long abandoned are brought forward anew in the reversion to a fatigue or endurance basis for structural design. It is now commonly accepted that steel structures are free from fatigue action, and no evidence to the contrary has been brought forward. Occasional tests of wrought iron and steel taken from structures long in service have in no case shown recognizable weakening or reduction of ductility of the material. Moreover, the large-scale experiments of recent years on fatigue endurance have established that in steel and other ferrous metals there is no fatigue at stresses lower than a certain limiting amount, the endurance limit, which is not far below the yield point even in case of complete reversal during the repetitive stressing, and probably very near to the yield point for one-directional stress repetition. Stresses permitted in structures are well below these limits, and the evidence is clear that such stresses may be repeated an indefinite number of times without effect on the metal. Thus, it would seem that there is no ground for the reversion to the concept of fatigue in connection with studies of working stress.

Further, some of the data on endurance phenomena contained in the paper are not in harmony with experimental results. The investigations carried out by Messrs. H. F. Moore and his associates, at the University of Illinois, D. J. McAdam, at Annapolis, Md., R. R. Moore, at Dayton, Ohio, and others, give

* Discussion on the paper by Henry S. Prichard, M. Am. Soc. C. E., continued from February, 1926, *Proceedings*.

† Author's closure.

‡ Associate Editor, *Engineering News-Record*, New York, N. Y.

no support to such an anomaly as the terminal drop of the stress-cycle curves in Fig. 3,* attributed to Bauschinger.

Certain unusual terms introduced by the author contribute to the difficulty of following an intrinsically complex argument. Such expressions as "remote fatigue" or "auto-stress" are unfamiliar. Even the term used in the title of the paper, "utilizable capacity," is not wholly clear in spite of a carefully phrased definition.

With respect to the latter term, few structural members are subject to only a single kind of loading, and it is, therefore, hardly possible to consider capacity, that is, the limits of service, without at the same time considering the conditions of service. The definition given and the term itself seem to have the sole effect of substituting for the concepts of breaking strength and yield point the unaccepted concept of structural endurance limit.

The revival of the term "Useful Limit Point", coined eight or ten years ago by the Society's Special Committee on Steel Columns and Struts, seems unfortunate. The term was the outgrowth, apparently of an attempt to displace the yield point by a compromise between yield point and proportional limit. It differs a little from the "apparent elastic limit" of the late J. B. Johnson, M. Am. Soc. C. E., an ingenious approximation to the proportional limit devised to give a uniform, unambiguous definition of that limit. No good reason has been advanced for modifying the Johnson limit and introducing still another point and name into the already confused terminology of the elastic behavior of structural steel—a material whose yield point is well marked and whose resistance to load is largely dependent on the yield. With respect to structural behavior, the establishment of an arbitrary point some distance below the yield point has no demonstrated justification.

Some interesting considerations suggest themselves in connection with "auto-stress." This new term presumably covers mainly initial and residual stresses. In such parts as gusset-plates and other vital details these stresses may be important even for uniformly ductile steel, especially when the load stresses are compressive. This fact constitutes one reason for estimating stresses in details rather liberally and maintaining a relatively large safety margin, since such concealed stresses are not determinable by analysis or test, but require empirical allowance. On the other hand, similar stresses within the solid body of a member do not appear to be important. In reasonably good fabrication and assembly they are certain to disappear before the strength of the member is overtaxed. This may be deduced consistently from large-scale tests, in which a uniformity of action is displayed that could not be realized if the local constraints arising from rolling, fabrication, and other sources of "auto-stress" were not ironed out by the growing load. It has been held by many authorities that initial stresses constitute one important element of the necessary margin between calculated working stress and testing machine load; with respect to main members, this is open to doubt.

The treatment of reversed-stress members, which are also considered by the author, has long been a doubtful point in specifications and structural prac-

* *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1743.

tice. Increased section is customarily provided for such members, but it is questionable whether there are tests or data from experience to support this rule; it rests rather on tradition. In so far as it seeks to provide resistance against the greater fatigue effect of alternating stress as compared with repeated one-directional stress, it cannot be defended in the light of modern endurance investigation. The loosening effect of reversed stress on connections should have consideration, and, as common rivet loads exceed the lower limit of frictional bond between the connected parts, the provision of excess strength in the end details of reversed-stress members is justified; but the members themselves ought not be penalized on this score.

It may be hoped that the author's study will bring about further attempts to define structural strength more precisely than has yet been done. This strength, it would seem, cannot be a fixed single quantity, but must be interrelated with service. The view that one point should determine the strength of a member is not in harmony with practical appraisal of service conditions. It is certain that some service resistances are related to the ultimate strength, that is, to the actual point at which a member breaks and other resistances to earlier stages of the elastic behavior of the material.

The understanding of these relations is slowly progressing. Several attempts have been made in recent years to approach the determination of factors of safety or of working stresses rationally, although so far without marked success. There are still too many variables, too many indeterminates, in the problem to permit of full rational analysis. Yet, even to-day, it is possible to set down some general conclusions from known facts and structural experience as a starting point. The following items may be noted, for example:

(1) Under steady, slow, and frequently occurring load, a member is likely to be deformed to destructive degree if the load passes its yield point.

(2) Stress intensification at points of changing section, holes, and flaws does not appear to be significant in ductile steel as long as the mean stress remains below the yield point, but at a higher stress it presents the possibility of failure, so that such stress has an importance greater than that due only to its deforming or crippling effect on the structure.

(3) The small, high-frequency variations or oscillations of stress which are present in all or nearly all steel structures should be considered as a possible tax on fatigue endurance when the main stress is beyond a certain limit. This limit, which is not yet known with certainty but probably is at or somewhat above the yield point, therefore is to be rated as the ultimate strength for some groups of stress.

(4) Plastic re-adjustment at the yield point acts to re-distribute load in most structures to a degree that makes this point not definitely critical, in addition to providing warning of over-stress.

(5) Under single, suddenly applied loading, a member has unimpaired resistance nearly up to its breaking strength.

(6) Stresses set up in elastic distortion, or secondary stresses, are in most (but not all) instances self-limiting in character, tending to disappear by a

smoothing out of the inequalities in stress when yield is approached and before they can contribute to failure.

Statements of this kind seem capable of providing a starting point for an approach to the desirable end of rational determination of working stresses and safety factors. For the present, the best that can be said is that the problem is not now amenable to mathematical calculation but must be tested by experience of long service, and to this extent the work of the immediate future seems likely to depart from the course outlined by the author.

HENRY S. PRICHARD,* M. AM. SOC. C. E. (by letter).†—The writer's reason for introducing the new term "auto-stresses" is that there is no other term that indicates precisely those stresses and only those stresses within a body which, considered by themselves, without regard to others from external forces, form a balanced system. The term, "initial stresses", is frequently used to designate auto-stresses but is not suitable. Strictly, initial stresses are those which exist in a body before an experiment or before an episode of loading. They include the auto-stresses existing beforehand, but they also include stresses from the body's own weight and those from slight loads applied in advance to take up slack but not considered in comparing measured stress with measured strain. Initial stresses, from the fact that they are initial, cannot include any changes that occur in auto-stresses during an experiment or episode. "Residual" ("remaining after a part has been taken away"), as applied to stresses, does not precisely take the place of "auto". The writer does not know of any other term that has been used or suggested for "auto-stresses".

The writer regrets that he has failed to enlighten Mr. Turner‡ regarding the fundamental facts on which the utilizable capacity of steel members of structures depends. He is confident that to most of the intelligent and thoughtful readers, and hopes that to the average reader, the presentation of the facts regarding auto-stresses will not (using Mr. Turner's phraseology) "prove more confusing * * * than blowing hot and cold with the same breath did to the satyr in the fable". Be this as it may, the facts themselves can not be altered because of confused ideas or preconceived notions; but it may be helpful to quote the previous clear presentation, or rather anticipation, of some of the facts by an eminent engineer and physicist.

In 1848, Professor James Thomson (the elder brother of Lord Kelvin) published§ the paper from which the following excerpts were taken. He assumes a case in which a round bar of malleable iron is twisted beyond its elastic resistance and then proceeds:

"* * * and during the progress of this process the outer particles will yield in virtue of their ductility, those toward the interior successively reaching their elastic limits * * *.

"If, after this, all external stress be removed from the bar, it will assume a position of equilibrium, in which the outer particles will be strained in the

* Analytical Engr., Am. Bridge Co., Pittsburgh, Pa.

† Received by the Secretary, March 15, 1926.

‡ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 337.

§ *Cambridge and Dublin Mathematical Journal.*

direction opposite to that in which it was twisted, and the inner ones in the same direction as that of the twisting, the two sets of opposite couples thus produced among the particles of the bar balancing one another. * * * Principles quite similar to the foregoing, are applicable in regard to beams subjected to cross strain * * *.

"When malleable iron has received its final conformation by the process termed *cold swaging*, that is, by hammering it till it is cold, the outer particles exist in a state of extreme compression and the inner ones in a state of extreme tension. The same seems to be the case in cast iron when it is taken from the mould in which it has been cast. The outer portions have cooled first, and have therefore contracted, while the inner ones still continued expanded by heat. The inner ones then contract as they subsequently cool, and thus they, as it were, pull the outer ones together. That is, in the end the outer ones are in a state of compression and the inner ones are in the opposite condition.

"The foregoing principles may serve to explain the true cause of an important fact observed by Mr. Eaton Hodgkinson in his valuable researches in regard to the strength of cast iron (Report of the British Association for 1837, p. 362). He found that contrary to what had been previously supposed, a strain, however small in comparison to that which would produce rupture, was sufficient to produce a set, or permanent change of form, in the beams on which he experimented. * * *

"It appears to me that the defects which he (Hodgkinson) has shown to occur even with very slight strains, exist only when the strain is applied for the first time, or, in other words, that if a beam has already been subjected to a considerable strain, it may again be subject to any smaller strain in the same direction without taking a permanent set."

This remarkable prediction has been supported by the subsequent experiments of Wöhler, Bauschinger, Bairstow, Moore, and others, except that it was found that while the first subjection "to a considerable strain", within limits, causes a great improvement in elasticity, it requires a cycle of such applications to perfect it. Professor Thomson proceeds by a pure effort of scientific reasoning, and without any tests on the subject to guide him:

"The considerations adduced seem to me to show clearly that there really exist *two elastic limits* for any material, between which the displacements or deflexions, or what may in general be termed the changes of form, must be confined. If we wish to avoid giving the material a set, or, in the case of variable strains, if we wish to avoid giving it a continuous succession of sets which would gradually bring about its destruction; that these two elastic limits are usually situated one on the one side and the other on the opposite side of the position which the material assumes when subject to no external strain, though they may be both on the same side of this position of relaxation; and that they may therefore with propriety be called the *superior* and the *inferior limit* of the change of form of the material for the particular arrangement which has been given to its particles; that these two limits are not fixed for any given material, but that, if the change of form be continued beyond either limit, two new limits will, by means of an alteration in the arrangement of the particles of the material, be given to it in place of those which it previously possessed."

Lord Kelvin thought so highly of his brother's paper that he incorporated it in its entirety in his own notable discussion on "Elasticity".*

Ewing, Bairstow, Staunton, Rosenhain, Beilby, Howard, and other eminent physicists and physical metallurgists (none of whom is mentioned by Mr. Turner) having the advantage of the previous investigations of Maxwell,

* In Encyclopædia Britannica, 9th Edition, Vol. VII, pp. 798–800.

Kelvin, Tomlinson, Wöhler, and Bauschinger (all of whom are mentioned by Mr. Turner), advanced the science of physical metallurgy considerably beyond the point reached by the earlier investigators. Some of Mr. Turner's statements regarding fatigue and the effects of strain are far from orthodox. Experiments by the foregoing eminent investigators will refute his views; some of their results, already quoted by the writer in a previous paper,* prove their contentions.

Mr. Turner states "that rupture of ductile metal under Wöhler's treatment takes place without plastic elongation". In cases of rupture under alternate compression and tension in bending, like that described previously by Ewing, rupture occurs with very little plastic deformation, but where a bar is ruptured by cycles of stress in tension there often is considerable plastic deformation. For instance, in Mr. Bairstow's valuable paper† on the range of the elastic limits, "experiments on an axle steel with a yield point of 24.9 tons per sq. in. showed that in repeated loadings from 0 to 23.2 tons per sq. in. no sign of want of elasticity occurred until after 6 000 loadings when a permanent extension in the order of a yield took place."

The writer wishes to go on record emphatically against Mr. Turner's statement that "these working stresses and proper allowances need be no lower for a range of stress from tension to compression than for one kind only". Intensity of stress that could be repeated indefinitely in tension without causing either rupture or undue permanent set is likely to cause rupture if it occurs frequently in alternate tension and compression. Moreover, alternate stresses are especially severe on end connections. The writer has seen pinholes that have been enlarged ¼ in. in members subjected to alternate stresses. The matter of alternate stresses should be considered in connection with possible increase in the live loads that produce them.

Mr. Turner is correct in his contention that columns are liable to twisting. The writer has seen a column that was perceptibly twisted by reason of its load; but for centrally loaded columns Mr. Turner attaches too much importance to this consideration. Columns should be designed with forms and proportions of cross-section that offer considerable resistance to torsion; but beyond this point tests do not seem to indicate that superior resistance to torsion appreciably adds to the resistance to direct compression for equal ratios of length to radius of gyration. The writer is pleased that "form of section" is included as a sub-division of the subject of "steel column research" outlined in the Progress Report of the Special Committee on this subject.‡

The last paragraph of Mr. Turner's discussion, the one in which he discusses faults in the theory of flexure, contains the amazing statement that the writer appears to suppose that the disregard of shear strain increases the maximum fiber stress at points of greatest deflection. In dealing with faults in the

* From writings of Bairstow, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1512; of Rosenhain, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1520. See, also, explanation of Ewing, Encyclopædia Britannica, 11th Edition, Vol. XXV, p. 1019.

† *Philosophical Transactions*, Royal Soc., Vol. 210, p. 35; see, also, discussion by Mr. T. E. Stanton, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1482.

‡ *Proceedings*, Am. Soc. C. E., March, 1926, Society Affairs, p. 146.

theory of flexure the writer did not mention or refer directly or indirectly or by implication to a point or points of greatest deflection.

The supposition, imagined to be the writer's, appeared to Mr. Turner and he combatted it with ideas regarding constrained and continuous beams; and when he had disposed of it to his satisfaction, he inquired, "Is not much of the mathematical discussion of the paper based on assumptions which are unrelated to the facts?" The writer's analysis was confined to beams simply supported and uniformly loaded either over the entire length or over any shorter distance symmetrical with respect to the center line; and for such beams both the maximum fiber stress and the maximum deflection occur at the center and by correct theory both are greater than indicated by the ordinary theory.

No attempt was made in the paper to analyze the true theory of flexure of restrained or continuous beams. The reactions indicated by the true theory would differ somewhat from the reactions indicated by the ordinary theory and in some cases of restrained and continuous beams this would tend to reduce the bending moment.

Mr. von Fabrice discusses "Beams"* in a paragraph in which he states: "It is difficult to understand why the old and approved theory of flexure should be revised for an empirical formula afflicted with 50% factor of error."

The ordinary theory of flexure obviously neglects the influence of shear on cross-sections. An analysis of beams and girders simply supported was made to ascertain whether, in consequence of this neglect, the ordinary theory was in any such cases seriously in error and, if so, in what cases.

Believing such an analysis and its results would be of interest to engineers it was published. The method of analysis was to determine approximately the percentage, P_e, which theoretically should be added to the intensity of the stress, as computed from the ordinary theory, to make it correct. Equations (20), (21) and (22)† give approximately the value of P_e.

The writer's attitude regarding P_e is well emphasized in the paper.‡ That a theory has long been approved and has stood the test of long experience is a good and strong argument for continuing to use it; but, as regards concentrated loads and short beams, the ordinary theory has not been approved, and as regards these points, it is theory supplemented by good practice which has stood the test of experience. In this regard Mr. von Fabrice has himself provided the answer to his criticism in his discussion of "Shear."§

With respect to that part of Mr. von Fabrice's discussion which has to do with his Fig. 5‖ (*a*), (*b*), and (*c*), the writer refers those interested (and all engineers should be interested) to a discussion¶ of a paper entitled "Effects of Straining Steel and Wrought Iron." This discussion contains descriptions of some instructive microscopic examinations of strained iron. It was shown that plastic elongation took place almost entirely by successive diagonal slippages over one another, to right and to left, of the layers within the

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 349.
† *Loc. cit.*, November, 1925, Papers and Discussions, pp. 1777 and 1779.
‡ *Loc. cit.*, p. 1780.
§ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 349.
‖ *Loc. cit.*, p. 348.
¶ *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 1503–1522, especially p. 1522.

crystalline grains, of which the metal is composed, without appreciably altering the inclination of the grains. There is no reason why such slippages should not similarly take place in plastic shortening under compression, and there is good reason to suppose that they do similarly take place; in other words, the physical phenomenon of plastic deformation under compression is in all probability similar to that under tension, except that each grain becomes shorter and wider under compression instead of becoming longer and narrower, as under tension.

Professor Moore,* after pointing out that Johnson's apparent elastic limit involves an increase in rate of strain over the initial value amounting to 50 rather than the 100% of the U. L. P., suggests that, "a more extended statement from Mr. Prichard as to the advantages of the U. L. P. over Johnson's elastic limit would be of value." Mr. Woodruff† inquires, "What is the 'utilizable capacity' of a structural member?" and he adds, "according to the author's definition, it would be permissible to stress a member to its 'utilizable capacity'." As a basis for discussing these points, suppose that a steel works which rolls shapes and plates and is known as "the Mill", retains a consulting engineer to design a sample column, of a given length, the purpose being to build a considerable number for testing and, if the tests are satisfactory, to build a large number, all like the sample, for a customer.

The Mill desires that when the columns are loaded with (*a*) a gradually increasing load of twice the working load; or (*b*) an oft-repeated load from zero to twice the working load; or (*c*) a long-continued static load of twice the working load; it will not (1) fail; or (2) acquire an undue permanent set; or (3) be seriously injured; or (4) deflect unduly. The Mill also desires that the columns shall have as little excess section as practical.

The consulting engineer is free to select all the material required from which to make specimen tests, and he is afforded every opportunity for making the specimen tests in any way he chooses. Doubtless he would make a number of specimen tests and a few tentative designs before finally deciding on the design from which the test columns will be built, and he would doubtless adopt or originate a method of procedure.

If in testing the full-sized columns, twice the working loads are applied under Conditions (*a*), (*b*), and (*c*) with the eccentricities to which they are liable in service and if they stand the tests as regards all the Requirements (1), (2), (3), and (4); and if they are again tested under slightly increased loads and fail to meet one or more of the Requirements (1), (2), (3), and (4), the consulting engineer will have accomplished his purpose, and the Mill will supply its customer with columns that, as nearly as can be foreseen, have a factor of safety of two.

The attention of engineers in general, and Mr. Woodruff in particular, is called to the fact that a factor of safety is necessary (although it may not be the factor of two used for illustrative purposes in the assumed case) for the reasons that in spite of the best intentions, the actual loading may exceed the

* *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 344.
† *Loc. cit.*, p. 345.

assumed working loading, and in spite of the consulting engineer's precautions, his determinations may be deficient.

The writer's reasons for preferring the U. L. P. to Johnson's apparent elastic limit, in determining utilizable capacity, are that he considers that the additional load permitted by the U. L. P. (as qualified in the paper) will not cause failure or serious injury and that the additional permanent set permitted is not undue.

In general, the consulting engineer's most difficult problem will be to determine the eccentricity of application of the loads (restraint of the ends being considered in terms of eccentric application), as it is a problem which, ordinarily, does not admit of close mathematical determination. Such being the case, it does seem to be somewhat irrational, as Mr. von Fabrice points out, to specify precise formulas for the influence of length on utilizable capacity or on permissible loading; but every engineer who writes specifications for the design of structures gives a formula or formulas for columns, and if he distinguishes between steels with different characteristic U. L. P.'s he should make the formulas consistent one with another. In a paper like the writer's, however, attention should be called to the fact (as it has been) that in determining, as nearly as may be, the utilizable capacity, such formulas should be used with judgment.

The writer considers that Table 12,* used with judgment, should be quite useful in determining the utilizable capacities of columns, and he hopes that engineers will find it useful in devising formulas for alloy and other steels with high U. L. P.'s (assuming that they will for a time continue the present practice of specifying the same formula for all steel of the same grade and chemical composition); but he realizes the justice of Mr. von Fabrice's criticism that "Table 12 cannot be readily understood"; hence he has supplemented the table with a diagram, Fig. 6, which, he trusts, will make it easier to understand.

After carefully considering Mr. Schmitt's discussion† it seems best to explain some matters in and regarding the paper. The first four parts were originally written as a monograph on the subject, "Perfecting of Elasticity of Steel Bodies by Modification of Auto-Stresses, and the Development, by Repeated Cycles of Stress, of Fields of Seemingly Perfect Elasticity of Chosen Range and Position". It was the original intention to present the monograph to the Society as a contribution to pure science without regard to its application to engineering practice. Its incorporation as a part of a paper on "Utilizable Capacity of Steel Members of Structures" was an afterthought. Tables 1, 2, 3, 4, and 5‡ therein give a collection of the results of experiments by others, and show for various cases the limits within which elasticity has been perfected. The explanation of the perfecting of elasticity by modification of auto-stresses is the writer's adjustment of the ideas, previously referred to, of James Thomson to the discoveries of Beilby, Ewing, Rosenhain, and others as to the nature of plastic deformation of metals.

* *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1762.

† See p. 963.

‡ *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, pp. 1744–1748, inclusive.

Fig. 3* illustrates the various ranges of seemingly perfect elasticity which can be developed in different specimens from the same piece of steel by repeatedly straining them in direct tension or compression through extreme ranges of stress with different upper and lower elastic limits. If the ranges were any greater and the upper limits any higher the specimens would eventually break. The terms and expressions used in the first four parts of the paper were chosen with reference to close reasoning and definite meaning.

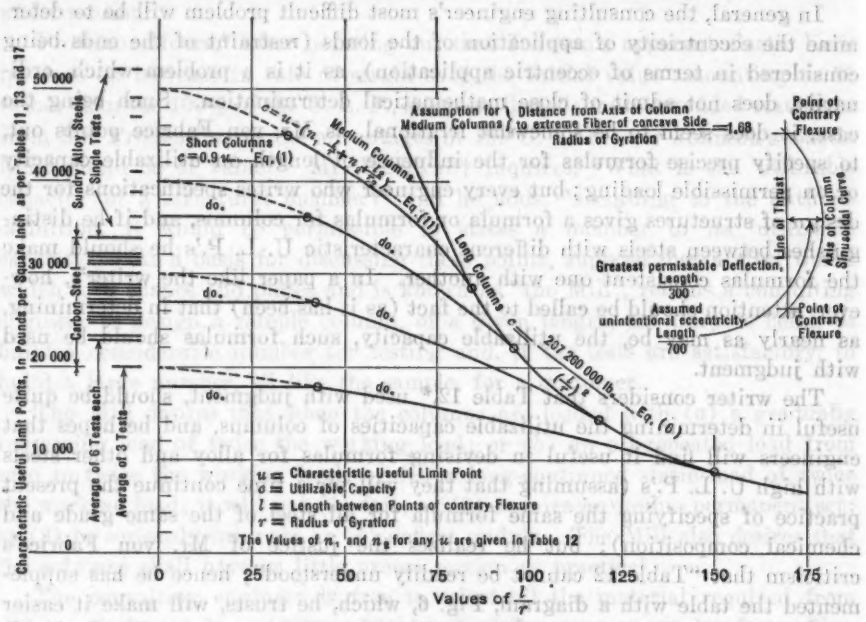


FIG. 6.—THE RELATIONSHIP OF LENGTH OF COLUMN, CHARACTERISTIC USEFUL LIMIT POINT, AND UTILIZABLE CAPACITY.

Among the experiments of Bauschinger, as given by Professor Unwin,† there were some, numbered 3, 6, 9, 11, 12, and 13 on Fig. 3, which indicate a strange anomaly in the diagram in the way of a decided terminal drop after seemingly perfect elasticity had been indicated through millions of repetitions. This is not in accord with the general indications of the other tests in the tables nor with the writer's explanation of the perfecting of elasticity. He considers it highly probable that there were special circumstances which caused this anomaly but he could not find information to confirm this view, so he included the tests in the paper, with qualifications as described therein. He was in hopes that some reader could contribute information regarding them. To distinguish the superficial indications of this anomaly from permanent fatigue, as previously defined, he used the term "remote fatigue".

After it was decided to incorporate the first four parts in a paper on utilizable capacity, the lines termed, "Critical Deformation Point in Stress", were added to Fig. 1.‡ This critical deformation point occurs somewhere in

* *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1743.
† "Testing of Materials of Construction."
‡ *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1740.

the zone of the yield which extends from the point, where the strain ceases to be closely proportional to the stress, to the yield point, when there is one. Sometimes the zone of yield is of zero length. Sometimes the yield shown in pulling tests is deceptive. In an extreme case (Test, 7934, Table 10*) soft steel with a maximum tensile strength of 48 240 lb. per sq. in. seemed in a careful pulling test, to be perfectly elastic up to 40 000 lb. per sq. in. and had a U. L. P. of 40 600 lb. per sq. in., or 84.2% of the tensile strength, and then the load dropped to 30 000 lb. per sq. in. Valuable information on this subject was obtained in one of Bairstow's experiments on axle steel, with a yield point of 49 800 lb. per sq. in., which in repeated loadings from 0 to 46 400 lb. per sq in., showed no lack of elasticity until after 6 000 loadings, when a permanent extension of the order of the yield took place. The first yield point observed was not a true criterion as the yield point under repeated stress was 3 800 lb. per sq. in. lower.

Some engineers consider the limit to approximate proportionality to be the critical deformation point, others consider the yield point as the critical point in deformation, whereas the Society's former Special Committee on Steel Columns and Struts† adopted the U. L. P. (the point where the plastic deformation just equals the elastic deformation) as critical. For specimens used as criteria for full-sized members the writer adopted 87% of the U. L. P., or 60% of the tensile strength, whichever is the smaller.

After Wöhler's experiments were published Launhardt and others developed formulas to indicate the loci of the upper elastic limit, under cycles of stress, beyond which failure would take place. In the past many engineers used these formulas as criteria on which to base working stresses, and some continue to use Professor Cain's modified Launhardt's formula. The writer for many years has consistently opposed such a practice and has advocated the use of the critical deformation point as the criterion, and this paper is in harmony with his record in this regard, but, in the light of experience, he now places the position of the point somewhat lower than formerly. If the critical deformation point is chosen somewhat too high a member under repeated one directional stress from zero to the point chosen may fail from permanent fatigue, while stress from a minimum tension or compression of appreciable intensity, to the point chosen as critical, may cause undue deformation. By adopting his method of selecting the critical deformation point, the writer considers it likely that he has escaped both these possibilities.

In the tests recorded in Table 2, it did not seem to make much difference whether the speed was 2 cycles per min., or 800. Above 800 the range and upper elastic limit were reduced. Regarding repetitions of stress to capacity under the conditions of actual service, it is conjectural whether the range and upper elastic limit would differ from those recorded, but unless the range would be less, it would not affect the criteria advocated in the paper for members subjected to compression only or tension only.

* *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1757.
† *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1618.

For alternate stresses attention is invited to the well-known specification:

"Members subject to alternate tension and compression shall be proportioned for both kinds of stress separately considered, but each kind shall be considered as increased by x per cent. of the lesser stress."

Suppose that, for purposes of analysis, the factor of safety is discarded; and that unit stresses for the various kinds of steel and iron termed Lots 1, 2, and 3 in Table 1 are chosen as follows: For charcoal iron, 32 500 lb. (it cannot be the 40 000 lb. indicated in the table, as that is greater than the yield point, which is 32 500 lb.); for unannealed axle steel, 48 000 lb.; and for unannealed Bessemer steel, 66 500 lb.; and that a different value of x is chosen for each kind of metal, as indicated in Table 24; and suppose further that the bars are proportioned for total stresses just equal to the stresses per square inch shown in Table 1; the tension per square inch plus the compression per square inch, which is the "range", will then be in each case up to, or nearly up to, the greatest range which can be indefinitely repeated without causing failure or exceeding the limits within which elasticity can be perfected as shown in Table 24. It should be noted that the analysis of the experiments recorded in Table 1, indicates, for every case of alternate stresses, (*a*) that an addition has to be made to the greater stress before the proper cross-sectional can be determined; (*b*) that the percentage to be added to the lesser stress varies in some way with the tensile strength; and (*c*) that, with the percentages of increase determined by experiments, the "well known specification" seems to be well supported by the experiments. The writer does not know who originated the "well known specification" or how he derived it, but it is shown in Table 24 that there are tests to support it. It is generally recognized that reversed stresses are severe on end connections of members. It seems reasonable to infer that they are also severe on the connections between the crystalline layers of the grains of the steel.

TABLE 24.

Lot.	x, in percentage.	Tensile strength, in pounds per square inch.	Yield point, in pounds per square inch.	Specified unit stress allowing no factor of safety, in pounds per square inch.	Total Stresses.		Tension plus x% of compression, in pounds.	Area required, in square inches.	Tension plus compression, in pounds per square inch.	Greatest possible range, tension plus compression, from Table 1, in pounds per square inch.
					Tension, in pounds.	Compression, in pounds.				
1	38.8	43.900	32.500	32.500	22.000	20.600	30.000	1.00	42.600	42.600
2	64.0	85.000	55.800	48.000	30.000	28.000	47.900	1.00	58.000	58.000
2	64.0	85.000	55.800	48.000	39.000	15.000	48.600	1.01	53.400	54.000
3	79.2	99.900	71.700	66.500	38.200	35.700	66.500	1.00	73.900	73.900
3	79.2	99.900	71.700	66.500	58.000	11.000	66.700	1.00	73.900	73.900
3	79.2	99.900	71.700	66.500	65.000	2.000	66.600	1.00	67.000	67.000

The writer is aware of the tendency to provide no additional cross-section to tension and compression members on account of reversed stress. When many engineers very properly discarded Launhardt's and similar formulas,

some of them also discarded (as the writer considers, without good reason) the "well known specification" cited.

The writer does not know of any modern endurance investigation that explains away the tests by direct stresses recorded in Table 1. Most if not all modern endurance tests are by bending, for which stress strain diagrams are very different from those of tests by direct stresses. Bending tests are so affected by auto-stresses developed in testing that inferences from them as to the behavior of members under direct stresses carry little weight as compared with tests by direct stresses. Such of the pertinent bending tests as the writer has analyzed, however, tend to support the conclusions reached, notably the extensive series of fine tests given in Table 5.

The writer would emphasize the fact that utilizable capacity can not be gauged from one "point in stress". The U. L. P. is sometimes so high that it can not be regarded as critical and some percentage of tensile strength should be substituted; the liability of failure from permanent fatigue is critical as regards members subjected to direct alternate stresses, but the liability to undue deformation is the consideration which limits the capacity of members subjected to stresses in only one direction. In some cases the degree of stiffness limits capacity. Details are supposed to be stronger than the member as a whole, but eternal vigilance is advisable in this regard. Some allowance is made for inevitable defects in workmanship, but good practice and intelligent inspection are highly important. Of course long endurance is dependent on protection of structural members from corrosion.

THE HEXAGONAL SLAB DESIGN OF CONCRETE PAVEMENT

Discussion*

By Lewis A. Perry, Assoc. M. Am. Soc. C. E.†

Lewis A. Perry,‡ Assoc. M. Am. Soc. C. E. (by letter).§—A review of the discussion indicates that the writer has wrongly made the paper too brief, assuming the reader's understanding of facts and theories implied, rather than presenting them.

Fig. 14 will serve to illustrate the reference to the added breadth at mid-slab of the fractional hexagonal slab, shown in Fig. 6.‖ It is surprising that this point has been missed or considered unimportant by some members. Slab (*a*), 10 by 20 ft., is typical of the design of the excellent pavements built by the Highway Department of the State of Washington; Slab (*b*) is identical with those shown in Fig. 6. In the design of the theoretical slab, indicated by the upper diagrams of Fig. 14, end concentrated loading was assumed, the slabs then being inverted and computed as simple beams. In both cases the sub-grade is assumed to be uniform, and resistance to flexure is not considered. There is also a fallacy of compensating bearing values at the ends, but as these conditions are common to both slabs, and as the loading and depths are equal, the resulting wide difference in the failure areas is a true indication of the superior value of Slab (*b*). This value, of course, will be realized first when "curling" occurs, and contact with the sub-grade at the ends of the slab is lost. The fact that the distance between the loads is not identical has no bearing as the distance between concentrations varies in actual practice. It should be borne in mind that these loads are not actually maximum safe loads, but are theoretical values used to compute moments. The actual safe load of Slab (*b*) is probably four times the load used here. In these computations plain concrete is assumed to be stressed to 300 lb. per sq. in. To simplify computations deepened outer edges are omitted, although in practice both designs include this feature; also, shear bar connections along the inner edge of Slab (*a*) are not considered because they do not serve the companion slabs when both are similarly loaded, and further because the effective life of shear bars, under these conditions, is limited.

It should be noted that the dimensions of Slab (*b*) might be varied to benefit the slab structure. It is a good rule, within reasonable limits, to make

* Discussion on the paper by Lewis A. Perry, Assoc. M. Am. Soc. C. E., continued from April, 1926, *Proceedings*.

† Author's closure.

‡ Constr. Engr., The Longview Co., Longview, Wash.

§ Received by the Secretary, April 1, 1926.

‖ *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1801.

the length of the diagonal center joint 1 ft. less than one-half the roadway width. In this case these joints would become 9 ft. instead of 10 ft. The writer has constructed some 20-ft. pavement by each method but as practically no cracking or breakage has occurred in either it is not possible to state which will show the better behavior. The shorter slab, with "dummy joints," is preferred by the writer.

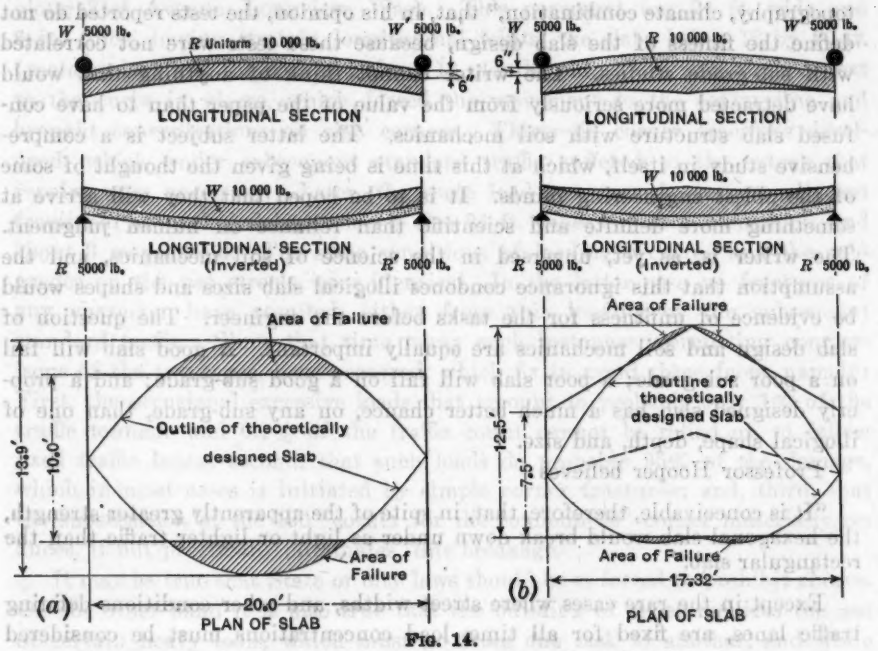


FIG. 14.

The lengths of joint filler in these two designs are:

Slab (*a*).....0.900 lin. ft. per sq. yd.
Slab (*b*).....0.909 lin. ft. per sq. yd.

Thus, Professor Hooper's objection* concerning the quantities of joint filler is correct only to this extent.

The question of the advisability of a grouping of free slabs has been discussed at considerable length by Professor Hooper, and the writer values his opinion, except that in his illustration buoyancy of water has been used to illustrate sub-grade fluidity or elasticity. This comparison does not seem fitting.

The writer believes that something more satisfactory than either the steel bar tie, or absolute slab freedom, exists in the "dummy joint" developed by Messrs. W. H. Tiedeman and H. F. Faulkner, Assistant City Engineer and Engineer of Tests of the City of Seattle, Wash., respectively. This "dummy," which is a narrow strip of joint filler, is inserted, with the aid of a T-bar template, into the plastic concrete after finishing, and penetrates about one-third of the depth of the slab; subsequent cracking is then controlled and

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 160.

does not show on the surface, and there is a frictional connection between slabs that the writer considers very valuable. Complete joints are placed at regular intervals to provide for expansion. This type of joint may be finished easily and is excellent in appearance; the writer is now using it in connection with the hexagonal design.

Professor Hooper has evidently meant by his reference to the "geology, topography, climate combination," that, in his opinion, the tests reported do not define the fitness of the slab design, because these tests were not correlated with sub-grade studies. The writer cannot think of anything that would have detracted more seriously from the value of the paper than to have confused slab structure with soil mechanics. The latter subject is a comprehensive study in itself, which at this time is being given the thought of some of the ablest engineering minds. It is to be hoped that they will arrive at something more definite and scientific than reliance on human judgment. The writer is, as yet, unversed in the science of soil mechanics, and the assumption that this ignorance condones illogical slab sizes and shapes would be evidence of unfitness for the tasks before the engineer. The question of slab design and soil mechanics are equally important. A good slab will fail on a poor sub-grade; a poor slab will fail on a good sub-grade; and a properly designed slab has a much better chance, on any sub-grade, than one of illogical shape, depth, and size.

Professor Hooper believes:

"It is conceivable, therefore, that, in spite of the apparently greater strength, the hexagonal slab would break down under as light or lighter traffic than the rectangular slab."

Except in the rare cases where street widths, and other conditions defining traffic lanes, are fixed for all time, load concentrations must be considered to be generally distributed as to transverse location; and that even if traffic is temporarily localized ("traffic" is generally understood to mean standard motor vehicles up to 12 tons, which, however frequent, should have no effect on a well-designed and well-built concrete pavement*), the great majority of initial failures are caused by the movement of occasional excessive loads, such as heavy construction equipment. These initial failures then progressively expand and become increasingly evident under standard motorized traffic so that the layman and sometimes the engineer fails to fix the responsibility on the right kind of "traffic". The point is that such occasional excessive loads do not necessarily follow, but, in fact, frequently avoid, traffic lanes, and if the improvement is designed with assumptions regarding locality of

* The writer has determined to his own satisfaction, from experiment, that a 6-in. hexagonal pavement slab will fail at about 12 tons, cantilever loading on each corner, or 24 tons total restrained load.

Conditions bearing upon this determination are:

Sub-Grade.—Uniform and sandy and not less than 1 500 lb. per sq. ft. bearing value, without deflection.

Concrete.—5 U. S. gal. per sack of cement (about 4 400 lb. per sq. in. compression).

Curing.—10 days under water, 18 days drying in air. Average daily temperature 60° Fahr.

Size of Slab.—16 ft. across opposite corners.

By stressing concrete one-third its ultimate strength this type of improvement is eminently fitted for 8 000-lb. wheel loads, but the writer would recommend heavier slabs under abusive traffic conditions, and, under all conditions, careful sub-grade treatment.

traffic—even admitting that a Utopia exists where the American motorist even temporarily drives where he is wanted, and the power shovel goes somewhere else, which is to be desired—the designer's plans have seriously miscarried.

The writer has in mind an instance that may be illustrative. In July, 1924, a 12-ton, gasoline-propelled road roller was moved three city blocks along 21st Avenue, Longview, Wash. This pavement was 24 ft. wide and 6 in. thick, having straight longitudinal joints, the slabs being 12 by 20-ft. (rectangular) and the concrete about 12 months old. Cars were parked next to the curbs in places, which forced the movement to the center line and brought concentrations on slab corners. Thirty-six corner fractures developed, which under subsequent standard traffic suffered to the extent that repairs were necessary. Later, the same load was moved over an adjacent fractional hexagonal slab pavement, also 24 ft. wide, but only 5 in. thick, and about 3 months old. The same conditions of loading occurred and the subgrades on the two streets were identical In the latter case no fractures of any character have resulted, either from this loading or the subsequent standard traffic. Since that time many such instances, permitting comparisons of the two types, have occurred, which fix in mind three facts, namely: First, the occasional excessive loads that amount to probably only 1% of the traffic tonnage and 0.1% of the traffic count cannot be relied on to follow fixed traffic lanes; second, that such loads do probably 95% of the damage, which in most cases is initiated by simple corner fractures; and, third, that the substitution of the 120° corner for the common 90° corner materially reduces, if not practically eliminates, this breakage.

It may be true that State or city laws should be enforced to prohibit abuses. On the other hand it is also true that the building of cities requires the use of certain heavy tools, which must go from one task to another, and while it is manifestly unjust to build with public funds improvements suitable for 0.1% of the traffic, it is also improper to disregard design refinements which materially increase loading capacities without additional cost.

Fig. 15 and 16 illustrate how unwise it would be for the designer to make assumptions concerning the transverse location of traffic on a wide street. The situation changes with parking restrictions, density of traffic, crown of street, and future widenings. It is correct, therefore, to assume that any weak spot between curbs will be found and punished; and to try to eliminate those weaknesses in design rather than to repair them later at considerable cost, and with sacrifice of street appearance. Fig. 16 also illustrates that, under the conditions shown, a rectangular slab pavement frequently comes under the objections raised by Professor Hooper to hexagonal slabs, namely, that the corners are generally in the path of traffic.

A pointed illustration of the fallacy of Professor Hooper's objection to the number of corners of hexagonal slabs would be to consider a circular slab (which is admittedly an ideal shape) the radius of which is, say, 8 ft. and the perimeter of which is cut into 1-ft. chords. It remains essentially a circle, with all its virtues, but with about fifty corners. If, however, a lim-

ited number of corners were desired, and the designer went to extremes, he might make slabs of equilateral triangles, having only three, instead of fifty, corners. A slab of this type, and of an area equal to the circle, would have sides about 21.6 ft. long, the corners of which develop less than 16% of the breadth of the corners of the former shape, yet this shape would use 27% more joint material. It is seen, therefore, that the number of corners and their locations are immaterial as long as each corner is structurally fit, and it is well to remember that breadth as well as depth of members fix their section moduli.

One discussor has referred to the "regular zigzag line or joint" as a "possible source of confusion to drivers." It is to be assumed that this observation was made without due consideration. It has been necessary to photograph the slab layout from a great height to show the joints.

The question regarding curling at corners due to temperature raised by Professor Hooper, but left "up in the air" by him, is important. The curl is proportional to the slab length. The "aggravated condition" is roughly proportional to the curl, and inasmuch as, for the hexagonal slab, the extreme cross-corner length is about 90% that of a square slab of equal area, it follows that the hexagonal slab suffers less than 90% of the "aggravated condition" of the square slab.

The question of appearances raised by Mr. Hare* is, as he states, "secondary" to that of utility, but is still important. In perspective, from the driver's eye, this design takes on a suggestion of uniform texture from curb to curb that is quite pleasing, and not possible with straight center points. On highways, and elsewhere, where lines for traffic division are desirable, they must be frequently re-painted for clearness, which is also true of the hexagonal design.

The writer thinks that Mr. Feld* might revise his statement that "there is no special advantage in having the intersections of the joints with the edges staggered," after careful consideration of Fig. 14; and that he intended the statement, "nor is the elimination of the square joints from the center line of great importance," to apply to highways or narrow streets. Even with this qualification the writer cannot agree. It is thought that each street width presents a new application of the principles, all of which have not been considered by Mr. Feld. The writer regrets that Figs. 14, 15, and 16 were not submitted with the paper for the better information of members in discussion.

Mr. Feld has referred to the "ordinary type of pavement" meaning probably highway pavements, and his approval of the hexagonal design for "large areas" leaves the writer wondering what his judgment would be of widths "in between," such as a street pavement from 30 to 100 ft. in width. Surely all common widths are "ordinary," and many are what the writer would call "large areas."

Mr. Root's discussion† has fittingly been a description of construction methods. This should be of great importance to engineers, especially since,

* *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 518.
† *Loc. cit.*, April, 1926, Papers and Discussions, p. 725.

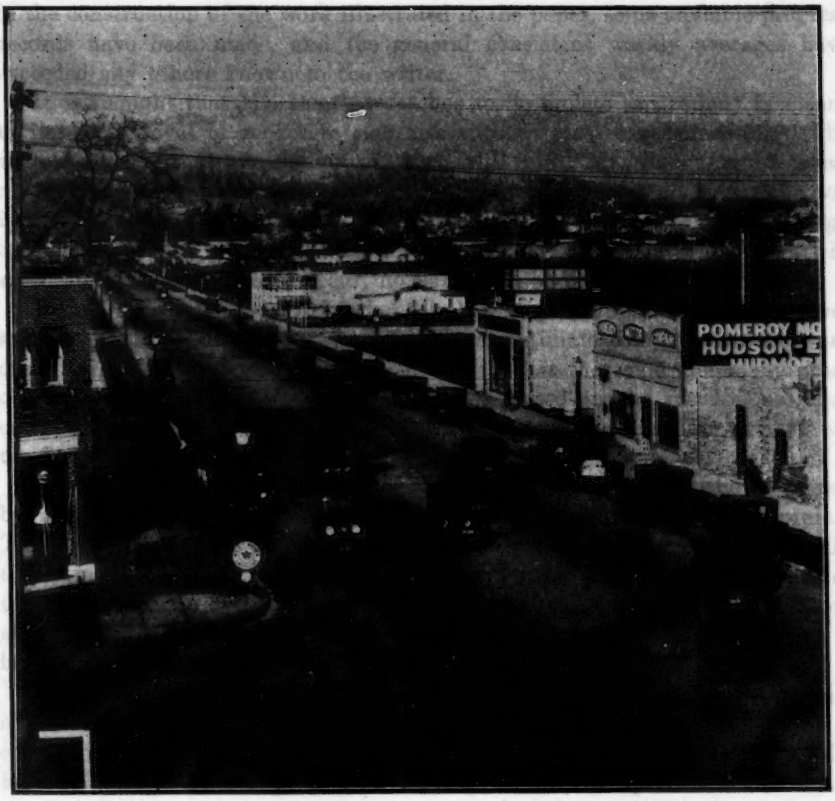


FIG. 15.—VANDERCOOK WAY, LONGVIEW, WASH., EXAMPLE OF 8-INCH RECTANGULAR SLAB PAVEMENT.

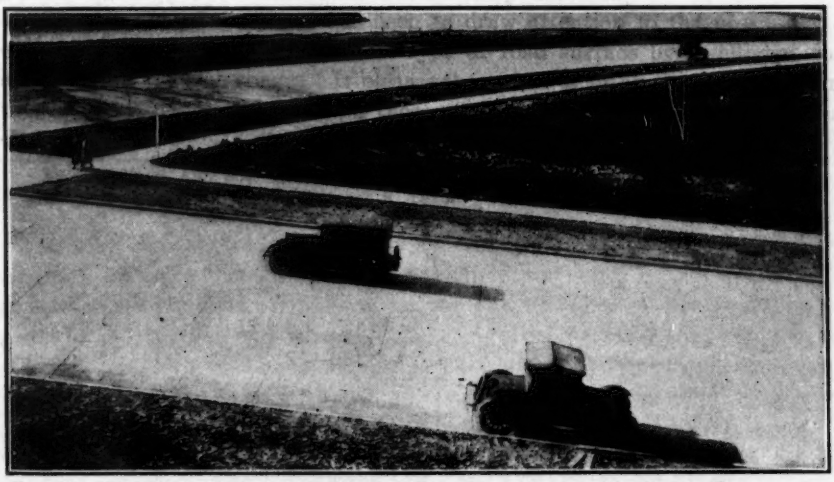
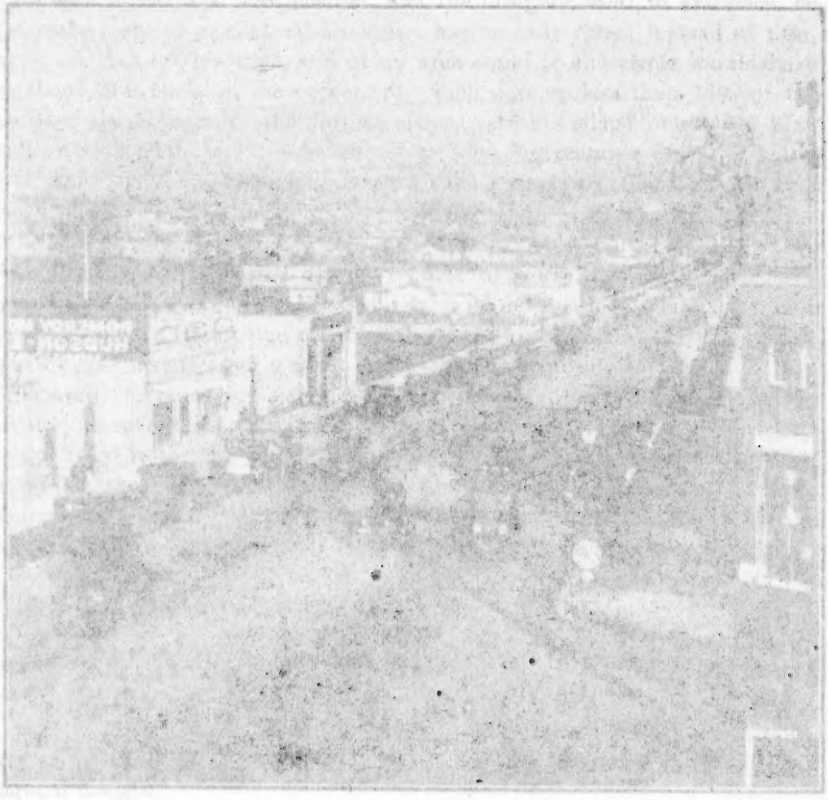


FIG. 16.—WASHINGTON WAY AND KESSLER BOULEVARD, LONGVIEW, WASH., EXAMPLE OF 6-INCH HEXAGONAL SLAB PAVEMENT.



in the construction of the work illustrated in the paper, some enviable progress records have been made, and the general consistent weekly averages have exceeded any others known to the writer.

It is thought that Mr. Root's reference to "complete separation" is meant to imply approval of moderate slab size without steel bar connection. The writer believes that the "dummy joint" will fulfill Mr. Root's requirement of complete separation. Certainly, the existence of joint filler for the top 2 in. of the slab will serve to "prevent spalling and the subsequent raveling" of the surface. It is not probable that the bottom of the controlled crack will spall; in the event this did occur, no harm will be done.

In his discussion,* Mr. Vandercook has a geographical advantage over some other discussors, in that he has been in intimate contact with the work described, from its first consideration, and in that the same and many other objections and questionings that have been raised in the discussions, were carefully considered by Mr. Vandercook before he sanctioned the writer's extensive use of the new design.

Mr. Vandercook's reference to the inadvisability of reducing the thickness, by reason of the hexagonal design, is the expression of a valuable opinion. This is a question of economic policy, to be determined by the local conditions, such as community finances, the traffic anticipated, and the nature of the sub-grades; and it is probable that the general trend in pavement construction will be toward higher safety factors, because in few cases can it be said that loading limits are fixed, or even anticipated, "for all time."

The "subsequent investigation" referred to by Mr. Vandercook was a series of tests of square and hexagonal pavement slabs under cantilever loading of corners.† Briefly, the average results may be reported, as follows: (*a*) 5-in. hexagonal slabs bore 23% greater loading than 6-in. square slabs of equal area; and (*b*) 5-in. hexagonal slabs bore 48% greater loading than 5-in. square slabs of equal area. The slabs were cast *in situ* on a water-settled sand subgrade. The individual breaks were remarkably uniform.

The purpose of the paper has been to draw attention to the possibility of enhancing the value of this logical type of road surface by intelligent consideration of the slab shapes. It pleases the writer to think that, in a measure, this has been done; and to thank those engineers whose discussions have furthered this purpose.

* *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 727.
† *Engineering News-Record*, October 8, 1925.

STREAM POLLUTION

A SYMPOSIUM

Discussion*

By Messrs. William Firth Wells and J. K. Hoskins.†

William Firth Wells,‡ Esq. (by letter).§—Consideration of the natural changes which pollution undergoes in water is as essential in controlling the quality of water as artificial processes of purification. Engineering practice has been principally concerned with artificial purification, but the present administrative tendency to treat water quality as a natural resource has shown the need of further study of the phenomena of self-purification. From this viewpoint the U. S. Public Health Service studies are particularly valuable. They separate the basic factors, develop methods of measure, determine the relations existing, and lead to a simple and logical system of expression.

The fundamental conditions treated are: (1) infection, the power of water to convey living forms which may directly affect persons, studied bacteriologically; (2) pollution, the condition produced by the decomposition of dead organic matter, determined biochemically; and (3) contamination, the alteration in quality by contact with foreign substances, tested chemically, as in the studies of trade wastes.

The oxygen studies have been discussed and the importance of the bacteriological work should also be recognized. From data representing years of work under diverse conditions, the determinations of the average daily number of *B. coli* contributed per capita, with the seasonal variation and the rate of disappearance from the streams, make a valuable addition to quantitative bacteriology. The utilization of logarithms in presenting the results is significant. Many advantages can be pointed out: First, it would be impractical to plot directly the ordinary variations in bacterial numbers, whereas the logarithm of the number per liter gives a convenient unit throughout the entire range, from drinking water to the most intense pollution; second, self-purification expressed in logarithmic terms approximates a straight-line function; third, Wolman and Streeter have shown that the ratio of the logarithm of the bacterial content for the influent to that for the effluent in purification processes is nearly constant; fourth, the distribution of these logarithms approximates the normal frequency law, and the average logarithm, therefore, approximates the median sample; fifth, the recognized scale of

* Continued from March, 1926, *Proceedings*.

† Author's closure.

‡ Biologist and Sanitarian, New York State Conservation Comm., Glen Cove, N. Y.

§ Received by the Secretary, March 24, 1926.

water quality is more directly represented by the logarithm than by the actual number of bacteria, and water-borne typhoid is more nearly proportional to the logarithm than to the actual number of *B. coli* in a water supply; and, sixth, the dilution method used in bacteriology actually gives a logarithmic scale by the simple numbering of dilutions.

The direct use of the highest positive dilution number, therefore, gives immediately the advantages of the logarithm in a rational system of quantitative bacteriology.

J. K. Hoskins,* Esq. (by letter).†—Mr. Jackson has brought out‡ a point that should be emphasized, namely, that excessive quantities of industrial wastes, especially those of a toxic nature, such as acids, metal salts, dyes, etc., materially affect the normal biological regimen of streams into which they are discharged. In such special cases obviously it is impossible to apply any general method of quantitative estimation of the bacterial content.

The Naugatuck River in Connecticut is seriously polluted with copper and brass factory and acid wastes practically throughout its length. Below Torrington, the uppermost city on the stream, where such pollution is less intense, the average concentration of bacteria for the summer months is 248 000 and for the winter 20 800. At points farther down stream, higher counts in winter than in summer might be expected to result because of the greater dilution of the toxic wastes effected by higher stream flows.

On the Hockanum River, above sources of serious industrial waste pollution at Rockville and Manchester, computed estimates, as presented by Mr. Jackson, compare quite favorably with actual observations (Table 17).§ However, at South Manchester, where silk mill wastes constitute 78% of the total sewage, the lack of correlation of computed and actual results is evident. Furthermore, the observations of the stream at East Hartford were made above the sewer outlet and as stated in the published report‖ of this work, "the results have not indicated the presence of this sewage in the samples collected." This would appear to justify the elimination of East Hartford results from consideration.

This same bactericidal effect of toxic wastes was observed during the course of the studies of Ohio River pollution, notably in the vicinity of Pittsburgh, Pa., where the Monongahela River carries large quantities of acid mine drainage. Here, the weighted average bacterial content of the Allegheny and Monongahela Rivers above Pittsburgh was found to be 4 120 per cu. cm., and of the Ohio River, 11 miles below the city, after receiving the sewage of 827 000 people, only 1 900 per cu. cm., or a reduction of 2 200 per cu. cm.

In closing, the writer desires to state again that justification for the acceptance of the empirical formulas presented in his paper¶ must depend on the results of further observations on numbers of different streams.

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* San. Engr., U. S. Public Health Service, Cincinnati, Ohio.

† Received by the Secretary, March 13, 1926.

‡ *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 2061.

§ *Loc. cit.*, p. 2063.

‖ First and Second Biennial Reports of the Industrial Wastes Board, State of Connecticut, Department of Health, Vol. I, 1918–21.

¶ *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1843.

NOTES ON SHEAR IN COMPRESSION MEMBERS

Discussion*

By Messrs. Henry S. Prichard, Anders Bull, and Ralph E. Goodwin.†

Henry S. Prichard,‡ M. Am. Soc. C. E. (by letter).§—The "frequently used allowance" mentioned by the author‖ is one of the requirements in the "frequently used specifications", of a particular company, as follows: "The latticing of compression members shall be proportioned to resist a shearing stress equal to 2% of the direct stress."¶ The writer does not know who was the first to specify the 2%, or whether any one has attempted to give the rationale of this allowance, or whether by any mathematical ingenuity it can be made to link the "beam analogy" of a uniform transverse load with any particular column formula, as was done for a certain rule for lattice and a form of the "straight line" column formula in many editions of "Modern Framed Structures"**; he does know, however, that it requires 33⅓% greater allowance for shear than, but is otherwise identical (although differently worded) with, a rule specified and used by the New Jersey Steel and Iron Company prior to 1891, referred to as Prichard's rule in the report of the Royal Commission on Collapse of Quebec Bridge††, which reads as follows:

"The lattice shall be so spaced that each channel between lattice connections shall be stronger than the column considered as a whole, and their size shall be obtained by treating the column as a lattice girder loaded at the middle with a load equal to 3% of the total compression on the column."

In their very able report on the collapse of the Quebec Bridge, the Commissioners state:‡‡

"The angle of inclination may vary from point to point along the column owing to the curvature of the column. This curvature may be due to original bends or to the action of the load or to both combined. If the curvature is sufficiently small the variation of inclination due to it will be negligible. There remains, however, the original inclination or obliquity which is due to the method of application of the loads at the ends of the column. If the eccentricity of application is the same at each end and in the same plane with the axis of the column, there will be no obliquity other than that arising from the curvature of the webs which may be negligible. If, however, the

* Discussion on the paper by Ralph E. Goodwin, Assoc. M. Am. Soc. C. E., continued from March, 1926, *Proceedings*.

† Author's closure.

‡ Analytical Engr., Am. Bridge Co., Pittsburgh, Pa.

§ Received by the Secretary, February 24, 1926.

‖ *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1941.

¶ Carnegie Pocket Companion, 1923, p. 98.

** "The Theory and Practice of Modern Framed Structures," 1896, p. 255.

†† Vol. I, 1908, p. 126; originally published in the *Lehigh Quarterly*, June, 1891, p. 159.

‡‡ Vol. I, 1908, p. 123.

eccentricities at the opposite ends are different or in different directions the obliquity may be of considerable amount. If the curvature of the column be negligible the obliquity arising from the eccentricity will be the same at every point. This obliquity causes a transfer of load throughout the whole length of the column from one web to another. This transfer of load is accompanied by longitudinal shearing stresses in the lattice system. The obliquity also causes transverse shearing stresses at every cross-section of the column."

Before the writer published his rule for lattice he realized in a general way all these facts (but he did not formulate or mathematically analyze any of them until shortly after the collapse of the Quebec Bridge when he wrote a paper on "Imperfect Butt Joints in Columns and Stress in Lattice Bars"*). From experience and observation (which included the exciting spectacle of a violent collision on a through span of a railroad pin-connected bridge which stood the shock), he knew that columns are liable to slips and falls in handling in shop and field, to impacts in transportation, to accidental side blows in service, as indicated by telltale dents and other injuries, and to usages for which no specific provision is made in their design. After considering the sundry and various possibilities, liabilities, and probabilities, he concluded, regardless of theory, that columns should possess substantial resistance to shear and torsion. From all these considerations and a diligent study of specifications and designs of other engineers, Prichard's rule was devised. It was not the formulation of a fine spun theory but a simple judgment, and probably represents about the average of the best practice of the time when it was devised. Later, in the "frequently used allowance", 33⅓% was added, presumedly for good measure.

The practice with respect to lattice at the time Prichard's rule was published remained about constant up to the date of the collapse of the first Quebec Bridge, except for a growing antipathy to batten plates at intervals in place of lattice, with little or no reason to regard the sizes of the lattice bars as inadequate.

It is the consensus of opinion that the latticing of the compression chords of the first Quebec Bridge was too light. The results of an analysis of the "frequently used allowance" in connection with the "frequently used" specifications, as applied hypothetically to Member *A*-9-*L*, is strong evidence that it is conservative for a large range of ratios of length to diameter of column. Member *A*-9-*L* had a cross-sectional area of 780 sq. in.; ⅞-in. rivets; a radius of gyration at right angles to the webs of 16.1 in. and parallel to the webs of 19.7 in.; an extreme width in the plane of the lattice of 67.5 in.; and a length of 684 in.; all as given in the report of the Royal Commission.† The arrangement of cross-section and lattice bars was as indicated in Figs. 4 and 5 (which are reproductions of a portion of a drawing of a model chord, the length and all the linear dimension of the portion shown being one-third of those of the full-sized member).‡

"The latticing between the center webs is inefficient. Intermediate latticing should have been used on these webs. One of the bars between the

* *Engineering News*, October 3, 1907.

† See pp. 130 and 199.

‡ Report of the Royal Commission on Collapse of Quebec Bridge, p. 112.

center webs in every panel on the upper face of the chord, *A*-9-*L*, has a net section of only 1.15 sq. in. at the center and 1.5 sq. in. for a length of about 4 in., whereas between the center web and the outside web the section is 2.48 sq. in. (gross)".*

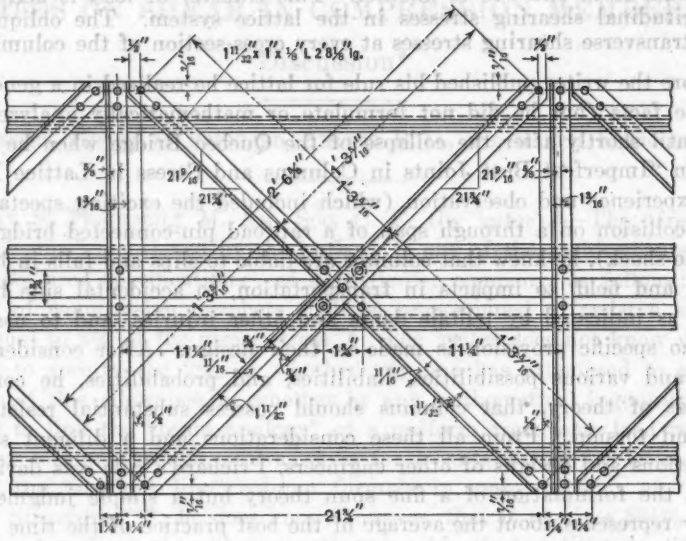

FIG. 4.

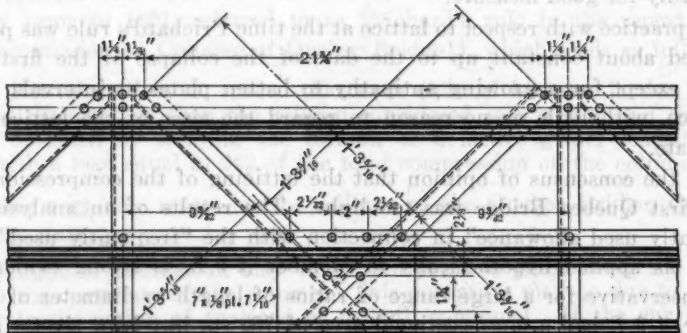

FIG. 5.

Between each panel of latticing, the chords of the member were connected by a cross-tie consisting of a 3½ by 3 by ⅜-in. angle. For this case, the "frequently used" specification with its "frequently used allowance" for shear

* Report of the Royal Commission on Collapse of Quebec Bridge, p. 133.

requires each of the main lattice bars to be connected to each outside built channel by ten rivets instead of two,* as shown by the following computation:

The member is composed of four units (built of web-plates and angles) of equal or nearly equal cross-sectional area; and the distance of the center of gravity of each outside unit from the center of gravity of the member is 4.5 times as great as the similar distance of each inside unit; hence the bending stress in the outside units is 4.5 times as great as in the inside units, and, therefore, the shear in the lattice between the outside and inside units, is $\frac{4.5}{5.5}$ of that between the inside units (assuming intermediate latticing between the inside units, to meet the criticism of the Commission).

The limiting load on the member is:

$$13\,000 \text{ lb.} \times 780 = 10\,140\,000 \text{ lb.}$$

The member should be proportioned for shear of 2%, or 202 800 lb.; $\frac{4.5}{5.5}$ of 202 800 lb. between outside and inside channel equals 166 000 lb.

The lattice-bars, of which there are four per panel or four cut by a cross-section, make an angle with the channels of slightly more than 45°, with a secant of 1.42:

The direct stress in each bar from the shear in the member = $(166\,000 \times 1.42) \div 4 =$	58 900 lb.
The direct stress in the member transferred to each lattice-bar as explained hereinafter =	9 200 "
Total	68 100 lb.

The number of rivets required = 68 100 ÷ 7 200 (the permissible shearing value of each rivet) = 9.46 or, say, 10.

The two rivets by which each lattice-bar was actually connected to each outside unit, or web, did not shear until the computed load on the member exceeded an intensity of 16 800 lb. per sq. in. The lattice intersected at points well outside the center of gravity of the outside webs, thereby producing bending stresses in the webs and lattice and tending to increase the shear on the rivets, but the amount of the increase would doubtless have been trivial in a 10-rivet connection.

The effect of the tie at the panel points of the latticing was to force the lattice to take part in the direct compression in the member. For each lattice-bar (4 by 3 by $\frac{3}{8}$-in. angle), each tie ($3\frac{1}{2}$ by 3 by $\frac{3}{8}$-in. angle), and each lattice making an angle of 45° with the webs, the writer has shown, in a discussion dealing with latticing,† that the theoretical intensity of the stress in the lattice from this cause is 28.4% of the intensity of the direct compression in the member, or 28.4% of 13 000 lb., which, applied to the gross area, 2.48 sq. in., gives 9 200 lb. as already used.

The fact that a member of the first Quebec Bridge, with a 2-rivet connection at each end of each lattice, collapsed after the computed compression in the member exceeded 16 800 lb. per sq. in., is no reason for doubting the ade-

* Correspondingly, the "frequently used allowance" requires larger lattice, but the number of rivets affords the simplest comparison between the "frequently used allowance" and the actual provision.

† *Engineering News*, October 3, 1907, p. 368.

quacy of a rule which, as applied to the lattice-bars of the member, requires a 10-rivet connection.

The other important questions which the author raises are: (*a*) The position of the eccentricities which will induce the greatest lateral reactions and, therefore, shears from lateral reactions; (*b*) the amount of unintentional eccentricity to which a member is liable from "poor workmanship"; and (*c*) the increase which deflection causes in shear.

In those cases in which the ratio of shear to direct compression is greatest, the computed increase which deflection causes in shear is small in comparison with the shear from the lateral reaction, even when the deflection, instead of being correctly computed, is arbitrarily chosen much too great, and "the theoretical deflection curve has been assumed to be a parabola", as illustrated by the author's computation* of 4 800 lb. of shear from the lateral reaction and 1 400 lb. increase from deflection "in a standard latticed channel column, 25 ft. long, composed of two 10-in. by 30-lb. channels."† The author has high authority for assuming a parabola‡ but for the purpose of determining the increase in shear, it does not give a close approximation. By correct theory the increase is much smaller than that computed by the author and may be neglected without serious error, as will be shown.

For a definite longitudinal load and definite eccentricities in application at opposite ends of the column, the lateral reactions and shears therefrom are greatest when the eccentricities are on opposite sides of the axis and are as indicated by a simple formula given by the author, which may be expressed as follows:

$$\text{Lateral reaction} = \frac{\text{Load} \times \text{sum of eccentricities}}{\text{Length of column.}}$$

This simple formula strikingly shows that, other things remaining constant, the lateral reaction, and, therefore, the shear from the lateral reaction, increases inversely as the length of the column.

The unknown quantities in the formula are the eccentricities. There may be eccentricities caused by unsymmetrical variations in cross-sectional area, slight unsymmetrical variations in the modulus of elasticity, and slight curves or bends in the axis, all of which are unintentional and may be important; they will be referred to again after a discussion of eccentricities caused by "poor workmanship". The discussion of eccentricities from poor workmanship is restricted to cases in which the longitudinal load, or the major portion of it, is received, through contact of surfaces intended to be plane and intended to make a known angle, usually 90°, with the axis. The poor workmanship, where it occurs, consists in the surfaces not being plane, or in the angles not being true, or in unspecified lateral forces being applied so as to open or to tend to open the joints between the ends of the columns.

Contracts for fabrication and erection of framed structures should be awarded only to concerns which are properly equipped to face the ends of

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1944.
† Carnegie Pocket Companion, 1923, p. 225.
‡ C. C. Schneider in his report, "Quebec Bridge Inquiry," 1908, p. 195.

columns so that they will be smooth and square or true to angle and to properly erect the structures of which the columns are a part, and which employ engineers and superintendents who understand the importance of these matters, including the proper position and setting of the shoes and base-plates on the foundations, and the proper design, fabrication, and placing of the pieces which connect the columns. Structural engineers probably understand and are alive to these matters to a greater extent than the author realizes. For instance, certain large compression members of a comparatively recent structure were designed and faced so that they were in contact for about the middle-third and only about the middle-third of their depth. This was accomplished by a slight flare from the middle-third toward the edges.

It is not disputed that it would be well to exercise more discretion in the award of contracts, more intelligence in their execution, and greater care in inspection, but the writer does not see the need for modifying specifications to provide for any such unintentional eccentricities as the author assumes in the suppositious case of "a standard latticed channel column" (previously referred to) to examine the matter "somewhat in detail." He assumes an eccentricity of 4 in. at each end of the column and to opposite sides of the axis at opposite ends. For this, say, the first case (with $P\ A = 180\,000$ lb., $L = 25$ ft.; $A = 17.6$ sq. in.; $I = 323.25$; $R = 4.29$ in.; and $V = 6.75$ in.),* using the method of computation and significance of symbols to be indicated subsequently in this discussion, the computed maximum shear (which occurs at the middle of the column) is 5 180 lb., or 2.88% of the load, as compared with the "frequently used allowance" of 2%, and a computed maximum stress in the extreme fiber (which occurs at the ends) of 25 260 lb. per sq. in., as against 16 000 lb. permitted by the "frequently used" specifications. If the 4-in. eccentricities were on the same side of the axis for, say, the second case, the computed maximum shear (which, for this case, occurs at the ends) is 2 349 lb. and the computed maximum stress (at the center) is 29 070 lb. per sq. in. It is almost needless to say that none of the well-known specifications provides for such eccentricities, and structures are not designed for them, yet failures have been exceedingly rare notwithstanding that loads on bridges have steadily increased.

Structures and the methods of erecting them vary so greatly that no extended discussion of erection will be attempted and attention will be called to only a few points.

A compression member in the course of its erection, held in the approximate position while connections with enough temporary bolts are made to hold it, stands more or less chance, as there is clearance in the holes, of being placed with, and temporarily held by, the friction of bolt-heads and tightly screwed nuts, with a slight V-shaped opening where it joins the adjacent member, a condition which may seem alarming to a novice; but such bolts will not hold much load without some slippage.

Even the best workmanship is not made with the precision required in astronomical and other scientific instruments. It is not surprising, therefore,

* Carnegie Pocket Companion, 1923, pp. 109, 113, 156.

that after the successive columns in buildings and chord members in bridges have come to a bearing they are likely, collectively considered, to form a slightly inclined, or bent, or wavy or curved line. The extent to which this occurs may perhaps be judged by the provision in the "Code of Standard Practice", October 1, 1924, of the American Institute of Steel Construction:*

"In the setting or erecting of structural steel work, the individual pieces shall be considered plumb or level where the error does not exceed 1 to 500.

"For exterior columns and columns adjacent to elevator shafts or multiple-story buildings, the error from plumb shall not exceed 1 to 1 000 for the total height of the column."

Unsymmetrical yielding of foundations under eccentric pressure, elastic deformation of columns, and, on occasion, plastic deformation of small portions of their outer and nearly outer fibers and crushing of their bearing edges, may be useful in effecting adjustments which bring adjacent surfaces to a bearing. In a short properly proportioned column with unintentional eccentricities to opposite sides at opposite ends, buckling of the column as a whole will not occur, shortening from deformation will be trivial, and plastic deformation will relieve the column of the excessive stress which caused it. Elastic deformation of the lattice-bars will also help and plastic deformation may help; but, on the other hand, excessive stress in the lattice may shear the rivets, or break or severely cripple the bars.

For short columns particular attention should be given to the problem of shear. Columns subject to heavy loads in buildings are usually built with cover-plates instead of lattice; hence the problem of the sufficiency of the latticing in very short columns in buildings does not often arise. In columns and compression members in outdoor construction exposed to the elements it is desirable that the interiors shall not be inaccessible for painting. It is also highly important that accessibility for purposes of fabrication, erection, and painting shall be achieved in such a way that an employee engaged in such service, if overcome by exhaustion, accident, heat, cold, fumes, or otherwise, will not be in an almost closed interior from which he can not be readily rescued. To make adequate provision for the greatest possible shear in short members requires a high degree of practical as well as engineering knowledge. Concerns that understand the subject in all its phases take particular pains in such cases to reduce the possible shear from poor workmanship to a minimum.

The engineer may ask how does he know that the designedly abutting surfaces of compression members actually come to a good bearing. The fact is that, except for the cases he has personally examined and the testimony of others regarding the cases they have personally examined, he does not really know; but failures of such members are exceedingly rare, and he can infer from this fact that sufficient bearing is achieved to keep the stresses from the actual loads within safe limits; and he knows, if he is experienced, that in many cases the actual loads sustained by structures in service have been greater than those for which they were designed. The writer knows of only

* See p. 11.

one failure, the first Quebec Bridge, where either the latticing or the butt joints are under suspicion. In this bridge, the bottom chord, *A*-8-*L*, extended beyond the panel point 10.5 ft. (almost one-fifth of the panel length) to a butt joint with the member, *A*-9-*L*. This long distance to the joint was occasioned by a strut which ran from the panel point making an acute angle with *A*-8-*L*. The details of *A*-8-*L* between the panel point and the butt-joint are not shown in the report of the Commission previously mentioned, but it seems to the writer from the data in the report, that the out-of-line distance of 1.5 in. observed at the joint before the collapse should be regarded as the individual outward bending of insufficiently connected webs loaded at their ends rather than the deflection of a compression member between panel points.

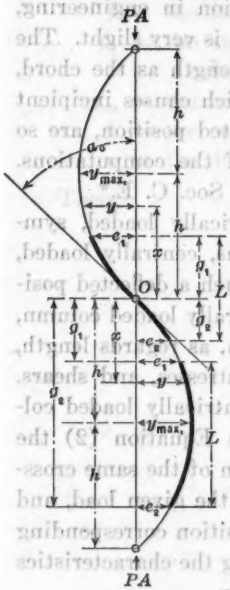

FIG. 6.

For short columns, the possible unintentional eccentricities, from unintentional variations in cross-section, slight variation in modulus of elasticity, and slight bends or curves in the axis, are all small as compared with the eccentricity assumed by the author for the "standard latticed column 25 ft. long", and do not require further discussion.

For short columns, the theoretical maximum shear, from eccentricities to opposite sides at opposite ends, is only slightly greater, than the theoretical lateral reactions (the computation of which is much simpler). To justify the formulas on which this statement is based, and from which the writer's computations were made, and to clarify ideas regarding the theory of eccentrically loaded columns, the following general analysis is given.

Fig. 6 shows the axis of a centrally loaded, continuous, symmetrical column of constant cross-section (made of perfectly elastic steel) with three points, of no flexure, one at each end and one at the middle, where it is held against lateral movement by frictionless pins which offer no resistance to the rotary movements developed in the column in deflecting. Owing to the continuity of the two halves the deflections in each half have to be equal and to opposite sides of the axis; otherwise, each half behaves precisely like an independent column and will support the same loads and have the same elastic line.

Let A = the cross-sectional area;
$2h$ = the length of each half; and
E = the modulus of elasticity.

For steel, considering the pound as the unit of stress and the inch as the unit of length, $E = 30\,000\,000$.

Let p = the load per unit of area; and
R = the radius of gyration.

As shown by Euler and numerous mathematicians since Euler, and as demonstrated in nearly all textbooks on Mechanics, the curve of incipient

deflection of such a column is a sinusoid and the load which will cause incipient deflection is given by "Euler's formula":

$$p = \frac{\frac{\pi^2 E}{4 h^2}}{R^2} \quad \ldots\ldots\ldots\

a_2 = the angle which a tangent to the column axis at the lower end of the eccentrically loaded column makes with the line of thrust;

a_0 = the angle which a tangent to the column axis, at origin point, O, makes with the line of thrust;

S = the transverse shear at any cross-section;

S_1 = the transverse shear at a given point the ordinate of which is e_1 and the abscissa, g_1;

S_2 = the transverse shear at a given point the ordinate of which is e_2 and the abscissa, g_2;

S_0 = the transverse shear at origin point of contrary flexure, O;

f = the stress intensity in the outer fiber at any cross-section;

f_1 = the stress intensity in the outer fiber at the upper end of the eccentrically loaded column;

f_m = the maximum stress intensity;

A = the cross-sectional area of the column; and

v = the distance from the axis to the extreme fiber.

Then,

$$y_m = \frac{e_1}{\sin\left(\frac{\pi\, g_1}{2\, h}\right)} = e_1 \operatorname{cosec}\left(\frac{\pi\, g_1}{2\, h}\right) \quad \ldots\ldots(3)$$

$$\tan a = \frac{d\, y}{d\, x} = \frac{\pi\, y_m}{2\, h} \cos\left(\frac{\pi\, x}{2\, h}\right) \quad \ldots\ldots(4)$$

$$\tan a_1 = \frac{\pi\, y_m}{2\, h} \cos\left(\frac{\pi\, g_1}{2\, h}\right) \quad \ldots\ldots(5)$$

$$\tan a_0 = \frac{\pi\, y_m}{2\, h} \quad \ldots\ldots(6)$$

$$S_1 = p\, A \tan a_1 \quad \ldots\ldots(7)$$

$$S_0 = p\, A \tan a_0 \quad \ldots\ldots(8)$$

$$f_1 = p + \frac{p\, e_1\, v}{R^2} \quad \ldots\ldots(9)$$

$$f_m = p + \frac{p\, y_m\, v}{R^2} \quad \ldots\ldots(10)$$

Calling eccentricities to opposite sides of axis at opposite ends of column, Case I; and eccentricities to the same side of the axis at opposite ends of column, Case II; then,

For Case I, for $e_2 = e_1$,

$$g_2 = g_1 = \frac{L}{2} \quad \ldots\ldots(11a)$$

and for $e_2 = 0$,

$$g_1 = L \quad \ldots\ldots(12)$$

For Case II, for $e_2 = e_1$,

$$g_1 = h - \frac{L}{2} \quad \ldots\ldots(11b)$$

For any other relative value of e_1 and e_2, the values of g_1 and g_2 have to be obtained by trial, which can be readily accomplished by using Equations (13) and (14*a*) or (14*b*), with the aid of a table of sines.

$$\frac{e_1}{e_2} = \frac{\sin\left(\frac{\pi g_1}{2h}\right)}{\sin\left(\frac{\pi g_2}{2h}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

For Case I:

$$\text{Angle}\left(\frac{\pi g_2}{2h}\right) + \text{angle}\left(\frac{\pi g_1}{2h}\right) = \text{angle}\left(\frac{\pi L}{2h}\right) \quad \ldots\ldots\ldots (14a)$$

For Case II:

$$\text{Angle}\left(\frac{\pi g_2}{2h}\right) - \text{angle}\left(\frac{\pi g_1}{2h}\right) = \text{angle}\frac{\pi L}{2h} \quad \ldots\ldots\ldots\ldots (14b)$$

Euler, and other mathematicians, who have developed Equations (1) and (2), neglected the influence of shear distortion.

Let t = the deflection due to a shear of unity in a length of unity.

The writer has previously shown* that when t is considered, Equations (1) and (2) become Equations (15) and (16):

$$p = \frac{\pi^2 E}{\frac{4h^2}{R^2} + \pi^2 E A t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$h = R\sqrt{\frac{\pi^2 E}{4p} - \frac{\pi^2 E A t}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Equations (15) and (16) clarify theory by bringing it into line with the common sense idea that even under ideal conditions potential resistance to shear is essential to capacity to support load and is really an element in determining the maximum load; but, for all cases which arise in practice, the effect of substituting Equation (16) for Equation (2) is so small it may be neglected.

The special case in which the eccentricities to the same side of the axis are equal at opposite ends, has been developed by a number of engineering authors, including Anson Marston, M. Am. Soc. C. E.,† who agree in their results. The general analysis here given leads to Marston's equation as a special case.*

For the 10-in. channel column, 25 ft. long, previously cited, the results recorded were obtained, by applying Equations (2), (3), (5), (6), (8), (9), and (10), as follows:

For both the cases previously cited:

From Equation (2), $h = 364$ in.

For Case I:

From Equations (11*a*) and (3), $y_m = 6.633$ in.

From Equation (5), $\tan a_1 = 0.02283$

* *Engineering News*, February 25, 1909, p. 206.

† *Transactions*, Am. Soc. C. E., Vol. XXXIX (1898), p. 108.

From Equation (6), tan $a_0 = 0.02862$

From Equation (8), maximum shear = 5 160 lb. (which is 360 lb. greater than the 4 800 lb. lateral reaction obtained by both the author and the writer).

From Equation (9) the stress in the outer fiber at the ends (which for this case is the maximum) = 25 260 lb.

For Case II:

From Equations (11*b*) and (3), $y_m = 5.0144$ in.

From Equation (5), tan $a_1 = 0.1305$

From Equation (7), the shear at the end (which for this case is the maximum) = 2 349 lb.

From Equation (10), the maximum stress in the outer fiber, $f_m =$ 29 070 lb., which is the same as would have been obtained by applying Marston's formula.

For Case I with the eccentricities on opposite sides of the axis, which the author chose to "examine somewhat in detail", he assumed, and illustrated in his Fig. 3,* the vertices to the elastic line as occurring about half way between the ends of the column and the point, *O*, of contrary flexure, whereas the writer's analysis shows them to occur at 364 in., or 30 ft. 4 in., from the point, *O*, in prolongations of the elastic line 17 ft. 10 in. beyond the ends. If the length of the column were, say, 63 ft. instead of 25 ft., and all other conditions, including load, were the same, the vertices would lie between the point, *O*, and the ends of the column; 30 ft. 4 in. from Point *O* and 1 ft. 2 in. from the ends.

ANDERS BULL,† ESQ. (by letter).‡—To apply the method outlined by Mr. Goodwin to column-lattice design, a value must first be assumed for the deflection, or, what amounts to the same thing, for the eccentricity due to imperfect workmanship or other causes, the former being a function of the latter.

The beam analogy furnishes a simple empirical rule by which this unknown factor is taken into account, whereas Mr. Goodwin would entrust its evaluation to the judgment of the designer. This would seem to be dangerous practice because it would leave too much room for guesswork. The deflection is too complex a function of loading conditions, length, and section characteristics to be divined, even by the most experienced of designers; it is far more involved than the allowable unit stress, which is always determined by a column formula.

Neither do the examples given by Mr. Goodwin support his claims. For the column shown in Fig. 3,* the deflection, *f*, is a function of the distance, *x*, from the end, or, approximately,

$$f = \frac{P_1 e}{4 E I}\left(\frac{L x}{3} - x^2 + \frac{2 x^3}{3 L}\right) \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1943.

† Asst. Engr., Board of Transportation of the City of New York, New York, N. Y.

‡ Received by the Secretary, March 17, 1926.

in which, E is the modulus of elasticity and I the moment of inertia of the column section, the other notations being those used by Mr. Goodwin.

The graph of Equation (17) is a line the curvature of which increases evenly from the middle section toward the ends. By equating to zero the first differential, the maximum deflection is found to occur at a distance $= 0.211\ L$ from either end. Substituting this value for x in Equation (17), the maximum is found to be:

$$f_{max.} = 0.008 \frac{P_1 e L^2}{E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The angle of inclination of the column axis to the vertical is greatest at the ends, where:

$$\tan G = \frac{P_1 e L}{12 E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

the inclination at the middle section being just one-half as great.

Substituting in Equation (18) the values for the column selected,

$$f_{max.} = 0.008 \frac{180\,000 \times 8 \times 90\,000}{30\,000\,000 \times 323} = 0.106 \text{ in.}$$

This is only 0.035% of the length, as compared with the 0.1% which Mr. Goodwin takes as his starting point.

The shear, S_1, due to flexure, at the middle of the column, is:

$$S_1 = P_1 \tan G_1 = 180\,000 \frac{180\,000 \times 8 \times 300}{24 \times 30\,000\,000 \times 323} = 335 \text{ lb.}$$

as against the 1 440 lb. obtained by Mr. Goodwin.

The flexure shear at the ends is twice as large, but acts in the opposite direction and, therefore, counteracts the shear, S_2, due to eccentricity. The latter shear being 4 800 lb., there results accordingly:

$$\text{Total shear} = S_2 + S_1 = 4\,800 + 335 = 5\,135 \text{ lb.}$$

Before comparing this result with that obtained by the beam analogy it should be noted that an eccentricity of 4 in., as assumed by Mr. Goodwin, is too large for the column section selected, giving a combined stress of:

$$\frac{180\,000}{17.64} + \frac{180\,000 \times 4 \times 7}{323} = 10\,200 + 15\,600 = 25\,800 \text{ lb. per sq. in.}$$

If a maximum stress of only 19 000 lb. per sq. in. is permitted, as indicated by the column formula,

$$\frac{P}{a} = 19\,000 - c \frac{L}{r}$$

the eccentricity must be reduced in the proportion, 8 800 : 15 600, which gives:

$$\text{Total shear} = 5\,135 \times \frac{88}{156} = 2\,895 \text{ lb.}$$

This is equal to only $0.016\ P_1$, as compared with Mr. Goodwin's figure, $0.035\ P_1$.

The beam analogy applied to the same column formula, gives for a column depth, d, of 14 in.:

$$\text{Shear} = \frac{8\,c\,a\,r}{d} = \frac{8 \times 100 \times 17.64 \times 4.28}{14} = 4\,315 \text{ lb., or } 0.024\,P_1$$

Being thus 50% in excess of the actual amount as found previously, the result is well on the safe side.

The Quebec Bridge failure cannot be ascribed to any shortcoming of the beam analogy, as Mr. Goodwin seems to imply, but to a number of concurrent causes, among which the most fatal were: (1) The excessive unit stresses specified; (2) failure to make allowance for added weight; and (3) the inadequate provision made for eccentricity in designing the compression members.

In regard to the stresses in the chord member, A-9-L, it must be considered that the member was in the act of collapsing when the measurements given in Fig. 2* were made. Care should be exercised in drawing any conclusions from these observations, since there is no way of determining how much of the computed stress was transferred at that time to the opposite chord member, A-9-R.

RALPH E. GOODWIN,† ASSOC. M. AM. SOC. C. E. (by letter).‡—In closing, the author desires to summarize the purpose of his paper as it relates to the discussions which have been submitted. It has been his experience that many engineers have only a specialized understanding of shear and fail to comprehend it when it assumes a disguised form. This is especially true in the case of columns for which the axis may be curved and the direction of the shear not constant. The purpose of the paper was, therefore, first, to show that the applied shear is the normal component of the applied load; and, second, to call attention to the satisfactory mathematical foundation which exists as a basis for the determination of empirical shear allowances. That such allowances must be empirical is due to the fact that the initial curvature and accidental eccentricity of a column are unknown quantities, for which values corresponding to the most dangerous shear condition likely to occur in practice must be assumed. For determining what conditions of curvature and eccentricity will produce the most dangerous shear condition, the principles reviewed in the paper will be found useful.

Since Mr. Walker§ states that the basis of treatment is not new, the writer would call attention to his acknowledgment of this fact in the Synopsis.‖

Mr. Prichard¶ objects to the shape of the deflection curve and to the value of the deflection assumed for the case illustrated by the writer's Fig. 3.** Since the objection is based on theoretical grounds the writer wishes to state with emphasis that the application of the method should not be limited to theoretical deflections and theoretical deflection curves. The shear arising

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1941.
† Instr., Civ. Engr., Coll. of the City of New York, New York, N. Y.
‡ Received by the Secretary, March 23, 1926.
§ *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 537.
‖ *Loc. cit.*, December, 1925, Papers and Discussions, p. 1939.
¶ See p. 986.
** *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1943.

from initial curvature may be greater than that due to deflection caused by bending, and no method of analysis can be considered satisfactory which fails to make allowance for this factor. The general method (shear = $P \sin G$) is applicable to any curvature, regardless of the cause which produced it. The deflection assumed in the illustrative problem may be considered as arising from a combination of initial curvature with bending deflection, and as such does not appear to be unreasonable. Any deflection and any curvature that may occur in a practical column are legitimate cases for investigation.

Using the method presented in Mr. Prichard's discussion, the writer finds the maximum bending deflection (measured from the dotted line axis of Fig. 3(*b*)) to be about 0.11 in., leaving 0.19 in. to be accounted for by initial curvature. This is 0.0006 L, or 0.0013 $\frac{L}{2}$. As Mr. Prichard shows, the shear due to deflection is small in comparison with that due to eccentricity. Nevertheless, it is the amount of deflection assumed by the writer which forms the basis of his criticism. The eccentricity and direct stress are admittedly high in the illustration cited, however, and, as Mr. Prichard states, the case falls outside the limits set by specifications.

Mr. Prichard's criticism in regard to the position of the vertex of the elastic line, in the same illustration, is somewhat misleading. When he states that the vertex lies 364 in. from the point of contrary flexure, he is speaking of the vertex determined by the axis of his equivalent centrally loaded column, which axis is inclined at an angle of about 1½° to the dotted-line axis of the writer's Fig. 3(*b*). If the writer's axis is applied to Mr. Prichard's elastic line, the point of maximum column deflection is found to be 85 in. from the point of contrary flexure, or 10 in. beyond the middle point of the half length, and Fig. 3(*b*), which alone formed the basis for computations, is, therefore, essentially correct in this respect, even when the possible effect of initial curvature is omitted from consideration. Fig. 3(*a*) was obtained by applying the inclined thrust line to Fig. 3(*b*), and its distortion with respect to that line results unavoidably from the exaggerated curvature adopted for the sake of clearness in illustrating.

The matter of changing present specifications, if this should be found necessary, may well be left in the hands of the Society's Special Committee on Steel Column Research. It is to be hoped that this Committee will include a study of the initial curvature of practical columns and of the curvature of loaded columns in its program.

Mr. Bull's discussion* is essentially similar to that of Mr. Prichard in using a theoretical deflection curve without making any allowance for original lack of straightness, and in criticizing the writer's illustrative problem. The need of making an allowance for initial curvature has already been mentioned. It can be seen more readily if a centrally loaded column is considered. The ideal centrally loaded column has no deflection until subjected to the Euler load. Since practical columns carry less than the Euler load, no basis for shear computation exists, in this case, until allowances have been made

* See p. 997.

for initial curvature and accidental eccentricity. Mr. Bull states that the deflection is a function of the eccentricity due to imperfect workmanship or other causes. This statement is based on the assumption of a column originally straight, whereas Turneaure states that a column may have an original deflection of 0.001 L, and Godfrey, one of 0.0025 L. Even in the extreme case used by the writer as an illustration an initial deflection of $0.001 \frac{L}{2}$ proves to be greater than the deflection due to bending. The total deflection from all sources should be used for the computation of shear.

It is difficult to reconcile the objections raised by Mr. Prichard and Mr. Bull to the writer's illustrative problem on practical grounds with their purely theoretical treatment of deflection, from which the practical consideration of initial curvature is omitted. Possibly the oversight is traceable to the practice of treating the deflection of a crooked column as equivalent to an equal amount of eccentricity in a straight column. The maximum bending moment will be the same in both cases, but not the shear, owing to the fact that shear depends on the inclination of the axis. Shear due to initial deflection can not be treated on the basis of equivalent eccentricity.

Mr. Bull undertakes to bring the writer's illustrative problem within the limits of good practice by reducing the eccentricity until the extreme fiber stress is reduced to a reasonable figure. For this purpose he assumes that the sum of the average stress and the bending stress may be made 19 000 lb. per sq. in., apparently overlooking the fact that the column formula of the American Bridge Company is limited to a maximum of 13 000 lb. per sq. in.

Again, in computing the beam analogy, he assumes that the bending allowance of the column formula is $100 \frac{L}{r}$. The writer is not familiar with any specification which defines the use of the beam analogy in connection with this particular column formula, but he is quite certain that the term, $100 \frac{L}{r}$, is not all bending allowance. The formula is limited to a maximum of 13 000 lb. per sq. in., due to the fact that the elastic limit is the practical ultimate value in compression. This particular formula, then, assumes that the elastic limit controls the design up to $\frac{L}{r} = 60$, and that bending controls from that point on. It is more nearly correct, therefore, to consider the bending allowance to be $100 \left(\frac{L}{r} - 60 \right)$.

These, however, are minor details. The main point of interest lies in the idea of a treatment that is fundamentally sound and applicable alike to practical and theoretical curvatures. Owing to the empirical factors inherent in the problem, rules and specifications will always be necessary in the designing room, but in order that the designer may design intelligently and with understanding of the assumptions involved, he should be familiar with the correct theory of shear.

CO-ORDINATION OF IRRIGATION AND POWER

Discussion*

By Messrs. C. S. Jarvis and William Kelly.†

C. S. Jarvis,‡ M. Am. Soc. C. E. (by letter).§—The record of the Federal Power Commission's rulings as reported by the author‖ goes far to prove both the need for such an authority and the benefits that have already accrued under its administration.

When irrigation and power interests are in opposition, only a non-partisan, deliberate, and independent estimate of the whole situation will lead to an equitable adjustment. It appears that many cases of litigation have been avoided by the intensive analysis of each problem with a view not only to the recognition of initiated or vested rights, but also to the ultimate beneficial use under other projects barely conceived and not yet launched.

Experience indicates that the general policy should be to promote projects for stream regulation and diversion on the head-waters concurrently with plans for similar enterprises in the intermediate and lower valleys. In no other way can there be utilized (*a*) the maximum head for hydro-electric power plants; (*b*) the maximum irrigable acreage and variety of products; (*c*) the maximum underground storage with return seepage increasing the natural flow—an important factor during years of drought; and (*d*) the full measure of potential benefits.

An excellent example of co-ordinated power and irrigation development is furnished by the Salt River Project, in Arizona. Formerly, the power plants at the Roosevelt Dam could not be operated during the non-irrigation season, because the discharge from the turbines was lost to the system. This situation was recently relieved by the supplemental storage provided between the Roosevelt Dam and the intake of the main irrigation canals. This immediately replaced the intermittent power generation with constant, dependable service, brought the canal and reservoir systems into more intimate relation so that the flow at the head-gates could be varied on a few hours' notice, and made possible additional power developments nearer the place of use. As a considerable part of the demand is for pumping during the irrigation season, there is a special value in the secondary power at the lower sites.

* Discussion on the paper by William Kelly, M. Am. Soc. C. E., continued from January, 1926, *Proceedings*.

† Author's closure.

‡ Associate Highway Engr., U. S. Bureau of Public Roads, Washington, D. C.

§ Received by the Secretary, March 5, 1926.

‖ *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1946.

Such a favorable solution is readily attainable where both power and irrigation interests are under the same ownership. Progress toward harmonizing the control of streams has resulted from the attitude of the Federal Power Commission to the effect that, in the last analysis, the welfare of the citizenry in each drainage basin should govern the development of water resources. This seems to be the spirit and intent of the Federal Water Power Act. Wisely administered, it should provide the unified control so essential to thorough co-ordination of power and irrigation interests, and should promote the normal activities which make for sound industrial growth.

WILLIAM KELLY,* M. AM. SOC. C. E. (by letter).†—A few closing remarks seem to be called for by Mr. Tibbetts' discussion.‡ The Yuba River was not mentioned in the paper because no final action had been taken on it by the Federal Power Commission at that time and, therefore, it could not be cited as an example. The Federal Power Commission has since granted the application of the Nevada Irrigation District in which Mr. Tibbetts is interested.

The District lies in the foothills of Nevada County, California. It contains about 200 000 acres, of which, according to the State Engineer's estimate, not more than 75 000 acres are suitable for irrigation. About 3 000 acres are now under irrigated cultivation. The principal industries in Nevada County up to the present have been gold mining and cattle grazing. The greater part of the land proposed for irrigation is covered with a growth of timber and brush and it is conservatively estimated that the average cost of clearing will be as much as $100 per acre. The irrigable lands are scattered over the district so that to supply them with water will require practically the same length of ditch as if the entire 200 000 acres were to be irrigated. The project could be developed to the extent of about 15 000 acres by the construction of the Scotts Flat Reservoir in Deer Creek Valley, using the run-off of the water-shed within which the District lies.

For greater development, water must be diverted from adjacent water-sheds. The project which has been approved contemplates the development of storage on the head-waters of the Middle and South Forks of the Yuba and its diversion by tunnel and conduit into Bear River and a part thence to Deer Creek.

The Pacific Gas and Electric Company has entered into a contract with the District to take the diverted water for use in existing and proposed power plants and finally to release it near the boundary of the Nevada Irrigation District. The schedule governing the release is based mainly on the power values and, therefore, only about 36% of the water supply will be delivered for use during the irrigation season. The contract is a good one for the Power Company, and the promoters of the Nevada District claim that returns from the power use of the water will pay all carrying charges on the investment required for the storage and diversion works. In other words, if the estimates are not exceeded, the District will get a small initial water supply,

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† Received by the Secretary, March 19, 1926.

‡ *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 148.

adequate for about 20 000 acres, at its boundaries practically without cost, except for the maintenance and operation of the mountain works. It should be noted, however, that the property owners of the District and not the Power Company assume all the risks.

Mr. Tibbetts' criticism that the engineers of the Federal Power Commission were balancing kilowatt-hours against acres is hardly justifiable. The question is not one of balancing kilowatt-hours against acres, but of balancing a possible but doubtful 75 000 acres in the Nevada District, plus a certain amount of power, against a greater irrigable acreage in the Sacramento Valley, plus about 40 000 kw. more power, plus incidental benefits to flood protection on the Yuba, Feather, and Sacramento Rivers, plus the possibility of renewing hydraulic mining on the Yuba where it is estimated that more than $50 000 000 of gold lies still unmined.

In other words, using the water from the head-waters of the Yuba in its own water-shed would increase and not decrease the acreage of land economically irrigable in California. Moreover, diversion of water from the Bowman water-shed, combined with the development of local sources, would supply all the lands that are probably irrigable in the Nevada District and leave the water of the Middle Fork to be developed in combination with the remainder of the Yuba River. It was the diversion of water from the Middle Fork to the Nevada District to which the engineers of the Federal Power Commission particularly objected.

Again Mr. Tibbetts is not correct when he attempts to point out a great difference in viewpoint between Everett N. Bryan, M. Am. Soc. C. E., and the writer. In his paper, "Hydro-Electric Power as a By-Product of Agricultural Storage,"* Mr. Bryan deals only with foothill reservoirs similar to the proposed Pine Flats Reservoir on the Kings River, described by the writer. Power at such reservoirs is and should be purely incidental to irrigation, but one of the important functions of such reservoirs is that they leave the river above them free to be developed primarily for power. Such reservoirs, when properly combined with head-water storage, accomplish the maximum use of water for both power and irrigation.

Investigation of the Nevada Irrigation District, in its relation to the Yuba River, was made jointly by engineers of the State and of the Federal Power Commission. There was no material difference of opinion between them as to physical facts or economics. The State engineers, however, operate under State laws which grant rights on the basis of priority of filing, and they could give but little regard to the broad economics of the problem, whereas the engineers of the Federal Power Commission are charged with determining whether or not any project applied for makes the fullest possible use of the water for all purposes.

In granting the Nevada District application, the Federal Power Commission certainly has demonstrated its intention to co-operate with the States to the fullest extent possible in administering the Federal Water Power Act. The Nevada District case illustrates some of the factors which the writer

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 910.

had in mind when he stated that practice in the Western States "sometimes sets aside economic laws". The California law, providing for the organization of Irrigation Districts, has proved itself a good one, but no law has ever been so good that it could not be used in ways not intended.

The Nevada Irrigation District was organized through the efforts of energetic and competent promoters. Its formation was made possible by the votes of many non-property owning individuals. Owners of nearly 50% of the property in the District opposed the mountain development project. The owners of property in the Lower Yuba Valley, who have perhaps a better right than the Nevada District to look to the Yuba River for water, have not organized and thus stand to lose their rights. If the Yuba water-shed and the Nevada District combined were being developed by one owner as a business proposition, looking to ultimate results as well as to earning expenses during development, it is inconceivable that the diversion of all the head-waters of the Middle Yuba to Deer Creek Valley would form the initial step.

Mr. Tibbetts' statement that "empire building in an arid section is dependent wholly on the use of water for irrigation", is rather beside the point in the Yuba controversy because, leaving aside all other factors, diversion of the head-waters of the Middle Fork to the Nevada District will result either in less land irrigated than if that water were used in its own water-shed, or in greater cost if water for the Sacramento Valley lands tributary to the Yuba is sought elsewhere.

Such a diversion may result in the gain of a valuable asset by the Nevada Irrigation District, but it will be at the expense of reduced irrigation and power development in the State of California as a whole.

STRESSES IN HELICALLY REINFORCED CONCRETE COLUMNS

Discussion*

By Messrs. C. L. Eddy and E. H. Harder.

C. L. Eddy,† Esq. (by letter).‡—In their second statement of the Synopsis§ the authors state:

"(2) Before a plain concrete column can fail in shear, the imposed load must be sufficient to overcome the shearing resistance of the concrete and the frictional resistance of the material to incipient motion along the impending plane of rupture."

The writer fails to grasp the authors' conception as expressed in this statement. It would seem that there could be no frictional resistance until shear has occurred; that incipient motion assumes a plane of separation and, therefore, assumes that shear has occurred. It is clear that concrete is not a perfectly homogeneous material and that when it is subjected to stress, adjustment occurs between the different particles, producing permanent set, but it is also true that after this set has occurred there is a relation between stress and strain, and the writer's conception of the meaning of the elastic deformation of concrete is that the constituent parts of concrete are deforming according to some law.

If, therefore, along the plane of shear the stress in the concrete is approaching a maximum, deformations are occurring in the particles constituting the concrete but no slipping has occurred and cannot occur until failure takes place. This conception, therefore, is the basis for a statement that no friction can be developed until shear has occurred. Such a conclusion would preclude the acceptance of the authors' statement, which is the basis of their demonstration.

E. H. Harder,‖ Assoc. M. Am. Soc. C. E. (by letter).¶—The authors are to be commended for their paper on an old subject but one which will probably never be free from discussion. From a practical point of view, concrete is an exceptional building material, but from a purely philosophical one, it is disappointing, to say the least. Therefore, any attempt to develop exact theoretical formulas is bound to be futile. At no stage of its existence is concrete an absolutely fixed material. Shrinkage goes on for years; the value of E

* Discussion on the paper by A. W. Zesiger, M. Am. Soc. C. E., and E. J. Affeldt, Assoc. M. Am. Soc. C. E., continued from April, 1926, *Proceedings*.

† Prof., Dept. of Civ. Eng., Case School of Applied Science, Cleveland, Ohio.

‡ Received by the Secretary, March 11, 1926.

§ *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 3.

‖ Structural Engr., Concrete Steel Eng. Co., New York, N. Y.

¶ Received by the Secretary, March 27, 1926.

varies with the intensity of the stress, and with the age and quality of the concrete; consequently, n varies in the same manner, while m likewise varies with the intensity of stress and perhaps, also, with the age and quality.

As to the newness of the authors' method of theoretical analysis, twenty-two years ago Rudolf Saliger,* an Austrian, developed similar formulas but in different dress, and his formula is the basis of all design formulas used in Europe to-day. Countless experiments have been made in the interim to determine the values of the constants of his formula and, judging from the State specifications of the various continental countries in Europe, all investigators are not in absolute agreement.

Since this paper treats the theoretical stress formulas for concrete columns with helical reinforcement, it will be proper to point out the difference between the authors' investigation and that of Saliger.

Equations (16)† and (17)‡ are the theoretical expressions for the longitudinal and radial deformations of a concrete cylinder subjected to a longitudinal compression and a lateral (in this case, radial) compression due to external pressures. Saliger simplifies Equation (17) by dropping the term, $\frac{h}{m E_c}$. Not only does this omission simplify the final result, but it is justified because the theoretical expressions of Equations (16) and (17) presuppose a simultaneous action of the longitudinal and lateral or radial forces. Actually, the radial forces do not come into play until the longitudinal compression has caused the concrete to expand radially and thereby engage the helical reinforcement. If the spiral had been wound under tension on the concrete cylinder, conditions would be different. It will be sufficiently accurate, therefore, to write:

$$e_h = \frac{v}{m E_c} - \frac{h}{E_c}$$

for Equation (17).

Thus, the first half of the right member signifies the radial expansion of a plain concrete column due to the longitudinal force, while the second half signifies the inward deformation due to the unit surface stresses, h. For convenience, the former will be written, e_h', and the latter, e_h''.

Thus, if A_s' is the cross-sectional area of the helical reinforcement for a height, g, then the tension in such a ring of reinforcement will be,

$$H = f_s A_s' = \frac{d g h}{2}$$

in which, d is the diameter of the concrete core and h is the radial unit horizontal pressure caused by H. Hence,

$$\frac{h}{f_s} = \frac{2 A_s'}{d g}$$

* *Zeitschrift für Architektur u. Ingenieurwesen*, 1904, Heft 5; 1905, Heft 1; *Oesterreichische Wochenschrift für den öffentlichen Baudienst*, 1904, Heft 25; *Deutsche Bauzeitung (Mitteilungen)*, 1907, No. 16; *Rundschau für Technik u. Wirtschaft*, 1908, Heft 1–2; "Der Eisenbeton," 1920, Alfred Kroener, Stuttgart, Germany.

† *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 11.

‡ *Loc. cit.*, p. 17.

Furthermore, let $\frac{A_s' \pi d}{g}$ be the volume of steel in the helical reinforcement per linear inch of column and $A_c = \frac{d^2 \pi}{4}$ be the core area. Then the ratio of the helical reinforcement to the area of the core is $p' = \frac{4 A_s'}{d g}$. From this it is evident that $\frac{h}{f_s} = \frac{p'}{2}$.

Omitting all hyper-refinement alluded to in discussing Equation (17), $h = E_c e_h''$ and $f_s = E_s e_h$. It is easy to demonstrate the latter if one will remember that e_h is the horizontal deformation per inch of column diameter. Then,

$$\frac{h}{f_s} = \frac{E_c e_h''}{E_s e_h} = \frac{p'}{2}$$

or,

$$\frac{e_h''}{e_h} = \frac{n p'}{2}$$

but e_h is equal to the residual lateral deformation, $e_h' - e_h''$; consequently,

$$e_h'' = e_h' - e_h.$$

Substitute,

$$\frac{e_h' - e_h}{e_h} = \frac{n p'}{2}$$

or,

$$e_h = \frac{e_h'}{1 + \frac{n p'}{2}}$$

but $f_s = E_s e_h$ and $e_h' = \frac{v}{m E_c}$ and hence on substitution,

$$f_s = \frac{n v}{m \left(1 + \frac{n p'}{2}\right)}$$

or,

$$v = \frac{m}{n} f_s + \frac{m}{2} p' f_s$$

If there is little helical reinforcement, then p' approaches zero and the column will fail at a unit stress, f_c', which is the required stress to produce a shearing failure in a plain concrete column; consequently,

$$f_c' = \frac{m}{n} f_s$$

and,

$$v = f_c' + \frac{m}{2} p' f_s$$

By making use of the equivalent for $\frac{m}{n} f_s$, the second term of the equation for v becomes $\frac{m\,n}{2} p'\, f_c$ and the final formula is:

$$v = f_c' \left(1 + \frac{m\,n}{2} p'\right)$$

If longitudinal reinforcement is added and the ratio of its area to the core area is p the total load carried by such reinforcement is $np\, f_c'$. For the combined longitudinal and helical reinforcement,

$$v = f_c' \left(1 + n\,p + \frac{m\,n}{2} p'\right)$$

If n is taken at its usual value of 15 and if an average value of $m = 6$ is used,

$$v = f_c\,(1 + 15p + 45p')$$

This is the design formula used in Germany. The restrictions imposed on the designer are that the longitudinal reinforcement must be at least one-third of the helical and may vary from $p = 0.008$ to 0.030. The pitch of the spiral must be less than one-fifth of the core diameter and not more than 3 in., while f_c is limited to 500 lb. per sq. in. Also,

$$A_{\text{core}} + 15\,A_s + 45\,A_s' = 2 \times (\text{total concrete area})$$

It is interesting to compare this formula with American formulas as specified by several building codes and others.

In the Chicago ordinance,

$$v = f_c\,(1 + 14p + 37.5p' + 525pp')$$

in the New York ordinance,

$$v = f_c\,(1 + 14p + 30p')$$

and in the American Concrete Institute specifications,

$$v = f_c\,(1 + 14p + 60p')$$

In these formulas, p' may vary from a minimum of 0.005 to a maximum of 0.015, while the percentage of vertical steel cannot be less than p' but may be equal to 0.005.

It should be noted that in the American formulas, the coefficient of p is 14, whereas in the German formula it is 15. This difference is due to the custom, in the United States, of subtracting the area of the steel from the area of the concrete core. This practice is theoretically correct, but it is surely a hair-splitting matter when there is such a divergent opinion regarding the coefficient of p'.

If one were interested in the history of theoretical derivations, the statement that, until recently, writers have neglected friction in concrete, will need some amplification. Saliger also considered it in his analysis of the helically reinforced column. His equation may be obtained directly from the authors' Equation (15).* Let the left side of this equation be called f_s,

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 11.

the shearing stress on a plane making an angle $\left(45° + \frac{\phi}{2}\right)$ with the horizontal (Equation 5*); then solving for v,

$$v = \frac{f_z}{\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi} + \frac{\sin\alpha\cos\alpha + \sin^2\alpha\tan\phi}{\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi}\,h$$

Let ρ be equal to $\tan\phi$, the coefficient of friction; then by trigonometrical manipulation, the foregoing equation may be simplified and written:

$$v = \frac{2\operatorname{cosec} 2\alpha}{1 - \rho\cot\alpha}\,f_z + \frac{1 + \rho\tan\alpha}{1 - \rho\cot\alpha}\,h$$

This is the equation which Saliger published in 1904.

According to Equation (2),*

$$f_z = \frac{P}{A}(\sin\alpha\cos\alpha - \cos^2\alpha\tan\phi) = f_c' \times \frac{1}{2}\sin 2\alpha(1 - \rho\cot\phi)$$

$$= f_c' \times \frac{1 - \rho\cot\alpha}{2\operatorname{cosec} 2\alpha}$$

therefore, it is clear that the first member of Saliger's equation is equal to f_c'.

The coefficient of h in the second member may be transformed into into $\tan^2\left(45 + \frac{\phi}{2}\right)$. Then,

$$v = f_c' + \tan^2\left(45 + \frac{\phi}{2}\right)h$$

but,

$$h = \frac{f_s p'}{2} = \frac{n p' v}{2m\left(1 + \frac{n p'}{2}\right)} = \frac{n p' v}{m(2 + n p)}$$

or,

$$v = \frac{f_c'}{1 - \tan^2\left(45 + \frac{\phi}{2}\right)\frac{n p'}{m(2 + n p')}}$$

which agrees with Equation (21)† if the previously mentioned omission is made in Equation (17).

The equation,

$$v = f_c' + \tan^2\left(45 + \frac{\phi}{2}\right)\frac{f_s p'}{2}$$

agrees in form with

$$v = \frac{m}{n} f_s + \frac{m}{2} p' f_s$$

hence it may be concluded that,

$$\tan^2\left(45 + \frac{\phi}{2}\right) = m$$

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 7.
† *Loc. cit.*, p. 12

Also, from Equation (5),

$$f_c' = 2\, f_z \tan \times \left(45° + \frac{\phi}{2}\right)$$

consequently,

$$m = \left(\frac{f_c'}{2\, f_z}\right)^2$$

If the values of f_c' and f_z at the point of rupture can be established, a ready means of obtaining the value of m is provided. As the authors have correctly stated, there is a difference between what one may call the cube strength and the cylinder or prism strength of concrete.

Bach gives* the following interesting tests using prisms of varying heights.

Ratio: Height to breadth.	Ratio: Prism to cube strength.
0.5	1.41
1	1.02
2	0.95
3.7	0.87
8	0.86
12	0.84

Thus, taking for the value of f_c', the average, 0.85, of the cube strength and for f_z, 0.20 of the cube strength, $m = 4.5$ and $\left(45° + \frac{\phi}{2}\right) = 65$ degrees.

* *Zeitschrift für Betonbau*, 1915, pp. 57-58.

EVAPORATION ON UNITED STATES RECLAMATION PROJECTS

Discussion*

By C. S. Jarvis, M. Am. Soc. C. E.

C. S. Jarvis,† M. Am. Soc. C. E. (by letter).‡—Although the data summarized by the author had been widely known in fragmentary form during past years, it has been enhanced in value by his orderly assembling and clear presentation. Of special importance is the comparison between rates of evaporation within the same locality as affected by exposure to desert winds or to air currents tempered by contact with dense vegetation like grain or forage crops, the rate of transpiration of which may approach one-half, or more, of the evaporation rate for free water surfaces.

It is probable that shallow pondage, such as depths of 2 or 3 in., may promote greater evaporation than that recorded for either floating or land pans, due largely to increased temperature. It is not unusual, in the irrigation districts, to find alfalfa and other field crops "scalded out" by intense sunshine on the stagnant pools remaining for a few hours after each watering on dense clay soil, if the surface is slightly uneven or the drainage is imperfect. This danger is greater immediately after a crop of hay has been removed, leaving only the bare stubble. The preferable practice is to apply the water a few days before cutting, thus taking advantage of the protective shade and, at the same time, insuring a vigorous new growth without delay.

Where health seekers' colonies have been established in the arid regions, it has been demonstrated that the best results accrue from living above the canals and cultivated fields, because the humidity exhaled from the growing crops may have a depressing effect on the patient and prevent the healing of respiratory tissues.

In contrast with the cultivated vegetation, the native shrubs and plants of the desert have structures and covering which make each leaf, stalk, and root act as a reservoir. It is truly remarkable what sturdy growth is maintained under a few inches of annual rainfall, and how eagerly the pores open for incoming moisture only.

Investigations carried on by the U. S. Department of Agriculture and various College Experiment Stations have shown that selected crops may be matured by strictly conserving the moisture which normally falls even in semi-

* Discussion on the paper by Ivan E. Houk, M. Am. Soc. C. E., continued from April, 1926, *Proceedings*.

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‡ Received by the Secretary, March 5, 1926.

arid regions. One of the most successful expedients is to plant and to fallow during alternate seasons, thus utilizing two years' rainfall for maturing one crop. Where the total precipitation is 12 in. or less per year, and the normal evaporation rate from a free water surface or from wet gound exceeds this total in one summer month, it is evident that the dry farmer or the settler on an irrigation project must be vigilant and able to reduce or nearly eliminate the opportunity for evaporation. Otherwise, the transpiration process would cease and the plant leaves would wilt. A week's delay beyond the proper time for cultivating and renewing the mulch or dust blanket may transform fair prospects to partial failure of crops.

In view of the enormous rates of evaporation reported for the desert regions of the Southwest, usually from 60 to 100 in. per year, the reservoir losses together with those sustained in transit to the farm unit may readily constitute more than one-half the total storage.

One of the many recent developments in the Salt River Valley of Arizona is the installation of pumping units to draw on the underground reservoir. The plane of saturation had been gradually rising since the improvement of the irrigation system by the Reclamation Service, and had water-logged extensive areas, flooding basements, and reducing the productivity of large tracts. The wells driven at strategic points and equipped with pumps have lowered the water-table, reclaimed the areas in need of drainage, added nearly 100 sec-ft. of dependable flow for the canals near the place of intended use, and eliminated the fear of further water-logging. Positive control of the underground water has transformed it from a menace to a potential resource, more valuable than surface storage because it is not subjected to such heavy tolls.

The practical irrigationist on a reclamation project soon realizes that the important consideration is not the quantity of water delivered on his land, but the portion retained within range of root systems. Where a surface mulch is carefully maintained, the crop may be matured in three months or more with the same quantity of soil moisture as might have been evaporated from the uncultivated ground in a few days of midsummer.

If properly interpreted, rates of evaporation from free water surfaces or from wet ground are indices to the attainable limits of transpiration and the consequent growth for ordinary crops. The great problem is to maintain an ample reserve of moisture to insure against wilting, and, at the same time, to avoid saturation and compaction of the soil such as results from flooding. A far better expedient is sub-irrigation wherever the humus content or the porosity of the subsoil makes it practicable. On various projects where drainage systems have been installed, it has proved advantageous to regulate the underground water surface in midsummer by the intermittent use of flashboards in the silt chambers or in the open ditches. This process introduces the moisture exactly where it is needed, promotes capillarity to the rootlets but not to the surface, and thus maintains transpiration at a maximum rate and evaporation at a minimum. The same principle is utilized wherever shrubs or trees are supplied through sub-surface ducts or distributing pipes, and also

where deep furrows are prepared for the reception of irrigation water, and covered soon thereafter with a dust mulch.

With the construction of surface reservoirs and embankments to control floods and conserve the surplus flow, it is advisable to make investigations, tests, and basic provisions for the utilization of underground storage and also the underflow of streams during periods of drought. This, in turn, would provide additional capacity for dissipating flood crests, if adequate inlets are available.

On a limited scale, storm sewer outlets are satisfactorily functioning in coarse glacial drift or in the underlying sand and gravel strata of river valleys. This method of disposal not only saves the cost of deep trenching and large conduits, but conserves the water in a reservoir effectually roofed by the valley soil, and, therefore, immune from the usual losses. The City of Phœnix, Ariz., has utilized this method to great advantage.

Probably the underground waters would be more extensively used and depended on if there were some competent authority to prescribe regulations. The usual procedure thus far observed is to allow installations and pumping without restriction as long as the supply meets every demand, and without reference to depletion of reserve. However, after the water-plane has been lowered until the shallow wells are dry, the controversy naturally develops into litigation with its attendant delays, expense, and uncertainties. As a result, many meritorious projects have been abandoned with financial loss to those investors least able to afford such reverses; and the hazards of financing similar enterprises have multiplied.

If the underground water resources were administered by the same authority and with the same regard for the public welfare as now govern the use of surface streams, untold benefits would follow. This is especially true in regions where the evaporation and transpiration rates are excessive and where the chief problem of agriculture is to maintain the most favorable moisture content of the soil for promoting plant growth and fruitage, as on the typical successful reclamation project.

PIPE TUNNEL UNDER GOWANUS CANAL, BROOKLYN, NEW YORK

Discussion*

By Messrs. Robert Ridgway, J. C. Meem, John R. Slattery, Henry B. Seaman, and T. Kennard Thomson.

Robert Ridgway,† Past-President, Am. Soc. C. E.—It was a privilege to witness the launching of this tube and, later, to see the completed work. Not a drop of leakage was observed, the tube being bone dry, which speaks well for a structure so deep under the water. Altogether it was a successful piece of work, both in design and construction, and the designers, the field engineers, and the contractors deserve great credit for the accomplishment. The design seems to be admirably adapted for the conditions that had to be met at Gowanus Canal.

Some day a rapid transit tunnel may be constructed under Gowanus Canal, in which case it will be well to consider whether this type of structure is the most feasible and economical one to adopt notwithstanding the fact that such a tunnel would demand a much larger cross-section and, therefore, would require deeper and wider dredging. Certainly, the successful completion of the pipe tunnel brings it prominently to the attention of those who will have similar problems to face.

The speaker wonders whether a tube of similar design could be constructed under a wide stream like the East River where the traffic is very heavy and the tidal current strong. If the practicable length is limited, what would be the limit? Could the tube be built and sunk in sections, and the sections connected under the water? These are practical questions that would have to be answered if the method is to be considered for the crossing of large waterways.

All the East and Hudson River transit tunnels have been constructed by the shield method under compressed air, cast-iron rings being generally used for the lining, with an inner lining of concrete. This method has been highly developed as a result of the work under New York waters, and its good features as well as its limitations are well known. It gives a water-tight structure which is believed to be permanent and its construction does not interfere with navigation. It is costly to build, however, and increasingly so, due principally to the rising schedule of pay for compressed air workers. Whether this cost will become so high as to make it necessary to adopt some other method is a ques-

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tion that is being given serious consideration by engineers concerned with the rapid transit problems.

For the four-track Lexington Avenue rapid transit tunnel under the Harlem River—a waterway about 600 ft. wide between bulkhead lines—the open trench method was used, and the tubes were constructed and installed by the same general method as that used for the Michigan Central Railroad Tunnel under the Detroit River.* The Harlem River tubes had a total length of 1 080 ft. and were sunk in five sections, four of them each 220 ft. long, and the fifth, 200 ft. long. The steel in each 220-ft. section, together with the side lagging, which answered for the concrete form, weighed about 750 tons. The sections were built, as a ship is built, in a yard on the Bronx side of the Harlem River about a mile northwest of the site of the tunnel, launched, towed to the site, sunk in place, connected with the previous section, and then encased in concrete, deposited by the tremie method, the concrete enclosure forming a structure 76 ft. wide and 24½ ft. high. The maximum depth of the bottom of the structure below mean high water was about 57 ft. Due to the admirable way in which the contractor, McMullen and Hoff Company, executed its work, there was no hitch in the operations and all the construction steps were co-ordinated and completed in an orderly manner with little or no complication. The leakage is negligible. The report of the operating company, the Interborough Rapid Transit Company, for the month of December, 1923, indicated an average leakage into the four tubes, including some leakage from the land approaches, of 1.27 gal. per min. for the preceding five years.

Inside the steel tubes of the Harlem River Tunnel is a 15-in. lining of reinforced concrete, designed to be strong enough to resist the external pressures in case of disintegration of the steel shell or the external concrete. The speaker assumes that the concrete lining of the Gowanus Canal tube was designed with the same thought in mind. This seems to be a precaution that should be taken, although even in the sewer-laden salt water of the rivers about New York, the steel probably will retain its strength for many years to come, as there is no circulating water in contact with it.

J. C. Meem,† M. Am. Soc. C. E.—In connection with the author's statement that the cost of this work was within the guaranteed estimate, the speaker considers that this was largely due to the hearty and generous co-operation of the author himself.

As to the strength of the tunnel, both longitudinally and laterally, it is not believed that any method of tunnel construction adapts itself so admirably to the greatest strength in either direction at relatively less cost. This is because a tunnel thus built may be designed for a cross-section adapted to the meeting points of the curves of strength and economy without reference to any other function, such as resistance to shield operation, excessive depth to gain cover, etc. The further fact that the section may be flattened to meet the minimum requirement of head-room, adds materially to the economical phases of construction and gradient.

* "The Detroit River Tunnel," by W. S. Kinnear, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXIV (1911), p. 288.

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Regarding the possibility of the rusting away of the steel shell, the speaker believes that steel buried in soil under water will never rust out; but rather that the initial rusting will be confined by the soil and by thus forming a rust-proof coating will prevent further deterioration. However, it is possible to provide at small additional cost a protective coating of concrete or mortar, which will prevent for all time the possibility of deterioration.

As to whether the tunnel can be built in great lengths, the answer is affirmative. Here the method provides temporary buoyant shafts at each end of sections about 200 to 400 ft. long. These sections are sunk by flooding the bulkheaded spaces under the buoyant shafts and as the water rises in these spaces and shafts the section sinks as described in the paper. Water-tight diaphragms are placed at the ends of each section, and the end of the new section is brought by divers into full contact with the last submerged section and then bolted. A minimum number of light bearing piles, cut off to grade, enable the tunnel section to be brought to grade easily and with certainty. The temporary shafts are then removed and the tunnel is back-filled. Later, the diaphragms are burned out internally and the lining is completed. The only requirement is that the tunnel shall have sufficient internal or integral girder strength to overcome and submerge the remaining buoyancy of the unwatered tunnel section between bulkheads. The author's Fig. 7* shows how it was carried down on a perfectly even keel by its internal strength between bulkheads and by the proper manipulation of the valves flooding the shafts.

As to the adaptability of this method to a rock section, the speaker's answer would be that it is not well adapted to those trenches where rock forms a large percentage of the cross-section.

As to carrying this system into the approaches, the speaker believes that the experience had at the site described demonstrates conclusively that approaches may be carried by this method into actual city streets for considerable distances, if pipes and sub-surface structures are removed or by-passed.

It was demonstrated conclusively, not only in the case of the sheet-piling restraining the sides of the trench under the canal, but particularly in that of the west shaft, that the combined pressure of soil and water back of sheet-piling is not materially greater than that of the water itself, providing the soil is below the level of the lowest water at all times.

In the case of the canal piling, the master piles, 38 ft. long, were first driven 10 ft. apart, and the sheet-piling, 20 ft. long, was then driven between, the tops resting at an elevation 13 ft. below mean low water; 35-lb., straight web, Lackawanna piling was used. The master piles were 20-in., 65-lb. I-beams, with a standard sheet-pile riveted to the back flange. In the west shaft, the master piles were 58 ft. long, and the sheet-piling, 38 ft. long, driven to a penetration of 38 ft. below low water. The master piles were driven from 4 to 10 ft. apart according to conditions, and although this shaft was dredged to a full depth of 37 ft. below low water, no evidence of disturbing pressure was noted in the case of a single master pile. In spite of the inertia of the

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 73.

master piles and the fact that much cribbing and rip-rap, as well as hardpan, was encountered in driving, satisfactory results were obtained by using a No. 8 McKiernan-Terry steam hammer; in fact, it proved better able to do the work than a 3 000-lb. drop-hammer. Much difficulty was experienced, more especially on account of the restricted areas of operation, in dredging the hardpan, which appeared when dredged to be a clayey quicksand, but so dense in its natural bed as almost to exclude water. Even when jetted, it settled back into hardpan so quickly that the jet was not effective. The best results were obtained with a Hayward ¾-yd. multiple-power, clam-shell bucket, alternating occasionally with a Mead-Morrison, extra heavy, ¾-yd., orange-peel bucket.

All the canal piling was pulled out. The foreman of the dredge, Mr. Charles Hegstrom, was responsible for the rigging of an ingenious device for pulling this piling, by reversing the steam-hammer and driving the piling up.

To summarize: The completion of the tunnel as described shows conclusively that it is possible, under proper conditions, where it may be necessary to restrain the adjacent soil, to sheath and dredge a submerged trench without bracing; that it is possible to construct a tunnel of a strength and water-tightness not heretofore paralleled, and to submerge and set it easily to line and grade.

JOHN R. SLATTERY,* M. AM. SOC. C. E.—The constantly increasing wages that it is necessary to pay "sandhogs", and the constantly diminishing hours of labor under the various pressures that they work, make it of the utmost importance in the interest of economy to avoid the use of compressed air in building tunnels. The author's method supplies a satisfactory solution, it is believed, for under-river tunnels in many cases.

The Board of Transportation of the City of New York has now before it the problem of building tunnels under the East River at 53d Street, Fulton Street, and a little later, perhaps, at Rutgers Street. At 53d Street the tunnel will lie partly in rock; mostly in rock between Manhattan and Blackwell's Island. Between Blackwell's Island and the Borough of Queens, from what has been determined by boring so far, there will be little rock.

There is a distinct limitation in the possibility of constructing a tunnel by this method where rock is encountered at such depths as are met in East River tunnels. The U. S. War Department requires a depth in the East River of 40 ft. over the roof of a tunnel.

The removal of rock under water can be economically carried out at the present time only by means of dipper dredges. The dipper dredge has yet to be built that will handle rock at the depth necessary to excavate a trench for a tunnel under the East River. It may be possible to design such a dredge, but it would be quite a step in advance of any dipper dredges that are now afloat. It is possible to remove rock at any depth with orange-peel buckets but it is a heart-breaking operation. The best orange-peel bucket that has ever been built, will come up time after time with a piece of rock no bigger than a man's head. It is a serious question whether the cost of removing the rock by orange-peel

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buckets would not offset the other economies in the trench method of construction. In soft material it would be desirable to draw up specifications to permit of alternate bids being submitted for shield-driven tunnels and tubes laid in an open trench.

At Fulton Street there is another problem, that is, the current and the traffic in the river. The channel at that point is 1 200 ft. wide. It is intensively used and the current is swift. Dredges working in the East River, near the existing tunnels, are frequently damaged by towboats coming down the river, without apparent reason. The War Department would probably not permit the use of tunnel sections that would occupy more than one-third of the 1 200-ft. channel. That would reduce the possible length of sections to about 400 ft. The question is as to how safe the sinking of a 400-ft. section would be in the middle of that 1 200-ft. channel.

To carry out the plan safely, it would be almost essential to shut off traffic on the river while a section in the channel was being sunk. It has been stated that it required only 2½ hours to pull this section out of the coffer-dam into position and settle it. It might be possible to induce the War Department to shut off traffic for that length of time, but it is doubtful. Even if the War Department gave its consent, there would be some problem in actually stopping that traffic; towboat men are not easy to handle.

On Newtown Creek, where another tunnel is to be constructed, the boring sheets are not completed. Newtown Creek has probably the densest traffic of any stream in America, if not in the world. Nevertheless, this method could probably be applied there, for even if rock is encountered it will probably not be so deep but that it could be economically dredged.

There is one other problem in connection with this method that raises a question as to its economy; that is, whether approaches can be built without compressed air. If it is necessary to use a shield in the approaches, it is a question whether it would be economical to change the method for the river section. For any tunnels of such long lengths as those under the Hudson, it is believed that this method affords the best solution.

HENRY B. SEAMAN,* M. AM. SOC. C. E.—This paper describing so clearly the novel method of construction, is extremely interesting and instructive from beginning to end. Yet there is one element which deserves particular commendation, that is, the driving of the sheet-piling to this depth, and the use of master piles. This seems to have been the key to the successful execution of the plan. Other matters, also, such as the adaptability of the design to the purpose intended, and its skillful prosecution, deserve the highest commendation.

It would seem that the method here adopted could be applied to work of indefinite magnitude, and may lead to a wholesome revision of tunnel methods.

T. KENNARD THOMSON,* M. AM. SOC. C. E.—More than thirty years ago a bright inventor tried to get the President of the Long Island Railroad, the late Austin Corbin, to appropriate $7 000 000 to build a tunnel under the East

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River. It was to have two iron shells, one inside of the other, with an annular space of 15 in. between them, which space was to be filled with oil (his great patent) to prevent rusting! As Mr. Meem has stated, iron has lasted for ages under water without rusting; but he should have added, "provided the water is pure". Steel does not stand rust as well as iron, and certainly the waters of New York Harbor are anything but pure. In at least one case in New York Harbor the impurities, combined with the heat from a power station, were sufficient to quickly destroy several tunnel linings, successively, of concrete, brick masonry, and stone masonry, whereas wood lining lasted much longer. Elsewhere in this harbor, wooden tunnels have been completely destroyed.

In the West some one estimated that 10% of the cost of a dam was to be saved by building it of steel, instead of gravity masonry construction, but shortly after the steel dam was completed, it collapsed. It was then found that an 8-lb. hammer which had been lying in the water for a few months was completely corroded due to the chemical impurities of the water.

The author, the consulting engineers, and the contractors are to be congratulated on their ingenious design, careful calculations, and successful work. The speaker's only criticism would be that a slight change of design would have saved a large sum, without reducing the factor of safety. Time could also have been saved.

About twenty-five years ago, the speaker took out patents for a trench tunnel. The proposition was: (1) To drive piles and cut them off, one row at a time, below the grade of the tunnel; and (2) to build the tunnel on floating pontoons, in lengths of from 500 to 1 000 ft. each, towing each section to the site and sinking it on the piles, the silt being dredged just before the sinking. One feature of these patents was a pneumatic caisson enclosing each joint, so that all the concrete would be placed in air instead of in running water. Sometimes heat and chemical impurities will destroy concrete before it has had time to set, even more quickly than steel will be destroyed.

A concrete tunnel with the roof and sides as well as the bottom 6 ft. thick, and with six compartments, each 25 ft. wide by 16 ft. high, will cost much less than the two 20-ft. roadways of the Holland (Vehicular) Tunnel and would be a little heavier than water, so that it would not rise and fall, like the present Hudson River Tunnels, inversely as the tides, hour after hour, year after year. The 6-ft. water main under Toronto Bay carried Toronto's water supply safely for years, and then, suddenly, a mile of it floated to the surface. No one can prove that the present tunnels will not last 1 000 years, nor that they will last 1 000 min. Then, if the bigger and safer tunnel costs much less why not adopt it?

Regarding the length of sections that could safely be used, it would be just as easy to handle sections 500 to 1 000 ft. long as a large ocean liner.

Where part of the excavation would be in rock there would be an air-chamber under the tunnel sections for completing the excavation under compressed air. An improvement over the device of sinking caissons over the joints would be to have the caissons attached to the tunnel sections before sinking so as to telescope those already sunk.

The reason given for rejecting this design for the Vehicular Tunnel was that with six compartments of 25 ft. each it would have too great a capacity for one location. To the speaker it seems absurd to expect one 20-ft. roadway in each direction to be at all ample to connect Long Island and Manhattan to the remainder of the United States, through New Jersey, even if such tunnels were placed at nearly every block. Even yet the peak of automobile congestion has not been reached.

DETERMINATION OF THE DUTY OF WATER IN WATER-RIGHT ADJUDICATIONS

REPORT OF THE DUTY OF WATER COMMITTEE OF THE IRRIGATION DIVISION OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

Discussion*

BY MESSRS. E. A. PORTER AND LLOYD GARRISON.

E. A. PORTER,† M. AM. SOC. C. E.—This report covers one of the most complicated and vexing problems in irrigation engineering and operation. It is a problem for which a reasonable solution has been attempted, from time to time, since the beginning of irrigation in the arid regions. It is a subject which ever seems to meet with changeable factors and conditions and which arouses the wrath and suspicion of the old water user more quickly than any other in irrigation. For this reason the report is not only timely but should be of great value to those engineers who are interested in irrigation and kindred problems.

It is well known that any one entering into a discussion in an attempt to solve the proper duty of water for a given area, is likely to be severely criticized. The speaker has been connected with problems involving the reasonable duty of water for several years. Such experiences have taught him that the problem must be met in a spirit of tolerance and with a practical mind, and that the factors of water supply, soil conditions, and methods of use in combination with crop production, although of great importance, may all be changed by the important, but variable, factor of climate. The present season in Utah furnishes an example of this condition. The water duty for the season of 1925 may be quite out of place for another year. Those in charge of water-right adjudications should be able to appreciate the fact that the user's interest, although paramount, should be analyzed in connection with the interests of the Commonwealth in which he lives. It will only be a matter of time until a reasonable water-right will have assumed as definite a value as any other property right. Therefore, the water supply of the arid regions must ultimately be made to perform its maxmium duty. Realizing the ultimate insufficiency of water for all the irrigable lands within the arid

* This discussion (of the Report of the Duty of Water Committee of the Irrigation Division on Water-Right Adjudications, presented to the Irrigation Division, Summer Meeting, Salt Lake City, Utah, July 9, 1925, and published in April, 1926, *Proceedings*) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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regions it does not seem consistent that final decrees should be permitted in water-right determinations. Neither does it seem consistent that States should permit further appropriations on most of the streams until accurate data are available to show a surplus.

The results secured to date by the Committee have been ably presented. However, it appears evident after a review of the report that there still remains a vast opportunity for improvement in the collection of data to determine the proper duty of water under practical methods of use. All who have had experience in irrigation management or operation realize that in most instances throughout the arid West, some District Court will pass on the final recommendations for a determination suit. The Committee is right when it explains that the testimony of the actual user of water will carry more weight than that of the expert. This is probably a bold statement, but, nevertheless, it is true, for the reason that the expert is usually employed to help a client win his point. It is well known that some so-called experts often introduce as facts testimony which they know to be untrue.

Under the revised Water Rights Law of the State of Utah (1919), the State Engineer is empowered to redetermine the water-rights along any river system in the State after the water users under such stream have taken the necessary legal preliminary steps. At present, this official is engaged with the problems of a proposed determination for the entire Sevier River Basin in Southern Utah which data will later be submitted to the water users and District Court for confirmation. This river system is typical of most of the rivers in the Intermountain region, and for various reasons presents an ideal example for use in connection with other systems not involving priority rights for power development. There are no important power developments within the Sevier Basin. It is solely an irrigation stream for the reason that the early settlers filed on all the normal water supply for surface irrigation purposes. Power developments will probably come in later years, but such installations will require equalizing storage reservoirs.

At present, the upper valleys of the Sevier Basin are irrigated almost solely by natural flow, diverted either from the main channel or from tributary streams. The area served in these valleys is about 25% of the total area irrigated within the Basin. The remainder of the irrigated, or irrigable, area lies within the storage zone where the water supply can be accurately controlled by the aid of several storage reservoirs. The largest of these reservoirs has been created during recent years, prior to which the natural flow rights, now within the storage zone, had been filed upon, or used, for a long time. The Courts had adjudicated these waters to the rightful owners. Later, the storage reservoirs were created to conserve all the surplus water supplies. Prior to 1914 the Federal Government and State had established three or four water-measuring stations within the Basin, but in 1914 these two agencies, in co-operation with the water users, saw fit to determine accurately the water resources of the Basin at all critical points.

To-day, the engineer, water user, or public have available before them a mass of accurate data showing the seasonal head-gate diversion of every important canal in the Basin, the daily, monthly, or annual flow of the river at a

score of points and the approximate area of irrigated and irrigable lands under each canal system. Without doubt the stream-flow data on the Sevier System are as complete as those on any system in the country. With the aid of these data the factors of canal capacities, gross diversions, and return flow from important areas can be ascertained. There is, however, one weak link in the chain of data for this river system, and this is the duty of water, which is of such importance that it may delay the adjudication suit for years.

However, the importance of the reasonable water requirement for the irrigated lands had been appreciated by a few. The steps taken to secure data on this subject have been along practical lines. The first step was to persuade the older canal companies, operating under the former natural flow rights, to appreciate the value of a stored water supply as compared with their variable and uncertain decreed rights. In 1920 one of the older canal companies agreed to exchange, for trial only, its decreed right for a definite stored water supply for use as desired during the irrigation season. The result was so surprising that all suspicion against the plan was immediately dispelled, with the result that this particular canal has now pooled its former natural flow right with the stored water interests. Several other canal companies have followed the plan, after this experience.

This method of procedure has brought together many conflicting interests which stood in the way of the most efficient adjudication. It has demonstrated, in a practical manner and without any additional cost, that a stored water supply is superior to a fluctuating natural right. It has taught the older water users that successful crops, common to the Intermountain region, can be matured with 2 to 3 acre-ft. of water instead of 4 to 7 acre-ft. as formerly used on the land. These results have been secured almost solely by the Water Commissioner who has had field charge of water distribution and utilization. The Commissioner has operated under the supervision of the Court or State Engineer, and his policy has been to distribute the water in co-operation with the water users and storage reservoirs. In years of stress and seasons of plenty this Commissioner, who is a technical engineer as well as a diplomat, has been able to protect the interests of all satisfactorily.

It would appear that all water supplies within a storage zone should be made common and that the interests should be pooled on a priority basis. Data in connection with the determination of the duty of water should be secured in co-operation with the actual farmer or irrigator. Water-right decrees should not be made final as a general rule. Water commissioners should be selected carefully and after a selection is made, the commissioner should be given full charge of all field duties with the necessary authority and backing of a legal body. The length of the irrigation season should be definitely set prior to the start of each season.

LLOYD GARRISON,* ESQ.—One or two notes based on the speaker's experience in Utah may be of interest. In 1919, the Legislature of Utah passed a law which provided for the adjudication of water rights, and under this law all

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the streams of the State will eventually be adjudicated. It has been the practice of the State Engineer to make a determination of all rights as provided by the law and to submit his findings to the Court with the recommendation that jurisdiction of the case be retained by the Court for a period of five years. Duty of water in the determination is fixed by general experience and such local information as is available. During the five-year period it is contemplated that tests of duty will be conducted on representative tracts of land. These tests will include records of water used, crops raised, soil determinations, and such other data as will give as complete information as possible concerning the duty required. At the end of the five years it is hoped that sufficient data will be collected to permit the Court to render a final decree.

The speaker hardly agrees with Mr. Porter that the suit should be held open indefinitely. That leaves the settlement for too long a period. Five years should be sufficient for an accurate determination of duty. The law in the State of Utah is good and if given a chance will be very helpful to the farmers in effecting a permanent settlement of their difficulties.

INTERSTATE WATER PROBLEMS AND THEIR SOLUTION

Discussion*

By Frank C. Emerson, M. Am. Soc. C. E.

Frank C. Emerson,† M. Am. Soc. C. E.—The State of Wyoming shares with Colorado the position of sitting at the high point in the United States and watching the waters of interstate streams flow across the borders of the State to the four points of the compass. To the north the Big Horn River rushes into Montana to join the Yellowstone and eventually the Missouri; to the east the North Platte flows into Nebraska also to join the Missouri; to the west, the Snake River departs into Idaho as a tributary of the Columbia; and to the south the Green and Little Snake Rivers flow into Utah and Colorado as tributaries of the mighty Colorado. This situation provides Wyoming with many interstate water problems of importance. No State has made greater progress in meeting these problems than Wyoming and the speaker feels assured that the facts given in this discussion will bear out the assertion.

The interstate water problems of the arid West were just beginning to present themselves with force when the speaker assumed the office of State Engineer of Wyoming in 1919. This was due no doubt largely to the extremely dry season of 1919 combined with the increasing use of water throughout the inter-mountain country. During the speaker's six years' tenure of office the problems have been coming with ever greater force, and it has been necessary to take an active part in attempting to meet them to the best advantage of the State. It is hoped that this discussion may convey some information and ideas that may be helpful in the solution of the problems under consideration, which mean so much to the development of this great Western country.

A few thoughts will be given in support of the conclusions of Messrs. Hinderlider and Meeker that the best solutions of interstate water problems will be found in amicable agreements largely expressed through interstate compacts. Wyoming has certainly had convincing experience that litigation should be avoided whenever possible in favor of effort to solve questions with other States through intelligent conferences and mutual agreements. It was stated that the case of Kansas *vs.* Colorado has been in the Courts for a period of twenty years and is still undecided, and that the Wyoming *vs.* Colorado case respecting the use of water from the Laramie River was before the U. S. Supreme Court eleven years before decision was made. With the latter case,

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despite the fact that much time, energy, and money were spent by both litigants while this case was running the gauntlet of the Courts, the results could not be considered to be of material advantage to either Wyoming or Colorado when the decision finally came. It is true that the earlier priorities on the Laramie River in Wyoming were sustained in a general way according to the wording of the Court opinion, but in practical effect the verdict has resulted in little advantage to Wyoming.

As State Engineer and President of the State Board of Control, the regulation and distribution of water from the streams of the State come within the speaker's jurisdiction. During the season of 1922, when the flow of the Laramie River was considerably below normal, it was found that water rights in Wyoming of 1883 priority were forced to go without water although, under the decree of the Court, the diversion of water into the Laramie-Poudre Tunnel in Colorado was allowed, notwithstanding the fact that the Court recognized only the late priority of 1909 for the right. The Colorado right was being served while water users in Wyoming having rights of twenty-six years seniority on paper saw their ditches "bone" dry. Why this situation? Simply because the Court of last resort in its wisdom granted to Colorado rights a certain volume of water for use each year, regardless of the seasonal flow of the river. As a matter of fact, under the conditions that prevail at present the situation is such that the 1909 right for the Colorado tunnel diversion could never exceed the quota of water allowed this right by the Court, although Wyoming rights of more than a quarter of a century seniority will at times be without water unless they undertake a large program of reservoir construction. What was heralded as a victory for Wyoming was indeed an empty one.

How futile the time, effort, and expense devoted to this litigation, and, curious though it may be, Wyoming fought for the establishment of a principle in this case (which in itself only concerned the water supply of a comparatively small stream), that would work against the State in relation to practically every large stream in the State. What was to be gained by Wyoming as a State from the logical decision of the Court that no State could use the waters of an interstate stream to the material prejudice of rights established in other States. The small Laramie River happens to rise in the mountains of Colorado and flow into Wyoming; practically every large stream of the State flows from Wyoming to States beyond its borders. Again, the eminent members of the legal fraternity who presented the case for Wyoming, contended that water could not be legally diverted from one water-shed to another. The Court wisely decided that the inter-water-shed diversions were permissible as long as damage was not inflicted on established rights. An adverse decision would have been disastrous to a considerable development in Wyoming in sections where it has been found that conducting water from one drainage area to another has resulted in the most valuable use of the water supply available.

The foregoing comment is made on the Laramie River case by reason of the fact that it is probably the most important irrigation case decided by the U. S. Supreme Court. In this controversy Wyoming engaged in legal combat with its neighbor, Colorado, and after years of litigation the decision came without

bringing material gain to either State. The speaker ventures to assert that representatives of the two States in intelligent conference could have reached within two years an agreement that would have been reasonably satisfactory to both Commonwealths. It is not intended to infer that the decision of the Court was unfair or poorly founded. To the contrary, the findings of the Court are generally logical throughout and have decided clearly and definitely several mooted points concerning water-right jurisprudence. It was well in a way to have them decided but, at the same time, the particular matters concerned by this case certainly did not warrant the struggle in the Courts.

Other cases concerning interstate water problems of interest to Wyoming could be cited. Suffice it to say, however, that the usual experience in cases wherein appeal has been made to the Courts for solution of water-right questions, has proved that litigation is a poor method indeed. In contrast to this unfavorable experience when resort has been made to the Courts, is the record of accomplishment obtained through friendly conference and mutual agreements. Favorable progress toward the solution of interstate water problems has been made by Wyoming along what may be considered as three different lines of effort, namely:

1.—By what may be considered as a "gentleman's agreement", as exemplified by the North Platte River Co-Operative Investigations;

2.—By reciprocal statutory enactments and agreements between States, as is evidenced by the negotiations and agreements concerning interstate streams between Utah and Wyoming; and

3.—By interstate compact such as is now under negotiation in respect to both the Green and North Platte Rivers of Wyoming. Comment will be made on these three plans in order.

North Platte River Co-Operative Investigations.—In 1904, the State of Wyoming granted to the United States a permit for the construction of the Pathfinder Reservoir and the storage of more than 1 000 000 acre-ft. of water per annum of the North Platte River. The U. S. Reclamation Service forthwith proceeded to construct a monument of good engineering and by the Pathfinder Dam provided for the conservation of the waters of the North Platte. Although this great conservator of water filled a definite need, it proved for many years to be a curse rather than a blessing to the State of Wyoming which granted the right. With its large investment in the dam the United States became much concerned about the adequacy of the North Platte to supply more than the areas of land lying under the canals constructed by the Reclamation Service, which were looked to for the return of the cost of the reservoir. On the supposition, or the fear, that these lands, lying principally in Nebraska, needed all the water of the river for their proper irrigation, an affective embargo was placed upon any additional developments in Wyoming.

Development on the North Platte in Wyoming was effectively prevented for years in any large way by the Pathfinder situation. Carey Act segregations were refused, desert land entries were denied, rights of way across Government lands were not granted, and, in other ways, developments which proposed a use

of water in Wyoming from the North Platte River and its tributaries were blocked by the Federal Government. The situation became intolerable until in 1918 a co-operative investigation of the whole matter was agreed upon between the State of Wyoming and the United States. The field investigations were thorough and resulted in what is known as the Conkling-Meeker report, Harold Conkling, M. Am. Soc. C. E., representing the United States, and R. I. Meeker, M. Am. Soc. C. E., representing the State of Wyoming. On this preliminary report the Board of Review on North Platte Co-operative Investigations founded its report carrying the final recommendations and conclusions concerning the use and disposition of the waters of the North Platte River. The Board of Review was composed of E. A. Moritz, M. Am. Soc. C. E., for the United States, the speaker for the State of Wyoming, and Mr. L. G. Carpenter, of Denver, Colo., as the third party.

The primary finding of the Board was to the effect that there was sufficient water in the North Platte River, when properly conserved, for the reclamation of all lands practical of irrigation in Colorado and Wyoming and for additional extensive areas in the State of Nebraska. This led to the further conclusion that artificial restriction of developments on the upper river were entirely unnecessary.

The report of the Board of Review received the approval of the Secretary of the Interior, the Director of the U. S. Reclamation Service, and the Governor of Wyoming. With its adoption the embargo against further Wyoming developments from the water supply of the North Platte River was immediately lifted and the unfair situation relieved. The report of the Board of Review has been referred to as a "gentleman's agreement" and such it is in practical effect. Although not a real adjudication, the findings of the Board have been respected, and the strain has been removed from the condition that existed. By intelligent study and friendly conference this "gentleman's agreement" recorded real accomplishment. Negotiations are now under way to form a compact between the States of Colorado, Wyoming, and Nebraska, concerning the use of the waters of the North Platte River and it is proposed that the report of the Board of Review be the basis for the compact.

Reciprocal Statutory Enactments and Agreements Between States Concerning Interstate Streams.—The example afforded by effort along this line by the States of Utah and Wyoming is noteworthy, and what may be accomplished through such a plan, is well illustrated by the present agreements concerning the use and administration of the waters of interstate streams of interest between these two States. In 1921, the Legislature of Wyoming passed a bill proposing reciprocity with Utah in water-right matters. After this bill was enacted into law by the Wyoming Legislature, the speaker made a trip to Salt Lake City to work with the State Engineer of Utah in the matter and a bill similar to that passed by Wyoming was prepared and introduced in the Utah Legislature. Favorable action was taken by the Legislature and, therefore, similar statutes are now in force in the two States, providing for conference and mutual agreements concerning the waters of the Bear River and other interstate streams. For the information of the membership the Wyoming statute is, as follows:

"Co-Operation with Utah in Water Right Matters

"Sec. 1. That the state engineer is hereby authorized and empowered to receive and grant applications to appropriate water from streams in Wyoming where such water is to be conveyed and used for beneficial purposes within the state of Utah, and the board of control is hereby authorized and empowered to issue certificates of appropriation under such permits as may be granted by the state engineer of Wyoming upon certification from the state engineer of Utah that the waters appropriated have been put to the beneficial uses set forth in the permits; the state engineer and the board of control of Wyoming are also authorized and empowered to co-operate with the state engineer of Utah in the determination, supervision, regulation and control of all water and water rights on all Utah-Wyoming interestate streams; and to these ends the state engineer and board of control of Wyoming, by and with the consent of the governor, may enter into the necessary agreements with the state engineer of Utah to carry out the purposes of this act, provided that such agreements are not in conflict with the provisions of the irrigation laws now in force in this state; provided further that such authority shall not be exercised by the state engineer or board of control until after the state of Utah has passed a law granting its state engineer like authority to that granted to the state engineer and board of control of Wyoming by this act. (L. 1921, ch. 47.)"

Advantage has been taken of these statutes and certain plans have been put into operation under the provisions thereof. In May, 1925, the speaker took a comprehensive trip by automobile with George M. Bacon, M. Am. Soc. C. E., State Engineer of Utah, over the several interstate streams between Utah and Wyoming, for the purpose of obtaining information concerning the situation on the ground in order to work out forms of agreements between the States that would be fair to both. Certain definite arrangements were consummated whereby intelligent administration of the waters of certain streams is made possible where before no satisfactory system was in force. For example, take the situation on the Bear River, an important stream from which a large use of water is made both in the State of Utah and the State of Wyoming.

The Bear River rises in the mountains of Utah and flows across the line into Wyoming and traverses Wyoming territory for a distance of 30 miles in the region of Evanston; then passes back into Utah to supply water to a large acreage of land in that State; then back into Wyoming to supply water for the irrigation of the territory in the vicinity of Cokeville; then into Idaho and finally again back into the State of Utah. Many ditches have their points of diversion of water in one State with the irrigable lands situated in whole or in part in another State. It will be readily seen that interstate problems of importance come into such a situation. Through and under the provisions of the reciprocal statutory measures between Utah and Wyoming an intelligent and effective administration of the waters of the Bear River between these two States is being obtained.

To illustrate what has been accomplished, consider the authority that has been given to Mr. David Dean, Water Commissioner of Wyoming, in charge of the regulation and distribution of water from the Bear River in the Evanston District. In addition to authority over the head-gates located in Wyoming, Mr. Dean has been given specific authority by the State Engineer

of Utah over three certain head-gates which are in Utah, but which irrigate lands situated in Wyoming. These three ditches cover a large area of Wyoming lands and have comparatively recent priorities. Heretofore, in times of shortage of water in the river, the Wyoming Water Commissioner has been powerless to act, and has been forced to see large diversions of water for the benefit of these recent rights, while rights of many years seniority in both Wyoming and Utah have been forced to go without. As a Water Commissioner of Wyoming, Mr. Dean had no authority whatever over the diversions situated in Utah. Having been given the power to regulate these diversions, he was immediately placed in a position to take such action as might be fair in the premises and serve the proper interests of users of water in both States.

On another section of the Bear River an opposite situation exists to the one described. In this instance three certain canals divert water in Wyoming for the irrigation of lands largely situated in Utah. Mr. Dean has been instructed, as Wyoming Commissioner, to allow such quantity of water to pass into the head-gates of these ditches as the lands irrigated thereunder would be entitled to under their priorities, the quantity of water to be governed by the acreage on which beneficial use is now being made. On other streams of interstate nature between Utah and Wyoming agreement has been had whereby water commissioners in the two States co-operate with each other in the administration of streams in accordance with priorities established. This co-ordidination allows fair and intelligent administration of these streams where formerly no competent system of regulation was operative between the two States.

Thus, in these and other ways, the States of Utah and Wyoming are working together in the solution of vexing interstate problems. Intelligent consideration is given to them, the physical facts on the ground are taken into consideration, and otherwise the matter is approached in a manner that, with little exception, provides for decisions and plans that are to the equitable and best interests of parties involved. It is believed that much has been and is being accomplished through these mutual agreements made possible through reciprocal statutory enactments by Utah and Wyoming.

Compact or Treaty Between States.—To this plan the paper of Messrs. Hinderlider and Meeker has been largely devoted and an excellent presentation of the subject has been given. The writer wishes to present certain comment in regard to the now famous proposed Colorado River Compact as an outstanding example along the line of solution by interstate compact of interstate water problems.

Through the Colorado River Compact, agreement was sought between the seven States of the Colorado River Basin concerning the division and use of the waters of the Colorado River and tributaries. No other stream in the arid West affects so many States as the Colorado River. Extensive additional uses of water, beyond the important uses now established, are proposed. Another element of material importance entering into the problem is that of the necessity of flood-control measures in order to relieve the menace of floods to the valleys on the lower river. Conflict of interests between States in rela-

tion to these and other phases of the Colorado River problem has led to the adoption of the treaty plan of solution. Without an agreement between States there is little question but that the river system would be involved in litigation for years in event any large additional developments were attempted on any section of the river. By treaty between States, such as would become effective on the ratification by all the States of the proposed Colorado River Compact, or a modified form of such compact that might be agreed upon, the way would be clear for all practical developments that may be proposed.

The Colorado River question is one that has been discussed at length at various times and at different places. It may not be amiss, however, to call attention to the general physical situation which exists in the water-shed of this great river in view of the necessity of compact or agreement between States if the problems of the interstate use and division of the waters of the Colorado are to be properly solved to the best interests of all parties concerned. The speaker, therefore, takes the liberty of the following recital of physical facts which apply.

The Colorado River Basin in the United States naturally divides itself into two great basins separated by hundreds of miles of deep barren canyon cutting through high and rough plateaus. The Upper Basin embraces areas in the four States of Colorado, New Mexico, Utah, and Wyoming, and these States furnish about 85% of the flow of the river. Millions of acres of land are irrigable in the Upper Basin, and possibilities exist for large developments of hydro-electric power. Throughout the canyon region separating the two basins large power possibilities also exist, although it is impractical to divert water in quantities for irrigation. The Lower Basin comprises areas chiefly in the States of Arizona, California, and Nevada, supplying only about 15% of the water to the river, but which have extensive possibilities for the use of water for domestic, agricultural, and power purposes. The agricultural and economic conditions in the Upper and Lower Basins are entirely different. Development of the water supply for the benefit of the Lower Basin will be much more rapid than for the Upper Basin. Conflict of interest therefore arises between the two basins, and the logical plan of solution is an apportionment of water between them.

The Green River rising in the mountains of Wyoming is one of the most important tributaries of the Colorado River. The Little Snake is also a Wyoming tributary. These two streams drain about one-fifth the area of Wyoming, and furnish about 14% of the total flow of the Colorado River System. Present irrigation from these streams in Wyoming covers areas of about 435 000 acres. Additional irrigation possibilities to the extent of 765 000 acres are estimated. There is, therefore, a possible total development of 1 200 000 acres on Wyoming tributaries of the Colorado. The developments of the future will not be rapid, but will be none the less desirable as projects become feasible. Wyoming, therefore, has looked with much concern on the proposed large developments on the Lower Colorado River that would establish priorities to the use of water from the river that might well cause an embargo against future developments in this State.

Having its inception as an undertaking at the meeting of the League of the Southwest held in Denver, Colo., during August, 1920, the matter has gone far. Through the refusal of the State of Arizona to ratify the form of compact agreed to by its representatives on the Colorado River Commission, the Compact has thus far failed to become operative, although ratified in its original form by the other six interested States. California has apparently taken a backward step in the matter recently by attaching a reservation to its ratification of the Compact under the six-States plan, when the proposal was considered by the Legislature to make the Compact binding on the States of California, Colorado, Nevada, New Mexico, Utah, and Wyoming, regardless of the failure of the State of Arizona to approve it.

However, the fact that necessary enabling legislation to allow consideration of the matter of the Compact was first passed by all seven States in the Colorado River Basin and by Congress; that the representatives of the seven States and of the United States who formed the membership of the Colorado River Commission could unanimously agree on a definite form of compact; and that this form of compact should be ratified and approved by the Legislatures of six of the seven interested States, is in itself a marked accomplishment, whether or not the Compact ever becomes operative in fact. Although falling short of the final mark the speaker believes that the terms of the Compact will be found in the future largely to govern the use and division of the waters of the Colorado River and, therefore, that much indeed has been accomplished by this effort to solve the interstate situation on this great river.

Having represented Wyoming on the Colorado River Commission, the speaker naturally is convinced of the merits and fairness of the present proposed form of compact. It is possible that certain modifications or amendments might be made to meet the objections of the present opponents of the Compact. However this may be, it is his firm conviction that some such agreement must be had between States before the way will be properly clear for meritorious development of the great water supply involved. It, therefore, behooves all parties to wipe out as far as possible those natural prejudices which exist between conflicting interests on the river and seek to learn as much as possible of the viewpoint of the other fellow and of his aims and possibilities. For the purpose of obtaining first-hand information, it was the speaker's pleasure to make a rather comprehensive trip over the Lower Colorado River when the question of the Compact first came up for consideration. The serious flood menace to the Imperial Valley and other sections on the lower river was made plain; conditions were noted which witnessed the need for conservation of water for irrigation, power, industrial, and other uses; and opportunity was given the speaker to inform himself generally concerning the situation in the Lower Basin warranting certain developments at an early date. He wonders whether many of the representatives of the States of the Lower Basin understand those conditions in the Upper States that warrant the insistent attitude for preservation of their natural resources that prompts those States to demand an equitable agreement in regard to the use of water from the Colorado River before their support can be given to lower-

river developments. The speaker thinks not and, speaking for Wyoming, he wishes to make brief reference to facts concerning the State.

The average traveler passing through Southern Wyoming over the Union Pacific Railroad, on his journey across country, would obtain no real conception of the resources and possibilities of this great State. To give some idea of what may be expected from Wyoming Table 7, compiled from data from the U. S. Census Reports of 1880, 1900, and 1920, is presented to show the remarkable increase in the agricultural and livestock industries of the State, in twenty-year periods, beginning with 1880.

TABLE 7.—VALUATIONS OF FARMS AND FARM PROPERTY.

Year.	Land, fences, and buildings.	Machinery and implements.	Livestock.
1880	$835 895	$95 482	$5 007 107
1900	26 965 530	1 366 000	39 145 871
1920	234 748 125	11 777 949	87 884 516

The increase shown by these figures of from less than $6 000 000 in valuations in 1880 to more than $67 000 000 in 1900, and then to the comparatively large total of more than $334 000 000 in 1920, demonstrates that Wyoming is a rapidly growing State in relation to its agricultural development. This development is largely dependent on the great natural resource of water supply. It should not be handicapped by artificial restrictions that might come from early development on the Lower Colorado River. For many years, as has already been stated, Wyoming was seriously handicapped by the embargo placed on the developments from the North Platte River under the supposition that the water supply of that river was not adequate to take care of further extensions of irrigation in Wyoming. It was finally determined that the water supply of the North Platte was sufficient and that the embargo had indeed been suffered unnecessarily. No such repetition in relation to the water supply of the Green and Little Snake Rivers is desired as actually did obtain on the North Platte. Wyoming would welcome being placed in a position whereby support and aid could be given to all legitimate development for the benefit of the States in the Lower Basin of the Colorado River. In the present situation, however, the citizens of Wyoming cannot look with favor on large developments on the lower river and in just consideration of their own interests, their position must so remain as long as no agreement is in force to protect properly the future use of water in the Upper Basin. Litigation to solve the questions that are bound to arise in the future would certainly result in a waste of much time, energy, and money by all parties concerned. It is, therefore, to be sincerely hoped that amicable agreement between the States of the Colorado River Basin may finally be reached, whether through the present proposed Colorado River Compact or through a modification of it.

In the matter of interstate compact, engineers should feel called on to take a hearty interest. The political question of interstate relations seems to have received little attention. Why should engineers devote themselves so

industriously to considerations of a technical character to the practical exclusion of the larger political questions on interstate streams that would handicap, if not entirely preclude, any large development of the water resources until the political phases of the matter receive intelligent attention and solution. It should be as much the duty of engineers to give their attention and assistance to the solution of the political questions involved by interstate problems as to the engineering phases. Until they take such attitude they will fall far short of the mark of the service that they are in position to render.

In conclusion, the speaker wishes again to voice both his hope and firm conviction that the best solutions of interstate water problems will come through intelligent conference and amicable agreement. Litigation means that decision will need to be made from a mass of testimony, generally conflicting in the extreme and well flavored with prejudice throughout. Solution through interstate agreements, worked out in harmony through the careful consideration of all facts and equities involved, means that justice will be done as far as humanly practicable to all parties in interest, and with the greatest saving possible in time and expense. Engineers as a body should give their attention to the political phases of interstate questions without the solution of which all technical devotions may be largely in vain. Let the profession take its place as a leader in the solution of these admittedly complicated and vexing problems and it will certainly be the winner thereby.

STREAM REGULATION WITH REFERENCE TO IRRIGATION AND POWER

Discussion*

By Messrs. Edward H. Sargent, W. G. Hoyt, F. F. Henshaw, Barry Dibble, Ralf R. Woolley, and Arthur P. Davis.

Edward H. Sargent,† M. Am. Soc. C. E. (by letter).‡—These remarks are submitted, merely as a few thoughts in regard to stream regulation in relation to power development and flood control.

The statement has frequently been made that stream regulation for the combined purposes of power development and flood control is impracticable on the ground that for flood-control purposes a reservoir should at all times be empty, which condition would destroy its utility for regulation for power purposes. This thought has probably been due to the fact that frequently it is not feasible to build a storage reservoir of sufficient capacity adequately to control the flow of the water-shed at the dam site. The writer will discuss two projects with which he is familiar where it was feasible, principally because of the effect on power developments, to build reservoirs sufficiently large so that their operation for power purposes to a great extent controlled the flood flow.

In the preliminary studies of the so-called Portage Reservoir, made under the direction of John R. Freeman, Past-President Am. Soc. C. E., it was planned to build large orifices in the spillway section of the dam at such an elevation as would permanently reserve a large amount of storage for flood control—in other words, the top of the reservoir was to be made into a retardation basin, the orifices being of such capacity as to discharge under maximum conditions a flow not exceeding the normal flow of the stream. The remaining capacity of the reservoir was to be used in stream regulation for power purposes. At the time the studies were made (1907-08), only limited stream-flow data were available on the Genesee River, the stream in question. Since that time daily records of the flow of this river have been kept. Extended mass curve studies of these records have convinced the writer that practically all the flood control which would have been obtained under Mr. Freeman's plan would be obtained by using the entire reservoir capacity for power regulation, that reserving the top of the reservoir as a retardation basin would have been equivalent to throwing away that amount of storage which could have

* This discussion (of the paper by J. C. Stevens, M. Am. Soc. C. E., presented to the Power Division, Summer Meeting, Salt Lake City, Utah, July 9, 1925, and published in April, 1926, *Proceedings*) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Chf. Engr., Hudson River Regulating Dist., Albany, N. Y.

‡ Received by the Secretary, June 24, 1925.

been used for power purposes. The water-shed in question was 958 sq. miles in area and the proposed storage totaled 413 000 acre-ft., or 431 acre-ft. per sq. mile of water-shed. Stated in other terms, the storage capacity was approximately 52% of the average yearly run-off at the dam site. The economics of the project made it feasible to develop storage to furnish this high degree of control which was such that, in the period of 1908-24, had the reservoir been operated for power purposes, only once would the normal flow of the river immediately below the dam have been exceeded.

The Sacandaga Reservoir, now being built under the direction of the writer, is to serve the dual function of regulation of stream flow for power and for flood control. This reservoir has a water-shed at the dam site of 1 044 sq. miles and is the principal tributary of the Hudson River, which at the "center of gravity of power" has a water-shed of 2 780 sq. miles. The water-shed of the Sacandaga River lies in a region of much higher precipitation than that of the Genesee River and the Sacandaga Reservoir is being built to have an available capacity of 689 000 acre-ft., which is equivalent to 658 acre-ft. per sq. mile of water-shed, almost exactly 50% of the total run-off in an average year.

Between the dam and the confluence of the Sacandaga and Hudson Rivers there is a fall of 152 ft., which is now undeveloped, and on the Hudson below this confluence there is a fall of 548 ft., most of which is now developed. In making the first studies of this reservoir it was felt that, because of the comparatively large amount of fall immediately below the dam, the reservoir would have to be operated to provide the best flow possible at the dam site and that this would seriously impair its usefulness for flood control, inasmuch as with power plants immediately below the dam, water would have to be released from the reservoir when the Hudson River was in flood. It has since developed that the power which will be generated at the power plants between the storage dam and the Hudson River will be transmitted into the systems of large generating and transmitting companies. These companies either at their own plants on this and other streams, or at those plants from which they purchase energy, have available during the spring high-water months a surplus of electric energy, so that shutting down the plants situated immediately below the storage dam, in order to minimize conditions of high water on the Hudson River, does not seriously impair their usefulness. This condition makes it feasible to operate the reservoir to produce the best flow possible at the "center of gravity of power" on the Hudson River and at the same time makes it unnecessary to release water from the reservoir when the Hudson River is at high flow. This regulation will increase the minimum flow of the Hudson River at the so-called "center of gravity of power" from the present minimum of 900 sec-ft. to 3 000 sec-ft., increasing the firm or continuous power from 34 000 h. p. to 160 000 h. p., on a load-factor basis of 100 per cent. At the same time, this regulation, which in effect will subtract the flood flow from 1 044 sq. miles, will eliminate most of the damaging floods on the Hudson River immediately below its confluence with the Sacandaga River, and largely reduce those not eliminated. The Hudson River Regulating District is a public corporation created as a result of an Act of the Legislature which is

quite similar to the Miami Conservancy Act. The General Plan of the District calls for the eventual construction of sixteen storage reservoirs having a total capacity of 1 800 000 acre-ft. It is believed that these reservoirs, although they are to be constructed primarily for the regulation of the flow of the Sacandaga and Hudson Rivers and their tributaries for power purposes, will largely prevent floods on the Hudson River above its confluence with the Mohawk River.

Too little consideration is given in the design of spillways of storage reservoirs to the storage above the spillway crest. Not infrequently plans provide for as great spillway length and capacity as if the proposed dam were merely a power dam without a large flowage area. Engineers are prone to take from one of the well-known curves of flood flows of typical streams a maximum flow for the project in question and design the spillway accordingly. If the project is that of a storage reservoir with a large area, the effect of storage above the spillway crest would largely reduce the necessary spillway capacity.

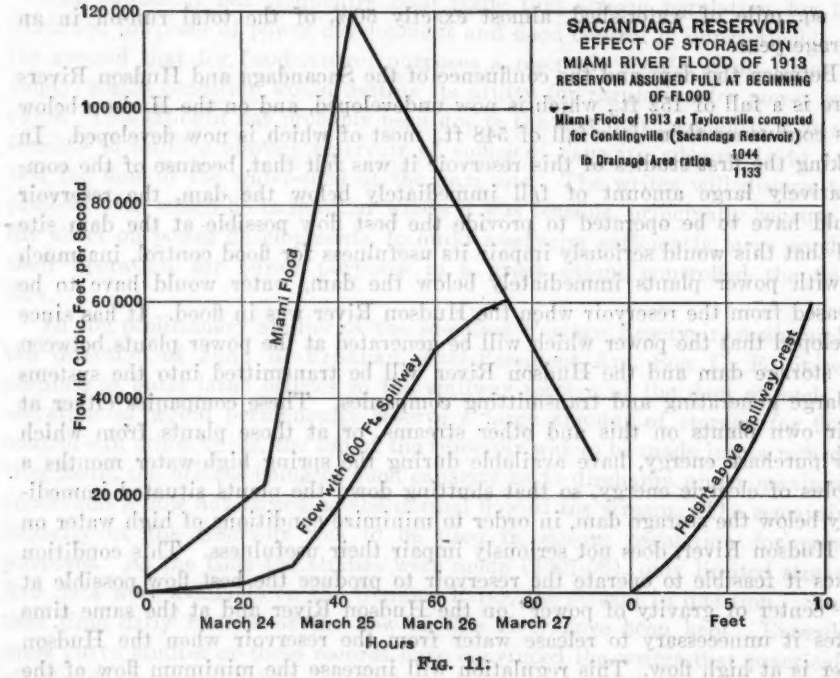

FIG. 11.

For example, in the design of the Sacandaga Reservoir, a flood flow on a square-mile basis equal to the Miami flood of 1913 was used (Fig. 11), assuming that such a flood occurred when the reservoir was full. The storage above the spillway crest would have reduced this flood from 120 000 to 60 000 cu. ft. per sec. Inasmuch as the topographic and geologic conditions at the dam site were such as to require a side-hill spillway the reduction in necessary spillway capacity materially reduced the cost of the dam and spillway channel.

Although it is obvious, yet it may be pertinent to note parenthetically that the storage above the spillway crest of the Sacandaga Reservoir would have a great mitigating effect on the floods of the Hudson River even if the reservoir were full at the inception of the floods. For example, the flood of March, 1913, would have filled the reservoir by April 1, on which date the natural

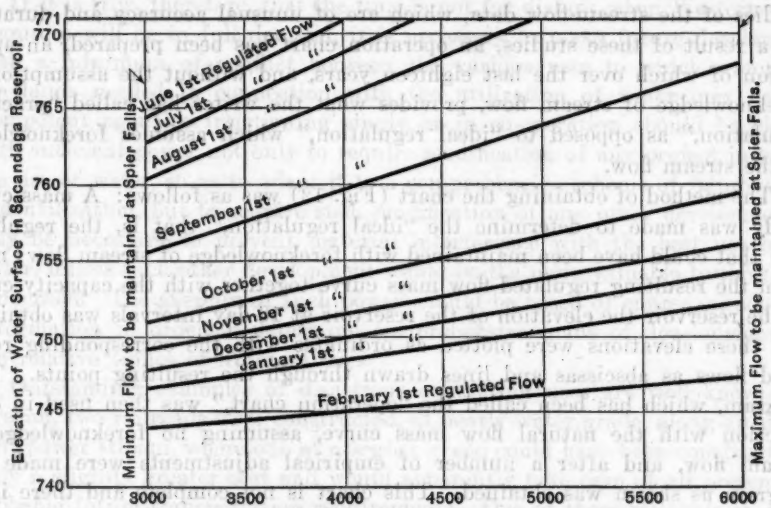

FIG. 12.—OPERATION CHART, SACANDAGA RESERVOIR.

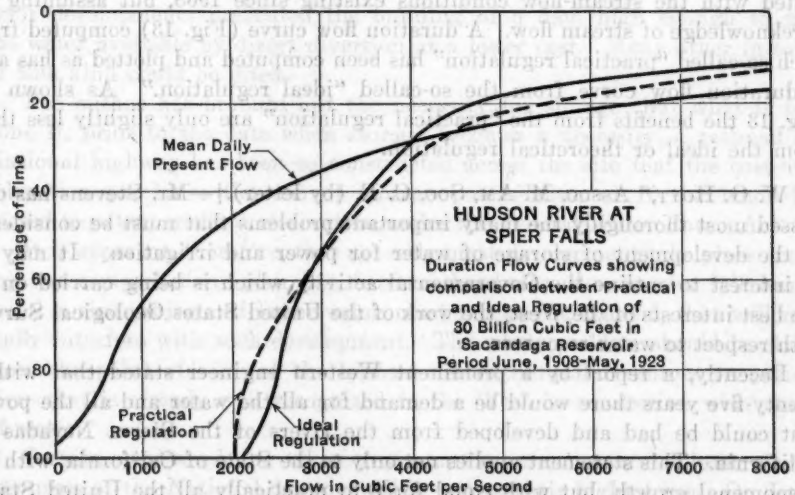

FIG. 13.

flow of the Hudson River was 33 800 sec-ft., of which 11 300 sec-ft. came from the Sacandaga River. The storage above the spillway crest would have reduced this 11 300 sec-ft. to less than 1 000 sec-ft., so that the flood flow in the Hudson River of 33 800 sec-ft. would have been reduced to 23 500 sec-ft.

The writer wishes to take issue with another generally accepted theory that storage reservoir operation which assumes variable rates of discharge for each dry season, to the end that most of the available storage can be used each season, is purely theoretical and cannot be carried out in practice. In connection with his studies of the Sacandaga Reservoir he has made extensive studies of the stream-flow data, which are of unusual accuracy and duration. As a result of these studies, an operation chart has been prepared, an application of which over the last eighteen years, and without the assumption of foreknowledge of stream flow, provides what the writer has called "practical regulation," as opposed to "ideal regulation," which assumes foreknowledge of the stream flow.

The method of obtaining the chart (Fig. 12) was as follows: A mass curve study was made to determine the "ideal regulation," that is, the regulated flow that could have been maintained with foreknowledge of stream flow; next, from the resulting regulated flow mass curve together with the capacity curve of the reservoir the elevation of the reservoir at 10-day intervals was obtained, and these elevations were plotted as ordinates with the corresponding regulated flows as abscissas and lines drawn through the resulting points. This diagram, which has been called the "operation chart," was then used in conjunction with the natural flow mass curve, assuming no foreknowledge of stream flow, and after a number of empirical adjustments were made the diagram as shown was obtained. This chart is not complete and there is in course of preparation a more elaborate chart for the use of the engineer who will operate the reservoir, but the chart in its present form has been tested with the stream-flow conditions existing since 1908, but assuming no foreknowledge of stream flow. A duration flow curve (Fig. 13) computed from such so-called "practical regulation" has been computed and plotted as has also a duration flow curve from the so-called "ideal regulation." As shown on Fig. 13 the benefits from the "practical regulation" are only slightly less than from the ideal or theoretical regulation.

W. G. HOYT,* ASSOC. M. AM. SOC. C. E. (by letter).†—Mr. Stevens has discussed most thoroughly the many important problems that must be considered in the development of storage of water for power and irrigation. It may be of interest to outline the Governmental activity which is being carried on in the best interests of the West, the work of the United States Geological Survey with respect to water resources.

Recently, a report by a prominent Western engineer stated that within twenty-five years there would be a demand for all the water and all the power that could be had and developed from the rivers of the Sierra Nevadas in California. This statement applies not only to the State of California, with its phenomenal growth, but with equal force to practically all the United States west of the Mississippi, with perhaps the exception of a narrow strip along the coast in Northern California, Oregon, and Washington, and other great concentrations of water resources such as are found along the Columbia and

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† Received by the Secretary, July 9, 1925.

Colorado Rivers and some of their principal tributaries. The time limit for this demand may vary to some extent in different parts of this great region, but inasmuch as many areas have already been retarded in development because of an inadequate supply of water, it is hazardous to set the time limit too far ahead.

If it is true, therefore, that the time is not far distant when Western water resources will be in full demand, it is necessary to make future development with a minimum of conflict between the various uses to which water and the lands needed in connection with the utilization of water may be put. Competent agencies functioning singly or in co-operation should be clothed with sufficient power not only to require modification of any project involving the use of water, so as to adapt it to a comprehensive scheme of improvement and utilization, but to require such modification of any other development as may be necessary to prevent material interference with the best use of the water, unless such other development would be the more valuable to the region concerned. The exercise of such power should be based of course on adequate information in order to determine comprehensive plans of development and the relative values of different uses.

Outstanding examples of disastrous conflicting uses are well known. In Oregon, two railroads were constructed, at nearly water grade, along an important power stream, where one at a higher grade could have been constructed at only a slightly greater cost and would apparently take care of all present and probable future transportation requirements. One of these railroads is, therefore, an added burden on power development. In Idaho, there is an irrigation district which must always meet a heavy assessment for pumping water, because prior developments prevented the building of a dam high enough to make the water available by direct diversion at a lower cost. Many other instances of this kind could be cited.

The author has brought out the necessity for storage; but what is to be done if, prior to the date when storage becomes a necessity, a railroad or a National highway has been so constructed across the site that the cost of its relocation precludes the use of storage possibilities, or if the lands have passed to patent without reservation under the Federal Water Power Act.

It is apparent, therefore, that not only must development take place according to a comprehensive plan, but, further, that before the time for full development is ripe projects of lesser value must not be constructed, that will materially interfere with such development. The whole problem should be studied from the point of view of the highest use of natural resources—a point of view that is already well recognized in the West in connection with the use of water.

Certain steps are being taken to guarantee, in a measure at least, that the resources of the National domain will be put to their highest use. On July 1, 1924, a total of 6 626 474 acres were embraced in withdrawals and classifications relating to water resources. A considerable part of this land is withdrawn from entry because it is covered by applications for permit or license under the Federal Water Power Act. The remainder has been investigated by the U. S. Geological Survey and found to have value in connection with

water utilization. It is possible that some engineers in their task of building the West, have been delayed and perhaps been put to a considerable expense and annoyance because of these withdrawals. The principle involved, however, is fundamentally right and the benefit that will eventually accrue to the western country because of these withdrawals will far outweigh any past inconvenience.

In making and administering these withdrawals the results of about 5 000 miles of river surveys have been studied and the records of nearly 1 000 gauging stations have been analyzed. Field studies have been made over a period of 17 years, since the first withdrawal of land for power, which covered 17 080 acres in Utah.

The value of land for the development of water power can not be estimated properly on the basis of the present limits to development, but must be determined by considering the probable limitations of a future day, when by reason of increased demand and increased value of such power a project that is now impracticable because of high cost will become thoroughly feasible. In classifying and administering public land with respect to the possibility of developing water power, it has been necessary to determine whether the land might not be more beneficially devoted to the use of water for domestic supply or irrigation than for power and whether its use for power would be prejudicial to the other uses at the particular site under consideration or at some other site more or less remote.

It has also been necessary to estimate in many cases whether the land is of more value for agriculture, or for a town site, or a manufacturing site than for a power site. Such considerations apply especially to lands that may be suitable for storage reservoirs. Where civic, industrial, or rural improvements have already been made on a reservoir site they largely control the decision on this point; but the sites which lack such improvements are still numerous. Therefore, the field of investigation has extended beyond the facts of the present day, and a forecast of prospective events must be made.

Those who have had to deal with these problems have not forgotten that human activities embrace many things besides the utilization of water—that a community which produces anything must, if its products are of value, have suitable means of transporting them to market. Consideration, therefore, has been given to the relative usefulness of a piece of land as a power site or as a right of way for a railroad. The bottom of a canyon that is suitable for a reservoir may also be an advantageous railroad or highway route. Although an alternate transportation route can usually be located, the cost of utilizing it may be so much greater as to outweigh the advantages that may accrue from the development of power along the stream. In determining such questions it is necessary to appraise the relative value and comparative cost of several possible methods of using the land and to choose the course that is, or is likely to become, of the highest human benefit.

The classification of land for water-power sites, therefore, requires a perspective so broad, and a comprehension of so many variable conditions and probable future changes in public demand that mistakes are easily made. With respect to the public domain, therefore, an attempt is being made to guide

future development according to what are believed to be far-seeing comprehensive plans. However, because of constitutional restrictions many States are still disposing of their remaining water resources on the policy, embodied in the proverb, "the early bird catches the worm." This was undoubtedly the best policy during the first stages of development, but viewed in the light of present-day conditions it seems erroneous and should be corrected.

In this discussion there is no intention to imply that present policies have acted to dissipate materially National water resources, but rather to draw attention to the fact that the problems of the future utilization of these resources will demand not only a careful and complete study of the drainage basins, but also the development of a sound economic policy.

F. F. HENSHAW,* M. AM. SOC. C. E. (by letter).†—The author has discussed the power and irrigation demands and their relation to regulation by storage for the normal streams of the arid and semi-arid West. He has given as examples six fairly typical streams having a concentrated fall in their headwater portions that is utilized for power development and having irrigable valley land at a low level, with storage capacity above the power plants and, on some of the streams, below the tail-race of the power plant and above the points of diversion for irrigation. A typical stream already developed in this manner is the Tuolumne River, rising in the Sierra Nevada and debouching into the San Joaquin Valley, California. This relative arrangement is normally found in a region of moderately mature topography.

A radically different condition prevails in the Pacific Northwest, where in Tertiary times the great Columbia River lava flow inundated like a flood a large part of Washington, Oregon, Idaho, and Northern California. The Cascade Range dates from an even more recent epoch, and adjustments of drainage have occurred during what may be considered a geologic yesterday. Another series of lavas, the andesites that form the summit of the Cascade Range, is one of the newest and most pervious of all geologic formations of any extent within the United States. As a result of this recent extensive volcanic activity there are in parts of the plateau area west of the Rocky Mountains a topography of extreme youth in which the surface is comparatively little distorted. The major streams have cut steep, narrow canyons in the borders of the plateau, but their head-waters flow near its surface. The later volcanic formations give rise to immense springs—some of the greatest in the United States if not in the world.

The typical stream of the lava area is the Deschutes River in Central Oregon, as mentioned by the author, but the Snake River in Southern Idaho, the Klamath River in Southern Oregon and Northern California, and the Rogue River in Southwestern Oregon, have many characteristics in common with the Deschutes. In the basins of all these streams most of the irrigable lands lie at a relatively high level, on the surface of the plateau. Water must be diverted in the upper portions of the streams before it has reached the deeper canyons. The power sites are below rather than above the sites for

* Dist. Engr., U. S. Geological Survey, Portland, Ore.
† Received by the Secretary, August 5, 1925.

irrigation diversion. The water available for developing power on most of the streams is augmented by inflow from springs at points below all possible sites of diversion for irrigation.

The natural flow of the streams in their head-water sections is, in general, that shown by Fig. 5.* With no storage regulation, or with partial storage, the flow below the point of irrigation diversion is little affected by the diversion from October to April, but is reduced materially during the summer. When partial storage is provided in the head-water section, the summer flow between the reservoir and the point of diversion of course is augmented, but the only effect on the power is to reduce it uniformly during the non-irrigating season.

The Snake River may be taken as an example. This great stream is fed by immense springs as well as by tributaries that have the usual characteristics of streams rising in high mountains. Its low-water flow from Lewiston, Idaho, to the mouth probably ranged from 20 000 to 30 000 sec-ft. before extensive irrigation was begun in Southern Idaho. The river discharges a fairly steady volume from about August 1 until sometime in the winter. Irrigation development has progressed until below Milner Dam, the lowest point of diversion from the main river to the plateau lands, the river is practically dry each summer. Not far from Milner the river plunges over Twin and Shoshone Falls and enters the series of canyons through which it flows to its junction with the Columbia at Pasco, Wash. Springs discharge large volumes of water into the river in this canyon section, and as a result the flow at the mouth has never been less than 10 000 sec-ft., even in recent years. This quantity is, therefore, available for continuous power generation.

Storage development has been started; Jackson Lake and Arrowrock Reservoirs augment materially the water supply for irrigation in the Upper Snake and Boise Valleys, respectively, but do not appreciably reduce the flow through the lower canyons of the Snake River. The American Falls Reservoir, now under construction, together with other proposed storage basins, will noticeably diminish the winter and flood discharges below the irrigated areas. The final result will be to produce during late summer and early autumn a fairly uniform flow of large proportions, say, 10 000 to 12 000 sec-ft, at the mouth and to leave available several hundred thousand horse-power, notably in the deep canyons between Huntington, Ore., and Lewiston. The lowest stages will continue no doubt to occur near the end of the irrigation season. The shortage of power during this time and the peak load produced by the requirements of irrigation pumping can be met in a measure by power generated from water released from storage reservoirs, and diverted at lower levels from irrigation. The utilization of such double sources of power may be deferred many years; it will probably await the construction of a super-power network of considerable capacity, but it presents important possibilities.

The Klamath River is almost unique in the lack of relief in the upper part of its basin and the contrasting extreme ruggedness of the lower part. The run-off characteristics harmonize with the topography, being uniform above Klamath Falls and flashy for the section below the gap in the Cascade

* *Proceedings*, Am. Soc. C. E., April, 1926, Papers and Discussions, p. 619.

Range. The land available for irrigation in this particular basin is considerably less than the available water will supply, and the irrigation demand will ultimately consume somewhat less than one-half the average annual run-off. Upper Klamath Lake, with an area at low stages of about 60 000 acres, presents a storage basin of large capacity. It has been developed to provide a controlled range of stage of 7.3 ft., storing somewhat more than 500 000 acre-ft. This range is limited by the cost of protecting by levees the valley lands bordering the lake. Ordinarily, it will be possible to maintain a fairly steady flow of 800 to 1 000 sec-ft. for power development, in addition to supplying all reasonable irrigation requirements. Severe shortages may occasionally occur during dry summers, but these can be provided for by developing the power available from water released from storage in the headwater portion of Deschutes River and elsewhere. Here, again, the utilization of such a double source of power must await the establishment of a super-power system of considerable capacity.

BARRY DIBBLE,* M. AM. SOC. C. E.—The importance of taking care of available water supplies has been mentioned by both Mr. Stevens and Mr. Hoyt, but the time comes when they are absolutely "extinguished", that is, so far as future development is concerned. In a great many places, particularly in California, it is not realized how far that "extinguishment" has already proceeded. Power development in California has had so much publicity that in comparing the statistics of that State with those of the United States as a whole, it has been rather surprising to learn that, whereas over a 10-year period the compounded annual increase in power consumption in California is 10.5%, in the United States as a whole it is 11.3 per cent. During the 10-year period California had a large increase in population so that the per capita use increased at the compound rate of only 6.6% per year as against 11.2% in the whole United States. The power consumption of Southern California dropped even lower with a per capita increase of 4.3 per cent. This decline in the growth of the California power business has undoubtedly been due to difficulty in keeping up with the demand by constructing hydro-electric developments of reasonable cost.

Throughout the world the consumption of electricity is held back because development follows the demand rather than precedes it. Power companies generally have not had to search for customers. They have operated in a "seller's market". There is no good basis for determining to what extent electricity would be used in a community if its sale were energetically and consistently pushed in all lines of application. There is no indication that the saturation point is being approached.

In the territory of the Minidoka Irrigation Project, in Idaho (120 000 acres and 17 000 population), the per capita consumption of power amounts to 2 800 kw-hr. annually, or four times the average in the United States. It has now stopped increasing because temporarily it has reached the limit of the power supply.

Compilations of estimates show that power company managements anticipate that the electrical business will continue to compound at the rate of 11%, which means that it doubles in less than 7 years.

* Cons. Engr., Redlands, Calif.

Mr. Woolley's remarks show that there is often not a great deal of difference between statements that may seem to be contradictory. No one will accuse the speaker of lacking interest in electric power and its application. The fault with Mr. Woolley's statement is that he has left the time factor out of consideration. Mr. Hoyt carefully included that factor. Already there are a number of instances—one at American Falls in Idaho and numerous others—where the economics of the situation have demanded the extinguishment of power rights in favor of irrigation. Apparently, the present plan of classification is to look to the future and prevent installations which may be economic at the moment but which, in the reasonably near future, may have to be extinguished to permit of better use of the natural possibilities. The present demands for power are making such developments feasible in connection with irrigation projects where a few years ago they were not economically possible. Power is becoming of considerable value in meeting summer peaks and in permitting the cutting down of expenditures for fuel. After all, there is no great difference of opinion between engineers on these questions.

RALF R. WOOLLEY, ESQ.*—Those who are interested in reclamation and agriculture do not hesitate to remark that agriculture is the basic industry; those interested in mining feel that mining is the basic industry; and the stock growers are sure that their's is a basic industry. In each case statistics are not wanting to support the claims of these various advocates.

There is no question but that agriculture, including irrigation, was the foundation on which practically all the permanent communities of the arid West were built. Some of these communities are still solely dependent on this factor for their existence, while others boast of additional forms of industry such as mining, manufacturing, etc. Communities supported by irrigated lands are no doubt more stable than those subject to the ups and downs of prosperity not uncommon to other industries, and, for this basic reason, public opinion in the West has defined certain preferential rights to the use of streams, placing irrigation superior to all other uses except domestic.

To those who have given some study to the question of the greatest economic utilization of Western streams, this fact raises a serious question; namely, whether or not it is advisable always to sacrifice the use of a stream for power in favor of irrigation regardless of the relative economic values of the two uses. With this question in mind this discussion is presented in the hope that thought may be stimulated among engineers interested and that other opinions may be expressed. It is only a general statement of a few of the facts which must be considered in arriving at any judicial conclusion.

Agriculture in the arid West is based on irrigation. Generally speaking its future growth is commonly expressed in terms of the total run-off of the streams and undeveloped acres without regard to the economic factor in the problem. The recent investigations of the United States Reclamation projects disclosed the fact that "the reclamation problem is to-day essentially an agricultural and economic problem". Dr. John A. Widtsoe, one of the members of this Fact-Finding Commission, makes the following statement:†

* Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Salt Lake City, Utah.
† *Proceedings*, Am. Soc. C. E, March. 1926, Papers and Discussions, p. 398.

"Irrigation is a complex art, involving the application of the best knowledge of agriculture, especially the relationships among soils, crops, and water; of engineering, especially of hydraulics; of rural economics and sociology; and of the life and response of human beings under the unique conditions always prevailing under the irrigation ditch.

"It is not surprising that so complex an art, at this time, when thoroughgoing changes are reshaping economic and social structures, should present many and serious problems. Such problems pertaining to the development of irrigation in the West do exist, and must be solved, if the great American experiment in irrigation shall become permanently successful. Yet these problems, essential to irrigation development, may be classified very simply into two groups:

"1. To establish on every irrigation project an acceptable economic and social environment.

"2. To extend gradually, as economic needs demand, the irrigated area by completing existing projects and by reclaiming new areas, until the water resources of the West shall be fully utilized."

It seems apparent from this statement that irrigation will become permanently successful only when it is practiced on a sound economic basis, and it should be extended "gradually, as economic needs demand". This is the one point that seems to be entirely overlooked by the irrigationist in his enthusiasm to develop irrigation, the result being that other uses of streams, such as for power, are made subject for all time, to any and all future demands for irrigation developments, regardless of their economic feasibility or the relative merits of the two uses.

Each stream is an individual problem, and should be considered as such in working out its complete utilization. Those communities that are solely dependent on irrigation, will only grow to the capacity of their available irrigable acres, and available water supply; and in many of the communities along the small streams throughout the arid West the limit of such growth already has been reached under present irrigation practice. If any further growth of these towns is to take place, it must either be the result of other industries coming in as, for example, manufacturing and mining, or the limited possibility of increasing the returns from irrigation by improving methods of farming.

Is it not possible, therefore, to conceive the following conditions in the growth of many Western communities that have begun with a small nucleus of irrigated land. As the community grew in population it extended its irrigated area, and continued for many years to supply all the demands of its inhabitants for a livelihood; finally, the limit of economical irrigation growth was reached because the railroad or other transportation agencies brought in supplies and foodstuffs more cheaply than the citizens could produce them. The community was fortunate enough to have a good geographical location for manufacturing industries, raw materials were close at hand, and vast mineral resources were discovered; the community began to grow industrially, and far beyond anything that could possibly have been expected from its irrigation resources. Hydro-electric power was easily developed, and, accordingly, industries of all kinds were encouraged. Is it not conceivable now that the curve of growth of this community based on irrigation is at some point crossed by the curve of

growth based on industry, and that henceforth, from this point, the community is not primarily an irrigation or agricultural community, but an industrial one? In all this development power is the one important factor; accordingly is it not the water power resources that will play the largest part in future industry and agriculture in the arid West?

How does power development effect agriculture? Without going into the details of the different combinations of co-ordination of irrigation and power uses of streams and the possible conflicts in use, it is interesting to note the following statements which no doubt are based on considerable study of the problem.

The Committee of Special Advisers on Reclamation to the Secretary of the Interior concluded regarding power that:*

"The place of power in modern agriculture has been well established. The electric current has become the servant of the farmer, by the aid of which one man is enabled to do several times the work that the farmer of old could do. The need for power on an irrigated farm is as legitimate as the need for water. Any future policy relative to the development of Federal reclamation should keep the power problem in mind. The water users, when they take over a project, should be allowed to operate the whole project as a unit, including the power plants that may be constructed; and, further, no reservations should be made regarding possible power sites, except as to the permament retention of existing power possibilities for the use of the project population."

The California Railroad Commission has stated† that:

"Any halt in the State's hydro-electric development will spell disaster. It is the Commission's firm belief that financially sound utilities, functioning economically and efficiently, will successfully carry the burden of California's development. It is obvious, that industrial and agricultural California cannot reach the peak of development unless there stride, a step in advance, utilities carrying service possibilities 'ready on demand' ".

Similar views are continually being expressed in the technical press. Some striking data have been published,‡ showing the relation between kilowatts generated and community development: "Each kilowatt of power developed makes possible the building of one home, the irrigation of 4 acres, employment of one person in an industry and means $6 000 to the community."

In conclusion, it is the speaker's opinion that water power really has an important place in the utilization of Western streams. It may be a by-product of irrigation, co-ordinate with it or even superior to it. Each project is an individual problem and should be solved according to the economic factors involved. It is a mistake to impose such restrictions on power projects as will postpone indefinitely or possibly preclude their development without a proper analysis of the whole problem. It would indeed be a short-sighted policy on the part of the person seeking to develop power if he did not foster all possible development of the community through irrigation; likewise, it is a short-sighted policy on the part of the person seeking to develop irrigation to put every obstacle possible in the way of developing power.

* Federal Reclamation by Irrigation (Presidential Message), Senate Doc. No. 92, 68th Cong. 1st Session, p. 135.

† *Electrical World*, October 30, 1920, p. 893.

‡ *Journal of Electricity*, January 1, 1921.

ARTHUR P. DAVIS,* PAST-PRESIDENT, AM. SOC. C. E.—Mr. Sargent has mentioned a plan proposed by Mr. Freeman, in which an automatic outlet or spillway was provided below the top of the reservoir in order that a certain amount of that storage would be reserved for flood control at all times. Every project must be stated in accordance with its own characteristics. A spillway that will discharge an amount that is not objectionable as a flood below the dam, and yet will discharge the water so fast that the storage back of it can be utilized for regulation of the next flood, is, in principle, exactly what Mr. Freeman proposed. Evidently, the capacity of his spillway in both cases differs with the head, so that he is not proposing a new principle in either case, but simply to dispose of the flood at a height at which it can be safely discharged. That differs greatly with the characteristics of each different problem. It differs greatly with the manner of flood discharge of the stream in its natural state. It differs also with the requirements of flood control in the stream below.

* Chf. Engr. and Gen. Mgr., East Bay Municipal Utility Dist., Oakland, Calif.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

JOHN LYELL HARPER, M. Am. Soc. C. E.*

DIED NOVEMBER 28, 1924.

The importance of hydro-electric power development was just beginning to be realized when Mr. Harper came to Niagara Falls, N. Y., in 1902. With his broad vision and ability and situated as he was at this mightiest source of power, he was destined to take the lead in an industrial achievement which remains unsurpassed anywhere in the world. At his death he was recognized as one of the greatest engineering minds of the age and the famous development in the Gorge below the Falls of Niagara will always stand as a monument to the genius of John Lyell Harper.

John Lyell Harper, the younger of the two sons of Joseph F. and Quintilla K. Harper, was born on a farm in a hilly section of New York State called Harpersfield, on September 21, 1873. Like most farmers' sons, he had to work long, hard hours in the fields as well as climb the mountain every day to attend the country school in the next valley; but the rigors of country life served only to develop his keen mind and foster in him a passionate love of Nature which he retained throughout his life.

He loved books and applied himself diligently to the pursuit of knowledge, very early showing more than ordinary interest in experimenting with running water. There were plenty of mountain streams around his home and it was his special study, with a degree of success beyond his years, to attempt to control the powers of Nature, damming up one stream and diverting the course of another to turn mills and water-wheels of his own invention. Thus, his boyhood was spent and at the age of twenty he had finished the course of study at Stamford Seminary; whence he followed his own particular star to Cornell University, from which he was graduated with the degree of Mechanical Engineer.

From Cornell, Mr. Harper went to Seattle, Wash., as a Draftsman with the Oregon Improvement Company, making designs for the repair and construction of railway equipment. After four months, however, he resigned this position to enter the employ of the Union Electric Company where the field of practical experience included the repair and testing of motors, transformers, and general electrical apparatus.

In June, 1898, he secured the position of Operating and Construction Engineer of the Twin City Rapid Transit Company, of Minneapolis, Minn., and remained with that Company a year. In this position his training was broader and more along the lines in which he was most interested. Among other things he had a great deal of experience in designing and constructing sub-

* Memoir prepared by N. R. Gibson, and O. D. Dales, Members, Am. Soc. C. E., and G. R. Shepard, Esq.

stations, installing underground cables, testing water-wheels, and similar work in connection with the operation of the 10 000-h.p. water-power plant of the Company.

At the end of the year Mr. Harper transferred his interests to the firm of I. W. Gray and Company and was responsible for the design of an electric railway and steam power plant for Mankato, Minn.

Steadily, he was increasing his experience and his value, and, in 1899, he took his first important position, as Field Representative for Messrs. Floy and Carpenter, Consulting Engineers, of New York, N. Y., on their project to build a hydro-electric power plant for the St. Croix Power Company of Wisconsin at Appleriver, Wis. He was in entire charge of the construction of this development which was completed in three years.

It was the turning point in his career when in 1901 he came East, to Buffalo, N. Y., to represent certain manufacturing and engineering interests at the Pan-American Exposition, for there he got into touch with the Niagara Falls Hydraulic Power and Manufacturing Company and in 1902 accepted a position with that Company as Assistant to the Engineer, the late Wallace C. Johnson, M. Am. Soc. C. E. It was there that his principal work began, and from that time until his death in 1924, the story of his life is intimately associated with the history of the development of power from the waters of the Niagara River.

This Company in 1881 had begun the construction of a power-house called Station No. 1 for supplying electric power for commercial purposes. In February, 1902, when Mr. Harper came to Niagara Falls, this plant had a capacity of 2 000 h.p., and a new power-house called Station No. 2 had been partly constructed, having at that time a capacity of about 20 000 h.p. In two years he became Chief Engineer and prepared plans for the completion of Station No. 2 and, in 1904, when this was accomplished, it had an installed capacity of 33 000 h.p. Station No. 1 was then abandoned and the construction of Station No. 3 begun, which was ultimately to have a capacity of 130 000 h.p. From that time rapid improvement was made and further plans for the future were under way when, in 1909, the Governments of Canada and the United States set a limit to the quantity of water that could legally be diverted for power purposes. Plans already made were carried out, but incentive to greater achievement was thus temporarily removed.

During the World War emergency, the Niagara Falls Power Company, Hydraulic Power Company, and Cliff Electrical Distributing Company, were merged under the name of The Niagara Falls Power Company, owning all developments on the American side and controlling the Canadian Niagara Power Company on the Canadian side. The combined plants had a total installed capacity of 350 000 h.p., and it was a fitting tribute to Mr. Harper's vision and ability that the developments of the consolidated companies were placed in his hands as Chief Engineer, and that, in 1919, the responsibility of Vice-President was added.

The shortage of power was serious in this period and when the War Department requested plans for the most economical development for utilizing the remaining water available for diversion, it was the Harper plan that was

accepted. This provided not only for immediately placing all developed power at the disposal of the Government agents for distribution to specified customers who were making the materials vital for war work, but also for the rush development of another 100 000 h.p. without sacrifice of permanency of work or efficiency of conversion.

These construction plans were being rushed to completion when the signing of the Armistice made speed less important; however, the whole 100 000 h.p. was put in operation within twenty-two months from the time it was authorized. Shortly afterward, the Government of the United States established the Federal Power Commission and its approval was given to the engineering plans of The Niagara Falls Power Company. A license for fifty years was issued for all the water permitted to be diverted from the American side of the Falls in accordance with the terms of the existing treaty, with the exception of 500 cu. ft. per sec., which was reserved for diversion through the New York State Barge Canal.

As no further water was available, the only source of additional power was the use of some of the water under increased head. Plans accordingly went forward for the construction of Station No. 3-C which was begun in April, 1921, and completed in two years. The water for this Station was brought from the upper river under the city in a tunnel 32 ft. in diameter. Each of the three units in this plant has a capacity of 70 000 h.p.—at present, the largest installation in the world.

In connection with the Niagara development, Mr. Harper was especially interested in the question of the preservation of the scenic beauty of the Cataract in its relation to diversion of water for industrial purposes. For the scientific engineering study of this problem he designed and had built a model of the Niagara Falls and Rapids on a scale large enough to determine with a reasonable degree of accuracy the effect of diversion on the Falls and Rapids and to actually demonstrate the feasibility of the various plans for remedial works which are necessary to prevent the present rapid and irregular recession of the Horseshoe Falls. He also wrote several papers on this subject and thus brought his opinions and the results of his research to the attention of the public.

Mr. Harper found time to interest himself in chemical and metallurgical research, particularly along the lines of the application in these industries of electric service. He was successful in inventing and patenting the Harper electric furnace, for the commercialization of which he organized, shortly before his death, the Harper Electric Furnace Corporation.

He also initiated, in 1924, the organization of the firm of Harper and Taylor, Incorporated, to carry on a consulting engineering practice in conjunction with his Engineering Staff at the Power Company.

In 1898, he was married to Linda E. Wheeler, of Ithaca, N. Y., who, with two sons and three daughters, survives him.

Mr. Harper was a man of high ideals and absolute integrity, a loyal friend, and devoted to his family. A lover of Nature and of the out-of-doors, his chief hobby was the cultivation of flowers and fruits and his gardens were always a delight.

Cut off in the prime of life, in the midst of plans and visions for the future, it was some consolation to those who knew him that he lived to see the completion of his two latest projects. Throughout his work he maintained the highest engineering and moral standards, and his premature death was a loss to the profession.

Mr. Harper was elected a Member of the American Society of Civil Engineers on June 5, 1907.

CURTIS HILL, M. Am. Soc. C. E.*

DIED OCTOBER 18, 1925.

Curtis Hill, the son of William Moberly and Ann (Gossett) Hill, was born near Independence, Mo., on September 4, 1870, on a farm acquired by his grandfather, Adam Hill, who came from Kentucky to Missouri on horseback in 1830. The Hill family is of sturdy pioneer stock and is one of the oldest in the United States, the first of that name having emigrated from England to America in early Colonial times.

Curtis Hill's early education was obtained in the public schools of Jackson County, Missouri, and at Woodland College, at Independence. He was graduated from the University of Missouri in 1896 with the degree of Bachelor of Science in Civil Engineering, and a year later received the degree of Civil Engineer from Cornell University. While in the University of Missouri he was prominent in athletics and played three years on the football team, serving one year as Captain.

Mr. Hill's professional work was connected chiefly with railroads, waterways, highway bridges, highways, and municipal construction. He was engaged as Instrumentman on railroad construction in Mexico in 1892, prior to his graduation from the University of Missouri, and after leaving Cornell University, in 1897, he was employed by the Illinois Central Railroad Company for a few months on track construction. He again engaged in railroad work in 1899, this time with the Chicago and Northwestern Railway Company.

From September, 1897, to August, 1899, Mr. Hill was with the United States Deep Waterways Commission, engaged in making hydrographic and topographic surveys and estimates for a proposed canal outlet for the Great Lakes. During the next two years he was employed by the Central Bridge Company, of St. Louis, Mo., as Designer, Contracting Agent, and Erection Superintendent.

Beginning in October, 1901, Mr. Hill spent almost six years with the Sewer Department of St. Louis, eighteen months as Assistant Engineer and the remainder of the time as Engineer of Sewers. In the latter position he had charge of all field surveys and construction work pertaining to sewers, the cost of the work done under his direction amounting to about $5 000 000.

In July, 1907, Mr. Hill was appointed State Highway Engineer of Missouri by Governor Folk, being the first to hold that office. Two years later,

* Memoir prepared by Victor H. Cochrane, M. Am. Soc. C. E.

on account of the highly satisfactory character of his work, he was re-appointed by Governor Hadley and remained in office until April, 1913. During his encumbency he did considerable pioneer work in connection with highway construction, laying the foundation for the extensive development later authorized by vote of the people. His valuable services in this office were commemorated by Mr. J. M. Lowe in the dedication of his book, "National Old Trails Road", as follows:

"To Curtis Hill, the first State Highway Engineer of Missouri, whose spirit rose to a full comprehension of the duties and obligations of his high office, and whose intelligence and lofty vision enabled him to grasp with full appreciation the mighty pioneer problems which he alone had to solve, this book is respectfully and affectionately dedicated."

In April, 1913, Mr. Hill became City Engineer of Kansas City, Mo., and during the next five years he had charge of a great variety of municipal construction aggregating about $10 000 000 in cost, and also made plans for proposed sewers, flood protection works, trafficways, and other improvements. For one year during this period he acted as Advisory Engineer to the Board of Park Commissioners of Kansas City.

On June 7, 1918, occurred the tragic event that caused Mr. Hill to be incapacitated for the remainder of his life. He was shot in the head by a disgruntled contractor, a man of violent temper and unsound mentality. Some time previously, this contractor had been awarded the contract for the construction of a small bridge, which he had failed to complete satisfactorily. Later, he submitted a bid for a much larger bridge, and because of the previous trouble and the fact that the bid seemed to be much too low, Mr. Hill recommended that the contract be not awarded to him; however, the contractor persuaded the Board of Public Works that he could build the bridge properly and expeditiously, and he was given the work. He was wholly without experience in bridge construction and had no adequate equipment. After he had worked several months without making any progress, the Board annulled his contract and gave the work to a responsible and experienced firm. The contractor was present at the meeting of the Board when this action was taken, and he immediately shot Mr. Hill.

At first it was thought that Mr. Hill's injury would prove fatal, but after undergoing an operation for the removal of a piece of the bullet from the brain, he seemed considerably improved; however, the wound had caused a partial amnesia, from which he never fully recovered. He was retained by the City as Assistant City Engineer until April, 1925, and during 1923 and 1924 he was able to work occasionally, although for only a few days at a time. In April, 1925, he was pensioned by the City. At this time his health was in such a critical state that death seemed imminent, but he lived until October 18, 1925.

During the entire period of his illness Mr. Hill was uniformly cheerful, and accepted his fate without bitterness and with a degree of fortitude that was the marvel of his friends.

On account of his pleasing personality, his kindliness, his helpfulness, and his fair-mindedness, Mr. Hill was well liked by his associates and respected by every one. He was a man of good intellectual attainments and excellent judg-

ment, and possessed the ability to analyze a given situation or problem and to determine the best course of action under the circumstances. His honesty and integrity were recognized by all who came in contact with him, and his ability and trustworthiness as an engineer and public servant were amply demonstrated by the fact that although he was a lifelong Democrat, he was retained in office by Republican officials as readily as if he had belonged to that party.

In 1899, Mr. Hill was married to Flora Lewis, of Missoula, Mont., who was the daughter of a pioneer missionary to the Northwest Territories. He is survived by his wife and two children, William Moberly Hill, an engineering student in the University of Missouri, and Elizabeth Hill Gordon.

He was a member of the Presbyterian Church, the Beta Theta Pi Fraternity, the Kansas City Engineers' Club, and was a Thirty-second Degre Scottish Rite Mason.

Mr. Hill was elected an Associate Member of the American Society of Civil Engineers on October 2, 1901, and a Member on January 3, 1907.

CHARLES EZRA PERKINS, M. Am. Soc. C. E.*

DIED JULY 17, 1925.

Charles Ezra Perkins, the son of Simon and Grace Ingersoll (Tod) Perkins, was born at Akron, Ohio, on May 7, 1850. His grandfather, Simon Perkins, went to Ohio from Connecticut in 1800 and settled at Warren, Trumbull County, as the Representative of the Western Reserve Land Company. His maternal grandfather was Judge Tod, father of Governor David Tod, a well-known name among Ohio jurists fifty years ago.

Mr. Perkins was educated at the public schools of Akron and at the Western Reserve Academy, at Hudson, Ohio. He also attended Rensselaer Polytechnic Institute, at Troy, N. Y., from 1868 to 1870, where he took a special course in Civil and Mining Engineering. In 1871, he studied at the School of Mines, Columbia University, New York, N. Y.

On leaving the School of Mines, Mr. Perkins was appointed Assistant City Engineer of Akron and served in that capacity until he was appointed City Engineer in the summer of 1871, to which position he was afterward elected, serving as such until 1877. His work in this connection comprised all kinds of municipal engineering, particularly grading and paving. He was also engaged in the settlement of street improvement claims against the city in process of adjudication. During his term of office preliminary surveys were made for the establishment of sewerage and water systems for the city.

From 1878 to 1883, he was engaged in private practice, including ditch work, bridges, and road improvement, but the greater part of this time was spent in the agricultural implement business.

In addition to his private practice Mr. Perkins served as Surveyor and Engineer of Summit County, Ohio, from 1883 to 1892, in charge of all County

*Memoir prepared from information on file at the Headquarters of the Society.

engineering work, highway bridges, ditches, highway improvements, and coal mine surveys.

In 1892, he was appointed to the official position of Chief Engineer of Public Works of Ohio, an office created by statute in 1878. As Executive Officer of this great system of waterways, consisting of about 600 miles of canals, he served the State faithfully for sixteen years, during which time the canal properties under his control proved to be of inestimable value to the people, and a distinct and important asset to the State.

After his retirement from his official position in 1908, Mr. Perkins engaged in private practice as a Civil, Hydraulic and Mining Engineer, with an office in Akron. He was a Republican in politics and a party leader in Ohio.

He was married, in 1880, to May Adams, daughter of Mr. Frank Adams, a prominent manufacturer of Akron, who survives him.

Mr. Perkins was elected a Member of the American Society of Civil Engineers on June 6, 1906. He was also a member of the International Association of Navigation.

HENRY NORLANDE RUTTAN, M. Am. Soc. C. E.*

DIED OCTOBER 31, 1925.

Henry Norlande Ruttan was born on May 21, 1848, at Cobourg, Ont., Canada, where he received his early education. His parents were Henry Jones and Margaret (Pringle) Ruttan. His grandfather was the Hon. Henry Ruttan, formerly Speaker of the Legislative Assembly of Ontario.

Without the advantages of a college education or a technical training, Mr. Ruttan started while quite young in what became his life work and profession. Beginning in 1867 on the Engineers' Staff of the Grand Trunk Railway Company, he had a varied experience for sixteen years in railway work, as follows: 1869, on Engineers' Staff, Intercolonial Railway Company; from 1870 to 1874, on construction, Intercolonial Railway, and, later, in charge of a Division on Baie de Chaleur, including much heavy bridge work; in 1874, on preliminary surveys, Canadian Pacific Railway, north of Lake Superior; from 1875 to 1877, in charge of a party locating the line of the Canadian Pacific Railway from the summit of the Rocky Mountains, eastward; from 1877 to 1880, Engineer for the Contractor on the construction of the Canadian Pacific Railway between Lake of the Woods, Ontario, and Winnipeg, Man.; and from 1880 to 1885, designed and built bridges over the Red River at Emerson, Man., and over the Assiniboine at Winnipeg. As Engineer and Contractor Mr. Ruttan built the first forty miles of the Manitoba North Western Railway, and as Contractor, the first fifty miles of the Manitoba South Western Railway.

On June 29, 1885, Mr. Ruttan was appointed City Engineer of Winnipeg, and held this position for twenty-nine years, resigning in May, 1914. Among the outstanding features of his work as City Engineer was the design and construction of the Artesian Well System in 1899, when the City purchased

* Memoir prepared by J. G. Sullivan, M. Am. Soc. C. E., and William P. Brereton, City Engr., Winnipeg, Man., Canada.

the plant of the Winnipeg Water-Works Company which obtained its supply from the Assiniboine River. In 1898, he remodeled the asphalt plant which the City had purchased from contractors, which plant has supplied all the requirements of the City since that time for paving purposes.

In 1905, Mr. Ruttan was instrumental in the installation of a Fire Service Water-Works System with a separate distribution system. This plant is capable of pumping 18 000 000 gal. per 24 hours at 300-lb. pressure. He also designed and constructed the sewerage system of Winnipeg, a difficult task when the flatness of the territory is considered. In 1906, he made the original survey for the hydro-electric system of the City of Winnipeg and, later, acted as Consulting Engineer.

His success as an Engineer is a good example of what can be accomplished by hard work combined with a good mind. Practically all his spare time was given to reading and self-education.

In addition to his engineering achievements, he had an enviable military record, as follows: He enlisted in the Cobourg Rifle Company of the 40th Regiment, Canadian Militia, in 1863, and served with that unit for seven years. On November 9, 1883, he was appointed Captain in the 90th Regiment (Winnipeg Rifles), and was promoted to the rank of Major on February 14, 1887. Subsequently, on December 25, 1895, he was appointed Lieutenant-Colonel in command of the unit, which command he held until December 27, 1900.

From 1900 to 1910, Colonel Ruttan was a Reserve Officer, but in April, 1910, he was appointed Lieutenant-Colonel to command the 100th Regiment (Winnipeg Grenadiers), in which appointment he remained until October, 1912, when he was selected to command the 20th Infantry Brigade, with which he served until September 5, 1918. He was appointed Temporary Brigadier General on July 8, 1916, and assumed command of Military District No. 10 (in addition to his duties as Commander of the 20th Infantry Brigade), during the World War period. General Ruttan was one of the first officers in Military District No. 10 to pass the Militia Staff Course.

His war service record is, as follows: Fenian Raid, 1866, for which he received the General Service Medal with one clasp (90th Winnipeg Battalion); North West Rebellion, 1885, actions at Fish Creek (April 24), and Batoche (May 9 to 12), and operations against Chief Big Bear's Band (June-July), for which he received a medal with clasp. He was made a Companion of the Order of St. Michael and St. George for services in connection with the World War (1914-18).

General Ruttan's military training developed a manner that was somewhat awe-inspiring to the young clerk, but his sense of justice and kindness of heart was such that he was admired and respected by all who knew him. He never attempted to shift any responsibility; on the contrary, he often assumed full personal obligation for the mistakes of his subordinates.

As is the case with all public servants he had adversaries and many men were elected to the City Council at Winnipeg and even as Mayor as an opponent to General Ruttan only to become his supporter in a short time.

In many cases the good judgment he displayed in disputed questions became apparent only after a long period of time, such as his decision to use municipal forces only in the construction of improvements during the boom days of 1905 to 1913, which saved the City of Winnipeg from a condition of over-development reached by many other towns in the Prairie Provinces at that time. If 50% of the public officials had the same character and determination to do what was right as General Ruttan, the public operation of utilities, especially those with a low operating ratio, would not be such a bugbear to the ordinary taxpayer.

General Ruttan is survived by his widow, Mrs. Andrian Ruttan, *nee* Barberie, and the following children: Maj. H. A. Ruttan, of Port Arthur, Ont.; Lieut.-Col. C. M. Ruttan, F. Ruttan, and Capt. J. D. Ruttan, all of Winnipeg; and Mrs. H. L. Wethey, of Neepawa, Man., Canada.

General Ruttan was elected a Member of the American Society of Civil Engineers on January 4, 1893.

GRANVILLE WHEATON SHAW, M. Am. Soc. C. E.*

DIED JANUARY 6, 1925.

Granville Wheaton Shaw, the son of Theodore Granville and Matilda (Ward) Shaw, was born in Chillicothe, Ohio, on April 27, 1849. He moved to Louisville, Ky., with his parents in 1851, where he resided for the remainder of his life. His education was obtained in the public schools of Louisville with the exception of five years spent in private schools before he reached his fifteenth year.

In January, 1868, Mr. Shaw entered the service of the Louisville, Cincinnati and Lexington Railroad Company, then building a line between La Grange, Ky., and Cincinnati, Ohio, under the direction of the Chief Engineer, the late I. M. St. John, M. Am. Soc. C. E., and was engaged as Rodman on location surveys and as Assistant Engineer on the construction of shops, roundhouses, etc.

In May, 1869, he went with General St. John to the Louisville, New Albany and St. Louis Railroad (now a part of the Southern Railway System) as Leveler on preliminary surveys of the line from New Albany, Ind., to Mt. Carmel, Ill. In 1870, he was employed as Assistant Engineer on the construction of a bridge across Licking River, at South Covington, Ky., and in building a railroad from that point through Newport, Ky., to connect with the Ohio River Bridge. He was also employed as Draftsman on a Government survey of the Upper Cumberland River under the late S. T. Abert, M. Am. Soc. C. E., U. S. Engineer. In 1872, he was appointed Assistant Engineer on the Louisville, Cincinnati and Lexington Railroad under General St. John and was engaged in the reconstruction of bridges and surveys for railroad extensions.

From 1873 to 1875, Mr. Shaw served as Assistant City Engineer of Louisville, under the late Thomas P. Shanks, M. Am. Soc. C. E., in charge of sewer

* Memoir compiled from information on file at the Headquarters of the Society.

construction and the extension of a short-line railroad along the river front to East Louisville, and had supervision of all city bridges. He subsequently went to Vincennes, Ind., in the employ of Cunningham and Mettesser, manufacturers of glucose, and for one season commanded the steamer *Vigo*, a Wabash River packet running in the interests of the firm. He also designed a drainage system for the property of the Company and rebuilt its plant.

In 1878, Mr. Shaw returned to Louisville and was employed as Assistant Engineer on the preliminary survey of the Falls of Ohio River with Mr Thomas H. McMichael and, in 1879, was in charge of a survey and cross-sections of the Falls of the Ohio River under Gen. G. Weitzel, Corps of Engineers, U. S. A., completing the field work by the end of the year.

Early in 1880, he accepted the position of correspondent for the *Louisville Commercial* at Frankfort, Ky., during the session of the Kentucky Legislature. When the latter adjourned, he was appointed City Editor of the *Commercial* and remained with that newspaper until the fall of 1882, when he established a Bureau for the Testing of Hydraulic Cement at Louisville From 1883 to 1885, he was engaged in various surveys and works in connection with the Louisville and Portland Canal and the Falls of the Ohio River under the late Col. William E. Merrill, Corps of Engineers, U. S. A., M. Am. Soc. C. E.

In March, 1886, Mr. Shaw accepted the position of U. S. Assistant Engineer in charge of improving the Falls of the Ohio River, at Louisville, and the enlargement of the Louisville and Portland Canal, under Maj. Amos Stickney, Corps of Engineers, U. S. A. He also constructed the new river wall, began the work of enlarging the harbor in front of the city, and was engaged in excavating a channel through rock. He remained in this position until December 31, 1894.

In December, 1895, he accepted the position of Chief Clerk in the U. S. Engineer's Office at Columbus, Ohio, under the late Brig.-Gen. H. M. Chittenden, U. S. A., M. Am. Soc. C. E. (at that time, Lieutenant, Corps of Engineers, U. S. A.), and was engaged in survey work for a canal from Lake Erie to the Ohio River. This work was finished in February, 1896.

Mr. Shaw then went to Milwaukee, Wis., to test cement for the Government and, later, to Missouri to inspect timber. In January, 1897, he was re-appointed by Capt. J. G. Warren, Corps of Engineers, U. S. A. (now Colonel, U. S. A. (*Retired*)), M. Am. Soc. C. E., to a position in the U. S. Engineer Office at Louisville, and again placed in charge of the improvements on the Ohio River. He remained on this work until his retirement in 1912, on account of failing health.

On January 14, 1873, Mr. Shaw was married to Adeline Orpha Wolford, the daughter of Henry Wolford, for many years City Treasurer of Louisville. Mrs. Shaw died on January 1, 1926. A son, George Wolford Shaw, survives him.

Mr. Shaw's standards of conduct were high, especially when pecuniary transactions were involved. To whatever work he was assigned he gave close attention, and did his duty to the satisfaction of those above him. He was a member for several years of the Engineers and Architects Club at Louisville,

helping it financially and taking part occasionally in the discussions of technical papers.

After entering the service of the U. S. Engineer Department, Mr. Shaw became interested in the local politics of Louisville, for which his previous employment under the City Engineer and as a reporter for a newspaper had prepared him. This intimacy, however, was not carried so far as to be detrimental to his work under the Engineer Department.

His social life found an outlet in his fondness for vocal music, especially when rendered by male voices. These musical activities were begun as a young man in church choirs, and were continued in wider circles until he was well beyond fifty years of age. Beginning with a quartet of singers and with a fifth man as leader and pianist, he watched a small organization grow to become a noted institution in the musical life of Louisville. This assemblage of young men had various names—the Arion Club, the Apollo Club, the Louisville Male Chorus—and of all of them he was President for a time. He took charge of arrangements for public performances and many other matters not connected with actual training for concerts, serenades, etc.

Throughout his life, Mr. Shaw was a devoted member of the Protestant Episcopal Church.

Mr. Shaw was elected a Member of the American Society of Civil Engineers on October 5, 1887.